AN ADAPTIVE ALGORITHM FOR BROADBAND FREQUENCY INVARIANT BEAMFORMING

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ABSTRACT

Frequency invariant beamforming is array processing in which the spatial response remains constant (with respect to frequency) within a wide frequency band of interest. In this paper we present a new algorithm for adaptive broadband beamforming which solves a minimum variance beamforming problem, with a structural frequency invariant beampattern constraint. This constraint allows us to reduce the dimension of the adaptation problem. The proposed algorithm is a block adaptive LMS algorithm which uses only a fraction of the parameters of a conventional fully adaptive array. Hence, the computational complexity is reduced and the convergence speed is increased. A simulation example is presented to demonstrate the new algorithm.

1. INTRODUCTION

Adaptive array processing techniques which minimise the contributions of strong interferers from unknown directions while passing signals from a chosen look direction are important in applications such as radar, sonar, and communications systems.

One algorithm which solves this problem is the constrained LMS algorithm of Frost [1]. For an array of N sensors each feeding an L tap FIR filter, Frost's algorithm uses L free parameters to constrain the response in the look direction, and the remaining $(NL-L)$ parameters to minimise the output power and thereby the contributions of the interferers. Hence, Frost's algorithm is a two-dimensional algorithm (over space N and time L).

It is well known that the convergence rate of the LMS algorithm is related to the eigenvalue spread of the data covariance matrix [10]. However, it is becoming apparent that the convergence rate is also influenced by the number of free parameters [11]. In this paper we show that, in broadband signal environments in which the desired signal and the interference signals cover approximately the same bandwidth, the minimum variance beamforming problem can be reduced to a one-dimensional problem through use of a frequency invariant beamforming structure. By reducing the number of adaptive parameters the computational complexity and convergence time are reduced. This is verified by a numerical simulation.

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2. FREQUENCY INVARIANT BEAMFORMING

A frequency invariant beamformer (FIB) is an array processing structure in which the resulting spatial response is constant (with respect to frequency) within a wide frequency band. Several methods of designing such a beamformer have been proposed, e.g. [2, 4–7]. We will use the method of [4], since this FIB is parameterised by a single vector of beam shaping coefficients which define the frequency invariant beampattern over the entire frequency band [8]. This property is demonstrated later, and makes this particular FIB well suited to adaptive implementation.

Consider a linear array of N sensors with a filter on each sensor. The filter outputs are summed and filtered to give the following spatial response for a farfield signal arriving from a direction θ (measured relative to broadside)

$$r(\theta, f) = H_s(f) \sum_{n=1}^{N} g_n \, H_n(f) \, e^{j2\pi f \tau_n(\theta)}, \tag{1}$$

where $\tau_n(\theta)$ is the relative propagation delay to the nth sensor, $H_n(f)$ is the *primary filter* on the nth sensor, $H_s(f)$ is the *secondary filter*, and g_n is a *spatial weighting* term to account for possibly nonuniform sensor spacings. The number and positions of the sensors should be chosen with regards to the specific bandwidth required; see [4] for details. We now indicate explicit constraints on $H_n(f)$ and $H_s(f)$ to achieve a spatial response which is frequency invariant within a wide bandwidth of interest.

The general idea of the FIB is that the active array size and shape should be kept constant in terms of operating wavelength to maintain a beampattern which is the same at all frequencies within the bandwidth of interest. The primary filters perform the role of maintaining a scale invariance property on the active aperture, whereas the secondary filter normalises the peak array response. The secondary filter response is $H_s(f) = \alpha f$, where α is a constant.

