

SENSOR ARRAY CALIBRATION USING MEASURED STEERING VECTORS OF UNCERTAIN LOCATION

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ABSTRACT

We present a maximum likelihood approach for calibrating sensor arrays in the presence of mutual coupling, channels gain and phase mismatch and array geometry uncertainties using measured steering vectors of uncertain locations. The estimated perturbation parameters is used to calibrate the array manifold, hence enabling many high resolution array processing algorithms to attain their potential advantages. We present two methods for optimizing the highly nonlinear and multimodal ML cost function. The first method is linearized local gradient search algorithm. The second method is derived from combining the fast local search of gradient methods with the nonlinear global search ability of Genetic algorithm. The resulting hybrid optimizer is both fast and globally converging. Simulation results are presented to illustrate the usefulness of the proposed approach.

1.. INTRODUCTION

To enable most high resolution algorithms (e.g.[1] and references therein) to reach their potential usefulness, array calibration is necessary to compensate the impreciseness of the array manifold. In this paper, we address the problem of calibrating array perturbations that results from array geometry uncertainties, mismatch in the channel gain and phase mismatch, and mutual coupling. Typically these are the main sources of array perturbations that can seriously degrade most high resolution algorithms performance.

Several researchers have addressed this problem and proposed parametric methods to calibrate the array manifold (e.g. [6][4][5]). Unfortunately most methods have limited applicability as they are formulated to deal with a few but not all of the array perturbations considered herein. Robust estimators based on Bayesian approach has been proposed recently and can be applied to very general array perturbation models[3]. However these methods require *a priori* knowledge of the perturbation parameters statistics. This may limit their potential usefulness. Moreover in the likely case of unidentifiable perturbation parameters, the limiting Cramer-Rao lower bound of the perturbation parameters are non-zero and increase with the variance of the perturbation parameters[2].

Recently another class of calibration algorithms was proposed. Array calibration is achieved by modeling the array perturbations as deterministic parameters and estimating them from a set of measured steering vectors. In the papers [7][8], methods for estimating the combined array

perturbations from mutual coupling and channel mismatch were reported. See and Ng in [10][9][11] extend these methods include the case of uncertainties in the array geometry. The approach of using measured steering vectors appears reasonable in many situations. In general, relatively small number of calibration sources are needed and the array calibration model is quite general and with physical justifications[13]. Good calibration accuracies are therefore expected.

So far these calibration methods hinge on the assumption of having precise knowledge of the calibration source locations. In this paper, we address a more general case where the calibration sources location are not known exactly. This generalization offers the flexibility to alleviate the need for accurate DOA of calibration sources, particularly in situations where such information are not readily available and remove inherent calibration inaccuracies from such errors.

2.. DATA MODEL AND PROBLEM FORMULATION

The observation vector at the array output in the presence of mutual coupling and array channel mismatch can be written as[13]

$$\mathbf{x}(t) = \sum_{i=1}^n \mathbf{C}\mathbf{a}(\theta_i, \Psi) s_i(t) + \mathbf{n}(t) \quad (1)$$

with $\mathbf{C} = \mathbf{M}\Delta$ termed as *Calibration Matrix*. $\mathbf{M} \in \mathbb{C}^{m \times m}$ describes the self- and mutual- coupling of the sensors and Δ is a complex $m \times m$ diagonal matrix depicting the effects from array channels mismatch.

In this paper, we assume the calibration signals to be continuous wave (CW) sources and the signal from each calibration sources are spatially and temporally disjoint. The steering vector measured using L independent snapshots from the i^{th} calibration source located at θ_i can be estimated using

$$\mathbf{a}_m(\theta_i) = \frac{1}{N} \sum_{l=1}^L \mathbf{r}(t). \quad (2)$$

The observation noise $\mathbf{n}(t)$ is assumed to be temporally and spatially uncorrelated zero mean Gaussian process of covariance $\sigma_n^2 \mathbf{I}_m$, we have

$$\mathbf{a}_m(\theta_i) = \rho_i \mathbf{C}\mathbf{a}(\theta_i, \Psi) + \mathbf{w} \quad (3)$$

where ρ_i is an unknown complex scaling factor and $\mathbf{w} = \frac{1}{L} \sum_{i=1}^L \mathbf{n}(t)$ is a zero mean Gaussian variable and has covariance $\frac{\sigma_n^2}{L} \mathbf{I}_m = \sigma_w^2 \mathbf{I}_m$.

