

# STOCHASTIC CONSTRAINTS IN NONSTATIONARY HOT CLUTTER CANCELLATION

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## ABSTRACT

This paper considers the use of spatio-temporal adaptive processing (STAP) in OTHR and SAR applications to remove nonstationary multipath interference, known as “hot clutter”. Since the spatio-temporal properties of hot clutter cannot be assumed constant over the coherent processing interval, conventional adaptive routines fail to provide effective hot clutter mitigation without degrading sub-clutter visibility for backscattered radar signals. The approach presented incorporates multiple stochastic constraints, previously investigated for spatial-only adaptive processing, to achieve effective elimination of hot clutter with distortionless coherent processing for backscattered radar signals.

## 1. STOCHASTIC CONSTRAINT STAP ALGORITHM

The stochastic constraints (SC) approach, previously introduced for spatial-only adaptive processing in [1],[2],[3] and [4] is generalised in this paper for the case of STAP. Specifically, let  $x_t(n)$  be an  $N$ -variate “snapshot” vector, registered at the output of an  $N$ -element antenna array at the  $n^{\text{th}}$  range gate in the  $t^{\text{th}}$  repetition period. We assume that this vector is a mixture of multimode hot clutter signals  $c_t(n)$ , “ordinary” backscattered clutter  $y_t(n)$ , white noise  $\eta_t(n)$  and possibly useful signal  $s_t(n)$  arriving from direction  $\theta_0$ . In accordance with STAP philosophy,  $Q$  subsequent range cells are to be involved in joint adaptive processing. The associated  $NQ$ -variate STAP data vector  $X_t(n)$  may be defined as;

$$X_t(n) = \begin{bmatrix} x_t(n) \\ x_t(n-1) \\ \vdots \\ x_t(n-Q+1) \end{bmatrix} \quad (1)$$

Usually, linear constraints are used to protect the useful (target) signal from distortions caused by temporal adaptivity [5]. The set of  $Q$  linear constraints typically imposed on the  $NQ$ -variate STAP weight vector  $W_t(n)$  were introduced in [5] as follows;

$$W_t^H(n) \mathbf{A}_Q(\theta_0) = e_Q^T \quad (2)$$

$$\mathbf{A}_Q(\theta_0) = \text{diag}[S(\theta_0), S(\theta_0), \dots, S(\theta_0)] \quad (3)$$

where  $e_Q^T = (1, 0, \dots, 0)$  and  $S(\theta_0)$  is the  $N$ -variate array “steering” vector for direction  $\theta_0$ . If one wants to make the output useful signal more robust against pointing errors the corresponding constraints on derivatives might also be imposed, for example;

$$W_t^H(n) \mathbf{A}_{2Q}(\theta_0) = e_{2Q}^T \quad (4)$$

with

$$\mathbf{A}_{2Q}(\theta_0) = \text{diag}[\mathbf{S}(\theta_0), \mathbf{S}(\theta_0), \dots, \mathbf{S}(\theta_0)] \quad (5)$$

and

$$\mathbf{S}(\theta_0) = \left[ S(\theta_0); S'(\theta_0) \right], \quad S'(\theta) = \frac{\partial}{\partial \theta} [S(\theta)] \quad (6)$$

The imposed set of linear deterministic constraints may vary and we may, without loss of generality, assume an  $NQ \times q$  matrix  $\mathbf{A}_q$  for the case of  $q$  linear constraints.

$$W_t^H(n) \mathbf{A}_q(\theta_0) = f_q \quad (q \ll NQ) \quad (7)$$

Since the spatial distribution of the ordinary backscattered clutter signal  $y_t(n)$  is relatively broad, any attempt to fix the receiving antenna pattern along each direction of ordinary clutter arrival is useless. On the other hand, the ordinary backscattered clutter  $N$ -variate snapshot vector  $y_t(n)$  may usually be described by a low order scalar-type auto-regressive (AR) model;

$$y_t(n) + \sum_{i=1}^K b_i y_{t-i}(n) = \xi_t(n) \quad (8)$$

where  $b_i$  for  $i = 1, \dots, K \ll N$ , are the scalar AR parameters and  $\xi_t(n)$  is an  $N$ -variate innovative noise vector.