The primary filters must satisfy a dilation property [4] and, to ensure the active array size remains constant in terms of wavelength, the nth primary filter should be bandlimited to $f_n = Pc/x_n$, where c is the speed of wave propagation and P is the active array size (measured in wavelengths). Equivalently, the input to the nth primary filter is bandlimited to f_n (to avoid aliasing), and the nth primary filter response can be expressed as [8]

$$H_n(f) = \sum_l h[l] e^{-j2\pi f T_n l} = \mathbf{h}^H \psi_n(f), \tag{2}$$

where T_n is the sampling period of the nth sensor, $\mathbf{h}$ is an L vector of beam shaping coefficients, and $\psi_n(f)$ is an L dimensional dilation vector. If the beam shaping coefficients $\mathbf{h}$ produce some desired primary filter response $H_{\text{ref}}(f)$ at some location x_{ref} with a sampling period T, then $T_n = T x_n / x_{\text{ref}}$. This formulation lends itself to implementation using multirate filtering.

Let $\beta(f) = H_s(f)[g_1\psi_1(f), \ldots, g_N\psi_N(f)]$ be an $L \times N$ matrix. The spatial response (1) can now be expressed as

$$\begin{aligned} r(\theta, f) &= \mathbf{h}^H \beta(f) \mathbf{a}(\theta, f) \\ &\approx r_{\text{FI}}(\theta),\ \forall f \in [f_L, f_U], \end{aligned} \tag{3}$$

where $r_{\text{FI}}(\theta)$ is a frequency invariant response, $[f_L, f_U]$ is the bandwidth of interest, and

$$\mathbf{a}(\theta, f) = [e^{j2\pi f \tau_1(\theta)}, \ldots, e^{j2\pi f \tau_N(\theta)}]^T$$

is the array direction vector. Note that (3) holds for any set of beam shaping coefficients $\mathbf{h}$.

Because the beam shaping coefficients $\mathbf{h}$ are independent of frequency, we have the following frequency invariance property

$$\beta(f)\mathbf{a}(\theta, f) \approx \beta(f_0)\mathbf{a}(\theta, f_0), \quad \forall \theta, \forall f \in [f_L, f_U], \tag{4}$$

where f_0 is a nominal frequency in the bandwidth of interest (which we take as the centre frequency for convenience).

The importance of this formulation is that there is a single set of coefficients which defines the spatial response over the entire bandwidth of interest. Thus, if the coefficients $\mathbf{h}$ are the parameters of an adaptive beamforming algorithm, the resulting beampattern is constrained to be frequency invariant at all steps in the adaptation process. Moreover, the number of coefficients is independent of the number of sensors. This is the basis of the algorithm presented in this paper.

3. OPTIMUM BEAMFORMER

The goal of adaptive beamforming is to preserve a desired signal (usually from a chosen look direction) while minimising the contributions from interfering sources. This is often achieved by minimising the beamformer output power while maintaining a chosen frequency response in the look direction. In this section we derive the optimum set of beam shaping weights, based on ideal knowledge of the second order statistics of the received array data.

Consider D wideband farfield source signals impinging on a linear array of N sensors from directions $\Theta = [\theta_1, \ldots, \theta_D]$. Assume that one of these signals is a desired signal which arrives from the look direction θ_1, and the remaining $(D-1)$ signals are treated as interference which should be rejected by the beamformer. We assume the look direction is known exactly, otherwise derivative constraints [9] could be included to provide robustness for look direction mismatch.

The received array data is divided into K blocks of M samples,[1] and an M point discrete Fourier transform is applied to each block. We assume that this produces J narrowband frequency bins in the bandwidth of interest, $f_i \in [f_L, f_U], i = 1, \ldots, J$. For the kth block, the array data in the ith frequency bin is

$$\mathbf{y}(k; f_i) = \mathbf{A}(\Theta, f_i)\,\mathbf{s}(k; f_i) + \mathbf{n}(k; f_i), \tag{5}$$

[1] For non-stationary sources, there is a fundamental trade-off between the block size and the tracking capability of the beamforming algorithm.

where $k = 0, \ldots, K-1$, and $i = 1, \ldots, J$. The $N \times D$ source direction matrix is

$$\mathbf{A}(\Theta, f_i) = [\mathbf{a}(\theta_1, f_i), \ldots, \mathbf{a}(\theta_D, f_i)],$$