Given measurements from n temporally and spatially disjoint calibration sources, $\{\mathbf{a}_m(\theta_i)\}, i = 1 \cdots n$, we have $\mathbf{A}_m(\theta) \sim CN(\mathbf{CA}(\theta, \Psi) \mathbf{\Lambda}, \sigma_w^2 \mathbf{I}_m)$. We can cast the sensor array calibration problem as the (maximum likelihood) identification of the unknown deterministic parameters by optimizing

$$\{\hat{\Psi}, \hat{\theta}, \hat{\mathbf{\Lambda}}, \hat{\mathbf{C}}\} = \arg \min_{\{\Psi, \theta, \mathbf{\Lambda}, \mathbf{C}\}} \|\mathbf{A}_m(\theta) - \mathbf{CA}(\theta, \Psi) \mathbf{\Lambda}\|_F^2. \quad (4)$$

where $\mathbf{A}_m(\theta) = [\mathbf{a}_m(\theta_1), \mathbf{a}_m(\theta_2), \dots, \mathbf{a}_m(\theta_n)]$ and $\mathbf{A}(\theta, \Psi) = [\mathbf{a}(\theta_1, \Psi), \mathbf{a}(\theta_2, \Psi), \dots, \mathbf{a}(\theta_n, \Psi)]$.

It is important to examine the identifiability of (4). We note the parameters $\mathbf{\Lambda}$ and $\mathbf{C}$ are identifiable upto a complex scaling constant. This indeterminacy is not important. In general a scaled version $\mathbf{C}$ and $\mathbf{\Lambda}$ will not affect the applications of the sensor array signal processing, *e.g.* direction finding, beamforming and to some extent signal copy.

From (4) it suffices to note θ and Ψ are not simultaneously identifiable since $\mathbf{A}(\theta, \Psi) = \mathbf{A}(\theta^\ddagger, \Psi^\ddagger)$ where $\{\theta_i = \theta_i^\ddagger + \delta\}_{i=1}^n, \delta \in (-\pi, \pi]$ and, Ψ is a rotated version of $\Psi^\ddagger$. This rotational ambiguity can be resolved by *a priori* directional information such as the DOA of one of the calibration sources or the direction from one sensor to another [12]. In this paper, we assume *a priori* knowledge of the DOA of one of the calibration sources. However, this will not limit the proposed calibration methods to this case. If other directional references are available, calibration can still be achieved by first estimating the sensor positions and calibration sources DOA (upto a rotation ambiguity). Directional reference will then be used to remove the rotational ambiguity.

A more complex issue of parameter identifiability is if among these competing parameters $\mathbf{C}^\ddagger \neq \frac{1}{\mu} \mathbf{C}, \mathbf{\Lambda}^\ddagger \neq \mu \mathbf{\Lambda}, \theta^\ddagger \neq \theta$ and/or $\Psi^\ddagger \neq \Psi$ there exist $\mathbf{CA}(\theta, \Psi) \mathbf{\Lambda} = \mathbf{C}^\ddagger \mathbf{A}(\theta^\ddagger, \Psi^\ddagger) \mathbf{\Lambda}^\ddagger$ where μ is some arbitrary complex constant. Condition for their existence are difficult to obtained as it involves analysing highly nonlinear and complex relationships among $\Psi, \theta, \mathbf{\Lambda}, \mathbf{C}$. However from our extensive computer simulations based on the maximum likelihood methods proposed in next section, we did not find any case of such non-identifiability. In this paper we assume the data model to be parameter identifiable.

The problem statement in this paper can be expressed as follows. Given sufficient number of unique measured steering vectors and θ_1 to be known, array calibration problem can be formulated as the maximum likelihood identification of the system parameters $\Psi, \theta, \mathbf{\Lambda}, \mathbf{C}$

$$\{\Psi, \check{\theta}, \check{\mathbf{\Lambda}}, \check{\mathbf{C}}\} = \arg \min_{\{\Psi, \theta, \mathbf{\Lambda}, \mathbf{C}\}} \|\mathbf{A}_m(\theta) - \mathbf{CA}(\theta, \Psi) \mathbf{\Lambda}\|_F^2 \quad (5)$$

with

$$\check{\theta} = [\theta_2, \theta_3, \dots, \theta_n], \quad \mathbf{\Lambda} = \begin{bmatrix} \mu & \mathbf{0} \\ \mathbf{0} & \check{\mathbf{\Lambda}} \end{bmatrix}. \quad (6)$$

Note if μ is known exactly, then $\mathbf{C}$ and $\mathbf{\Lambda}$ can be estimated exactly, otherwise upto a complex non-zero scaling constant. Following the tenet of number of equations versus number of independent observations, the array calibration problem considered herein is ill-posed unless the number of equation bound $n \geq \lceil \frac{2m^2 + 2m - 5}{2m + 1} \rceil$ where $\lceil \kappa \rceil$ is the smallest integer larger than κ . This condition is necessary for the existence of Cramer-Rao lower bound.