Now, the key point of the Stochastic Constraints (SC) approach is to protect sub-clutter visibility by approximating the condition;

$$\dot{y}_t(n) = W_t^H(n) Y_t(n) = W_0^H Y_t(n) \quad (9)$$

where  $W_0$  is some time-independent (reference) STAP weight vector and  $Y_t(n)$  is introduced similarly to  $X_t(n)$  in Eqn.(1). If the ordinary clutter model Eqn.(8) is accepted, then the following system of constraints in Eqn.(10) and Eqn.(11) might be introduced to ensure that an arbitrary alternating STAP weight vector  $W_t(n)$  satisfies condition Eqn.(9) exactly.

$$W_t^H(n) Y_{t-i}(n) = W_0^H Y_{t-i}(n) \quad \{i = 1, \dots, K\} \quad (10)$$

$$W_t^H(n) \tilde{\mathbf{R}}_{cl} W_t(n) = W_0^H \tilde{\mathbf{R}}_{cl} W_0 \quad (11)$$

In the STAP case, the  $NQ \times NQ$  matrix  $\tilde{\mathbf{R}}_{cl}$  corresponding to the ordinary backscattered clutter is constructed as;

$$\tilde{\mathbf{R}}_{cl} = \text{diag} [\mathbf{R}_{cl}, \mathbf{R}_{cl}, \dots, \mathbf{R}_{cl}], \mathbf{R}_{cl} = E \{ y_t(n) y_t^H(n) \} \quad (12)$$

For the  $K^{th}$  order scalar type AR clutter model in Eqn.(8), the system of constraints introduced in Eqn.(10) and Eqn.(11) ensures that the beamformed scalar *output* clutter signal  $\hat{y}_t(n)$ , processed by the alternating antenna weight vector  $W_t^H(n)$ , is also a stationary  $K^{th}$  order AR random process with the same temporal auto-correlation function.

In most cases the backscattered clutter spatial covariance matrix  $\mathbf{R}_{cl}$  is well defined; otherwise, for a rank deficient spatial clutter covariance matrix, obvious alternative approaches can be adopted. In this case the non-linear constraint in Eqn.(11) may be shown to be non-critical [1], as fluctuations of  $W_t(n)$  in the Euclidean sense are negligible. Moreover, even if we admit innovative noise beamformed output power fluctuations, they would not destroy the temporal structure of the whitening filter.

Thus, in order to simplify our problem, we may consider only the linear stochastic constraints presented in Eqn.(10), and formulate the problem of optimum spatio-temporal adaptive processing (STAP) for nonstationary hot clutter cancellation in the following way;

$$\text{Find: } \text{Min} \{ W_t^H(n) \mathbf{R}_{hc}(t) W_t(n) \}$$

$$\text{Subject to the constraints:} \quad (13)$$

$$W_t^H(n) \mathbf{A}_q = f_q$$

$$W_t^H(n) Y_{t-i}(n) = W_0^H Y_{t-i}(n), i = 1, \dots, K$$

where  $\mathbf{R}_{hc}(t)$  is the ‘‘slow time’’ dependent hot clutter spatio-temporal covariance matrix. The solution to this problem can be presented in closed form;

$$W_t(n) = \mathbf{R}_{hc}^{-1}(t) \mathbf{A} (\mathbf{A}^H \mathbf{R}_{hc}^{-1}(t) \mathbf{A})^{-1} f \quad (14)$$

with

$$\mathbf{A} = \left[ \mathbf{A}_q \ ; \ \tilde{\mathbf{Y}}_t^0(n) \right], \quad f = \left[ f_q \ ; \ 0, \dots, 0 \right]$$

$$\tilde{\mathbf{Y}}_t^0(n) = \left[ \mathbf{I}_{NQ} - \frac{\tilde{S}_0 W_0^H}{W_0^H \tilde{S}_0} \right] \tilde{\mathbf{Y}}_t(n) \quad (15)$$

$$\tilde{\mathbf{Y}}_t(n) = \left[ Y_{t-1}(n) \ ; \ Y_{t-2}(n) \ ; \ \dots \ ; \ Y_{t-K}(n) \right]$$

$$\tilde{S}_0^T = [S^T(\theta_0), 0, \dots, 0]$$

## 2. SIMULATION RESULTS

The processing efficiency of the introduced SC STAP algorithm is illustrated by computer simulation results, the Generalised Watterson model is used with typical OTHR parameters to simulate ionospherically propagated nonstationary hot clutter [6].

In this model the hot clutter  $N$ -variate snapshot vector  $c_t(n)$  may be represented by a superposition of  $P$  independent hot clutter signals with each signal having  $L$  independent modes;

$$c_t(n) = \sum_{p=0}^{P-1} \sum_{l=0}^{L-1} c_t^{(pl)}(n) \quad (16)$$

where

$$c_t^{(pl)}(n) = \alpha_{pl} \mathbf{S}_p(\theta_l) E_p(t, \theta_l) g_t^{(p)}(n-l) e^{j\Delta\omega_{pl}t} \quad (17)$$

In Eqn.(17)  $\mathbf{S}_p(\theta_l)$  is an  $N \times N$  diagonal matrix, whose elements are defined by the array steering vector, formed in accordance with the DOA of the  $l^{th}$  mode emanating from the  $p^{th}$  source. Similarly,  $\Delta\omega_{pl}$  is the regular component of the ionospheric Doppler shift,  $\alpha_{pl}$  is the RMS amplitude,  $g_t^{(p)}(n)$  is the transmitted far-field waveform.