$\mathbf{s}(k; f_i)$ is the D vector of source signals, and $\mathbf{n}(k; f_i)$ is the N vector of additive sensor noise (assumed to be uncorrelated with the source signals). The data covariance matrix is

$$\begin{aligned} \mathbf{R}_y(f_i) &= E\{\mathbf{y}(k; f_i)\mathbf{y}^H(k; f_i)\} \\ &= \mathbf{A}(\Theta, f_i)\mathbf{R}_s(f_i)\mathbf{A}^H(\Theta, f_i) + \mathbf{R}_n(f_i), \end{aligned} \tag{6}$$

where $\mathbf{R}_s(f_i)$ is the source covariance matrix, and $\mathbf{R}_n(f_i)$ is the noise covariance matrix.

Assume we apply a FIB to the received array data (5), giving a beamformer output

$$z(k; f_i) = \mathbf{h}^H\,\beta(f_i)\,\mathbf{y}(k; f_i). \tag{7}$$

The expected beamformer output power in the ith frequency bin is therefore

$$E\{|z(k; f_i)|^2\} = \mathbf{h}^H \beta(f_i)\mathbf{R}_y(f_i)\beta^H(f_i)\mathbf{h},$$

and the average expected output power is

$$E\{|z(t)|^2\} = \mathbf{h}^H \overline{\mathbf{R}}\mathbf{h}, \tag{8}$$

where

$$\begin{aligned} \overline{\mathbf{R}} &= \frac{1}{J}\sum_{i=1}^{J} \beta(f_i)\mathbf{R}_y(f_i)\beta^H(f_i) \\ &= \frac{1}{J}\sum_{i=1}^{J} \beta(f_i)\mathbf{A}(\Theta, f_i)\mathbf{R}_s(f_i)\mathbf{A}^H(\Theta, f_i)\beta^H(f_i) \\ &\qquad + \beta(f_i)\mathbf{R}_n(f_i)\beta^H(f_i). \end{aligned} \tag{9}$$

Because of the frequency invariance property (4), this can be written

$$\overline{\mathbf{R}} \approx \beta(f_0)\mathbf{A}(\Theta, f_0)\overline{\mathbf{R}}_s\mathbf{A}^H(\Theta, f_0)\beta^H(f_0) + \overline{\mathbf{R}}_n, \tag{10}$$

where

$$\overline{\mathbf{R}}_s = \frac{1}{J}\sum_{i=1}^{J} \mathbf{R}_s(f_i)$$

is the frequency averaged source covariance matrix, and

$$\overline{\mathbf{R}}_n = \frac{1}{J}\sum_{i=1}^{J} \beta(f_i)\mathbf{R}_n(f_i)\beta^H(f_i)$$

is the frequency averaged noise covariance matrix. Hence, the data covariance matrices may be averaged across the frequency band of interest while preserving the source direction information.

The optimum beam shaping weights are found by minimising the output power while preserving the desired signal from the look direction. This is formulated as the following constrained minimisation problem

$$\min_{\mathbf{h}} \mathbf{h}^H \overline{\mathbf{R}}\mathbf{h} \tag{11a}$$

$$\text{subject to } \mathbf{C}^H\mathbf{h} = 1, \tag{11b}$$

where $\mathbf{C} = \beta(f_0)\mathbf{a}(\theta_1, f_0)$ is a constraint vector which maintains a unity response in the look direction. Note that because the beamformer has a frequency invariant spatial response, a broadband unity response is imposed in the look direction by imposing a constraint at a single frequency.[2]

The solution to the well-known constrained minimisation problem (11) is

$$\mathbf{h} = \frac{\overline{\mathbf{R}}^{-1}\mathbf{C}}{\mathbf{C}^H\overline{\mathbf{R}}^{-1}\mathbf{C}}. \tag{12}$$

4. ADAPTIVE ALGORITHM

Having derived the optimum beam shaping coefficients in the previous section, we now describe an algorithm which converges to these coefficients in an environment in which no *a priori* knowledge of the locations of interfering signals is available.