3.. MAXIMUM LIKELIHOOD METHODS

Direct minimization of multi-dimensional nonlinear (5) entail searching a $2m^2 + 2m + 3n - 5$ parameter space. This is computationally prohibitive even for small number of sensors. A parsimonous parameter dimensionality can be obtained by *concentrating* $\mathbf{C}$ analytically with its maximum likelihood estimates

$$\hat{\mathbf{C}}_{ML} = \mathbf{A}_m(\theta) \hat{\mathbf{A}}^H (\hat{\mathbf{A}} \hat{\mathbf{A}}^H)^{-1} \quad (7)$$

where $\hat{\mathbf{A}} = \mathbf{A}(\theta, \Psi) \mathbf{\Lambda}$. The concentrated ML cost function can be expressed as

$$\hat{\xi}_{ML} = \arg_{\xi} \min J \quad (8)$$

where $J = \text{Tr}(\mathbf{P}_{\hat{\mathbf{A}}}^\perp \mathbf{Q})$, $\mathbf{Q} = \mathbf{A}_m(\theta)^H \mathbf{A}_m(\theta)$ and $\mathbf{P}_{\hat{\mathbf{A}}}^\perp = \mathbf{I}_n - \hat{\mathbf{A}}^H (\hat{\mathbf{A}} \hat{\mathbf{A}}^H)^{-1} \hat{\mathbf{A}}$. The vector parameters ξ is defined by $\xi = [\rho_R^T, \rho_I^T, \Psi^T, \check{\theta}^T]^T$ where $\rho_R = \text{Re}(\text{vecd}(\hat{\mathbf{A}}))$ and $\rho_I = \text{Imag}(\text{vecd}(\hat{\mathbf{A}}))$. Next we present two methods for optimizing (8).

3.1.. Modified Gauss-Newton Search Method

The modified Gauss-Newton performs an iterative linearized search ξ by

$$\hat{\xi}_{ML}^{i+1} = \hat{\xi}_{ML}^i - \nu_i \mathbf{H}^{-1}(\hat{\xi}_{ML}^i) \mathbf{V}(\hat{\xi}_{ML}^i). \quad (9)$$

$\mathbf{H}(\xi)$ and $\mathbf{V}(\xi)$ are the Hessian matrix and gradient vector, respectively. The step size $\nu = 0.5^k$ is set by where choosing k to be the smallest non-negative integer that minimizes $J^{i+1} - J^i$ to ensure J^i is monotonically decreasing and converge asymptotically to a local/global extrema.

An approximate Hessian matrix and gradient vector can be expressed compactly [14]

$$\mathbf{H}(\xi) \approx -2\text{Re}\left\{\mathbf{B}^T \left(\Xi \mathbf{P}_{\hat{\mathbf{A}}}^\perp \Xi^H \odot \Pi \mathbf{E}^T \Pi^H\right) \mathbf{B}\right\} \quad (10)$$

$$\mathbf{V}(\xi) = -2\text{Re}\left\{\text{vecd}(\Omega_1 \mathbf{W} \Omega_2^T)\right\} \quad (11)$$

respectively, where

$$\mathbf{B} = \begin{bmatrix} \mathbf{I}_2 \otimes \check{\mathbf{I}}_n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 \otimes \check{\mathbf{I}}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \check{\mathbf{I}}_n \end{bmatrix}$$

$$\Xi = [\mathbf{I}_n, -j\mathbf{I}_n, \mathbf{\Lambda}^H \mathbf{A}_x^H, \mathbf{\Lambda}^H \mathbf{A}_y^H, \mathbf{\Lambda}^H]^H$$

$$\Pi = [\mathbf{1}_2^T \otimes \mathbf{A}(\theta, \Psi), \mathbf{1}_2^T \otimes \mathbf{I}_n, \mathbf{A}_\theta]^T$$

$$\mathbf{E} = \check{\mathbf{A}}^\dagger \mathbf{Q} \check{\mathbf{A}}^{\dagger H}, \quad \mathbf{W} = \mathbf{P}_{\hat{\mathbf{A}}}^\perp \mathbf{Q} \hat{\mathbf{A}}^{\dagger H}$$

$$\Omega_1 = [\check{\mathbf{I}}_n, j\check{\mathbf{I}}_n, \mathbf{A}_x^T \check{\mathbf{I}}_m, \mathbf{A}_y^T \check{\mathbf{I}}_m, \mathbf{\Lambda} \check{\mathbf{I}}_n]^T$$

$$\begin{aligned}
\Omega_2 &= [\mathbf{1}_2^T \otimes \mathbf{A}(\theta, \Psi) \hat{\mathbf{I}}_n, \mathbf{1}_2^T \otimes \hat{\mathbf{I}}_m, \hat{\mathbf{I}}_n]^T \\
\mathbf{A}_X &= j2\pi \mathbf{A}(\theta, \Psi) \mathbf{A}_{\sin}, \quad \mathbf{A}_Y = j2\pi \mathbf{A}(\theta, \Psi) \mathbf{A}_{\cos} \\
\mathbf{A}_\theta &= j2\pi \left[\frac{\partial \mathbf{a}(\theta_1, \Psi)}{\partial \theta_1} \dots \frac{\partial \mathbf{a}(\theta_n, \Psi)}{\partial \theta_n} \right] \quad (12)
\end{aligned}$$

and $\mathbf{A}_{\sin} = \text{diag}([\sin \theta_1, \sin \theta_2, \dots, \sin \theta_n])^T$, $\mathbf{A}_{\cos} = \text{diag}([\cos \theta_1, \cos \theta_2, \dots, \cos \theta_n])^T$.

3.2.. Modified Gauss-Newton Genetic Algorithms Search Method (MGNGA)

MGN exhibits fast local convergence. However it has small region of global convergence and failed for initial estimates with *large* deviations. This is not surprising as the cost function is expected to be highly nonlinear and multimodal and linearized search such as MGN is likely to fail.

Nonlinear search techniques such as Genetic Algorithms (GA) appears to be a natural choice. It is well known that GA is asymptotically global convergent. However, it performs poorly in local search from the abrupt parent-child transitions. Also GA does not exploits the differentiability of (8). In this paper, we present a hybrid optimizer that exploits the fast local search capability of MGN and the nonlinear global search properties of GA. Based on the terminology in [15], the procedure outlined below amalgamate the fast locally converging property of MGN method with GA's global search capability to derive a fast and *almost* globally converging search procedure:

1. Supply a population P_o of N individuals
2. $i \leftarrow 1$
3. evaluate elements of P_i using GN search
4. $P'_i \leftarrow \text{Selection_Function}(P_i)$
5. $P'_i \leftarrow \text{Reproduction_Function}(P'_i)$
6. $i \leftarrow i + 1$
7. repeat step 3 until termination

At this time of writing, it came to our attention that a similar approach of combining gradient search and GA algorithm was reported in [16]

4.. SIMULATION EXPERIMENTS

We consider a nominal 7 element uniform circular array of $\frac{\lambda}{2}$ inter-sensor spacing. 13 calibration sources of equal strength are located over the field of view $[-\pi, \pi)$. The average sensor position error is 40% of $\frac{\lambda}{2}$ per sensor and $\mathbf{C}$ is a randomly generated full-rank matrix. The standard deviation of calibration source location error is 3° . In MGNGA, uniform mutation and arithmetic crossover are used and the selection function is based on roulette wheel.

Fig 2 shows the MUSIC spectrums for 3 uncorrelated sources at -10° , 10° and 30° . Note MGNGA calibrated array achieves accurate localization while the uncalibrated and MGN calibrated array fail to localize the 3 sources. The failure of MGN method is due to local convergence.

Fig 3 compares the convergence rate of the MGN, MGNGA and the Genetic Algorithms. It can be clearly seen that only MGNGA converge globally and quickly to the global minimum. While having the fast convergence rate, MGN fails to converge to the global solution. The

graph also shows the slow convergence of the Genetic Algorithms. Although not explicitly shown here, GA converges to the global solution asymptotically.