The constrained optimisation problem (11) is identical to that considered by Frost [1]. Frost developed an LMS algorithm to minimise the output power of a broadband array while maintaining a chosen frequency characteristic in the look direction. We may thus apply Frost's algorithm to the adaptive frequency invariant beamforming problem.

The proposed algorithm is summarised as

$$\mathbf{h}_0 = \mathbf{q} \tag{13a}$$

$$\mathbf{h}_{k+1} = \mathbf{Q}\left[\mathbf{h}_k - \mu\widehat{\mathbf{R}}_k\mathbf{h}_k\right] + \mathbf{q}, \tag{13b}$$

where $\mathbf{h}_k$ is the set of beam shaping coefficients to use for the kth data block, μ is the adaptation step size,

$$\mathbf{q} = \mathbf{C}[\mathbf{C}^H\mathbf{C}]^{-1}$$

is an L vector,

$$\mathbf{Q} = \mathbf{I} - \mathbf{C}[\mathbf{C}^H\mathbf{C}]^{-1}\mathbf{C}^H$$

is an $L \times L$ projection matrix, and

$$\widehat{\mathbf{R}}_k = \frac{1}{J}\sum_{i=1}^{J}\beta(f_i)\mathbf{y}(k; f_i)\mathbf{y}^H(k; f_i)\beta^H(f_i)$$

is an $L \times L$ matrix used to estimate $\overline{\mathbf{R}}$ for the kth block of data.

We will refer to this as the FIB algorithm.

5. EXAMPLE

We now present a design example to evaluate the performance of the FIB algorithm compared with the conventional Frost algorithm.

The design was for a linear array of $N = 12$ sensors with an inter-sensor spacing of $\lambda_U/2$ (where λ_U is the wavelength corresponding to the maximum frequency of interest) with $L = 8$ for both the Frost and FIB algorithms (hence there are 96 parameters for the Frost beamformer). The FIB was designed to have a frequency invariant response over the normalised bandwidth of $[0.35, 0.45]$. The signal environment consisted of two plane wave sources, with flat frequency spectra over the bandwidth of interest.

[2] For strong signals from the look direction a single frequency constraint may be insufficient to maintain a unity response in the look direction. The reason for this is beyond the scope of this paper, and will be treated in a future paper [3].

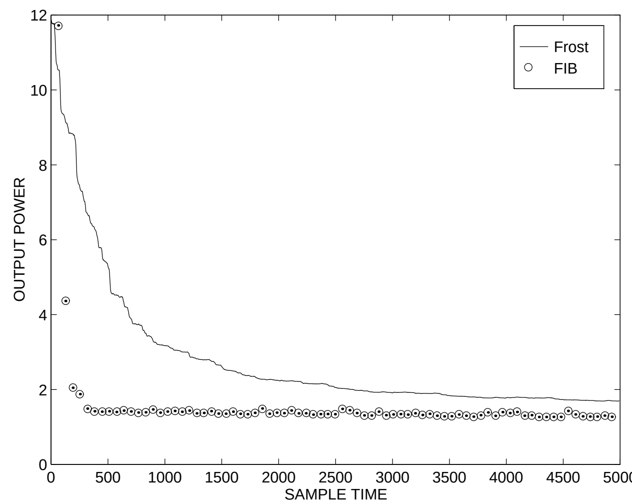


Figure 1: Comparison of output power convergence for example FIB and Frost algorithms (see text for description).

The desired signal was at $0°$ with an SNR of 0 dB, and an uncorrelated interferer with SNR of 30 dB was present at $30°$. White Gaussian noise was modelled at the input of each sensor. For the FIB algorithm, the received data was partitioned into blocks of 64 samples and discrete Fourier transformed to produce 6 frequency bins within the design band. The adaptation step size was $\mu = 1 \times 10^{-7}$ for both algorithms.