5.. CONCLUSIONS

In this paper we present a maximum likelihood approach for calibrating sensor array modeling due to unknown sensor location, mutual coupling and channel mismatch using measured steering vectors of uncertain location. We formulate the calibration problem as a deterministic parameter estimation problem in a maximum likelihood (ML) framework. We present two optimizer to deal with the highly nonlinear and multimodal ML cost function. Specifically we introduce a hybrid of MGN and Genetic Algorithms to solve this problem to deal with large deviations in the initial estimates. This approach provides an efficient, fast and globally converging search algorithm. Simulation results show the global convergence capability of the proposed optimizer. They also show the usefulness and effectiveness of the calibration approach in realising the potential of high resolution array processing algorithms.

6.. REFERENCES

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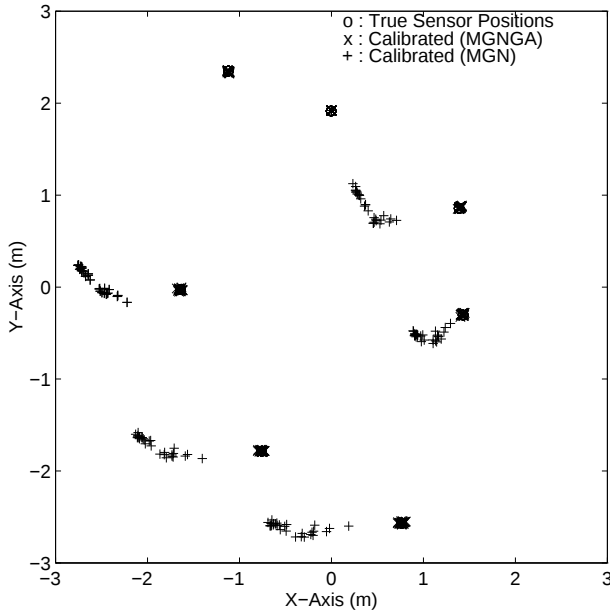


Figure 1. Calibrated Sensor Positions using MGNGA and MGN Algorithms

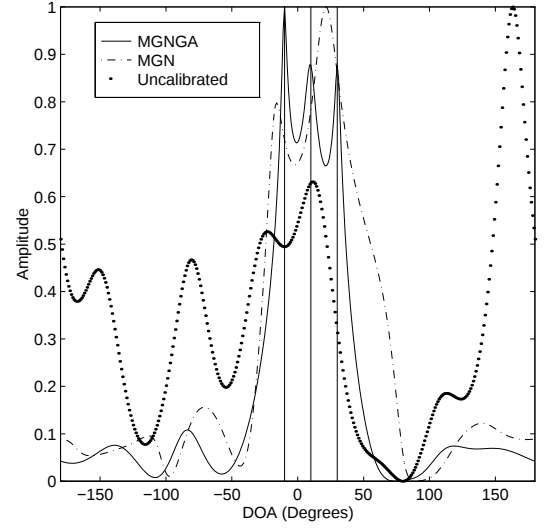


Figure 2. MUSIC Spectrums, 13 Calibration Sources, 400 Snapshots, SNR=10dB

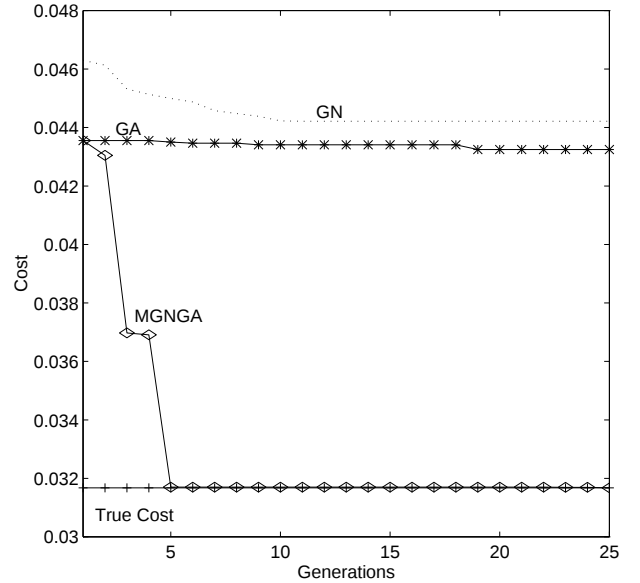


Figure 3. Typical Cost Function Trajectories. 400 Snapshots, SNR=10dB