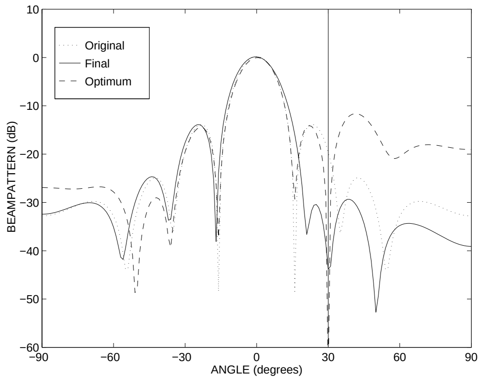
Figure 1 shows a comparison of the convergence rate for both algorithms. Note that the FIB algorithm is shown at discrete sample times, corresponding to the end of each block of processed data.

Figure 2(a) shows the beampatterns produced by the FIB algorithm initially (dotted), after 5000 data samples (solid), and the optimum beampattern (dashed). The final adapted beampatterns (after 5000 sample periods) are shown in Fig. 2(b) at 25 frequencies within the design band. The corresponding results for the Frost algorithm are shown in Figures 3(a) and (b). Observe that the FIB beampatterns exhibit little variation with frequency.

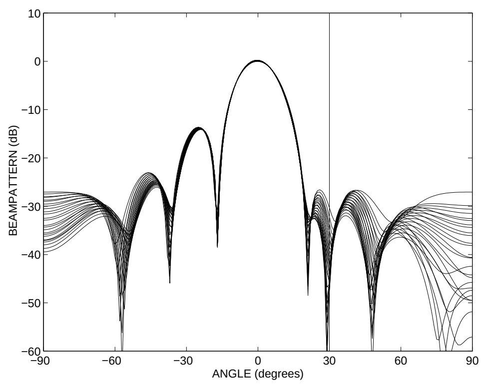
As a simple quantitative test of the relative complexity of the two algorithms, we counted the number of floating point operations required for the 5000 sample simulation shown in Fig. 1. It was found that the FIB algorithm required less than 1% of the flops required by the Frost algorithm.

6. CONCLUSIONS

In this paper we have presented a new algorithm for adaptive broadband beamforming. This algorithm solves a minimum variance beamforming problem, but with a structural constraint that ensures that the beampattern is frequency invariant at each step in the adaptation process. Simulation results indicate that the proposed algorithm has significant speed of convergence advantages over conventional broadband beamforming methods. A theoretical analysis of these quantities is currently underway and will be presented in a future paper [3].

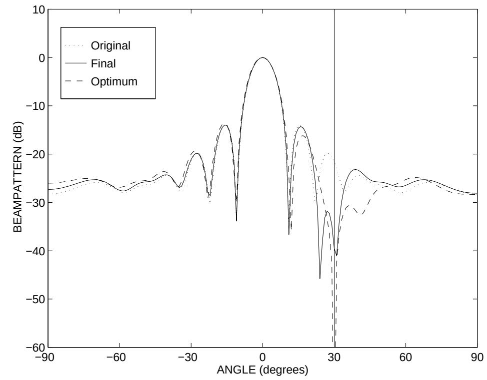


(a) Frequency averaged beampatterns.

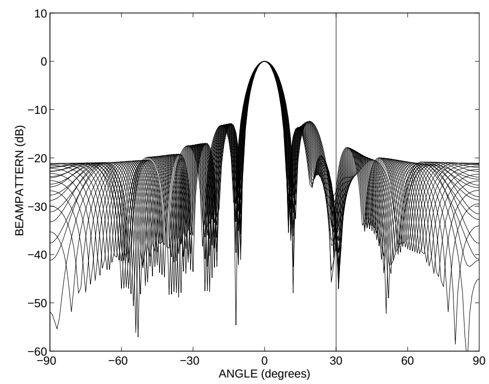


(b) Final beampatterns at 25 frequencies within the design frequency band.

Figure 2: Example of FIB algorithm (see text for description).



(a) Frequency averaged beampatterns.



(b) Final beampatterns at 25 frequencies within the design frequency band.

Figure 3: Example of Frost algorithm (see text for description).

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