Spatial and temporal fluctuations of the propagation media responsible for causing the hot clutter phenomenon are both represented by the two dimensional (spatio-temporal)  $N$ -variate complex random vector  $E_p(t, \theta_l)$ . We may simplify the notation by considering  $E(t, \theta)$  as modelling the spatio-temporal fluctuations experienced by some arbitrary nonstationary hot clutter mode.

The temporal sequence of  $E(t, \theta)$  is described by a multivariate scalar-type AR model;

$$E(t, \theta) + \sum_{i=1}^I \rho_i(\Delta t) E(t - i\Delta t, \theta) = \beta \epsilon(t) \quad (18)$$

where  $\Delta t$  is the sampling period, the scalar AR coefficients  $\rho_i(\Delta t)$  and  $\beta$  are defined according to the assumed characteristics of the Doppler spectrum for the given mode fluctuations. To introduce spatial fluctuations, the simplest Markov model can be adopted.

$$\epsilon_{i+1}(t) = r(d_{i+1} - d_i) \epsilon_i(t) + \sqrt{1 - |r(d_{i+1} - d_i)|^2} \gamma_{i+1}(t) \quad (19)$$

where  $(d_{i+1} - d_i)$  is the antenna inter-sensor spacing,  $r(d_{i+1} - d_i)$  is the spatial correlation coefficient for the given hot clutter mode and  $\gamma_i(t)$  is innovative noise with the following correlation properties;

$$E \{ \gamma_i(t_1) \gamma_j^*(t_2) \} = \delta(i - j) \delta(t_1 - t_2) \quad (20)$$

Figure 1 illustrates the improvement in signal-to-hot clutter ratio achieved by the standard quasi-instantaneous STAP algorithm (curve 1), without stochastic constraints, relative to the classical beamformer. Curve 2 and curve 3 in figure 1 illustrate the improvement when one and two stochastic constraints are incorporated respectively. Note that these curves are almost identical to curve 1.

Curve 4 in figure 1 illustrates the improvement in signal-to-hot clutter ratio that is achieved by the standard (time-independent) STAP beamformer, when the applied weight vector  $W(n)$  is derived from the hot clutter covariance matrix  $\mathbf{R}_{hc}$ , averaged over the whole CIT.

$$\mathbf{R}_{hc} = \frac{1}{T} \sum_{t=1}^T \mathbf{R}_{hc}(t), \quad T = 256 \quad (21)$$

One can see that the addition of stochastic constraints does not significantly degrade the quasi-instantaneous (intra-sweep) hot clutter rejectability, while an attempt to use the time invariant STAP algorithm leads to a severe degradation in nonstationary hot clutter cancellation over the CIT.

The Doppler spectra of the beamformed scalar backscattered clutter signal for the first and second order AR clutter models are shown in figure 2 and figure 3 respectively.

Curve 1 in Figure 2 and Figure 3 illustrates the output Doppler spectra for the stochastically unconstrained (quasi-instantaneous) STAP algorithm, curve 2 in figure 2 and figure 3 shows the Doppler spectra of the stochastically constrained STAP algorithms. While curve 3 in the same figures illustrates the Doppler spectra when the classical beamformer is used in the absence of hot clutter.

Clearly, the stochastic constraints have protected the ordinary sub-clutter visibility regardless of the time-alternating nature of the receiving antenna pattern. It is evident that the consequent degradation in sub-clutter visibility when no special means are undertaken to compensate for the antenna pattern fluctuations is in turn devastating.

### 3. FINITE SAMPLE EFFECTS

Of course, in practical applications  $\mathbf{R}_{hc}(t)$  should be replaced by its sample estimate  $\hat{\mathbf{R}}_{hc}(t)$ . The number of hot clutter samples  $M$  needed to estimate the sample covariance matrix  $\hat{\mathbf{R}}_{hc}(t)$  in order to achieve better than 3dB average losses is shown in [7] to be  $M \geq 2NQ$ . In our example, we note that the dimension of the STAP weight vector  $W_t(n)$  is significant ( $NQ = 256$ ), therefore apart from computational problems, the number of hot clutter samples  $M$  required for proper estimation of the quasi-instantaneous spatio-temporal covariance matrix  $\hat{\mathbf{R}}_{hc}(t)$  becomes critical. This issue is specifically important in HF OTHR applications, where the number of range cells available for covariance matrix estimation is limited.

However, as demonstrated in [8] and [9], by appropriate diagonal loading this value may be reduced to  $M \geq 2K_{sig}$ , where  $K_{sig}$  is the signal subspace dimension of  $\mathbf{R}_{hc}(t)$ , indicated for our example by the eigenvalue spectrum of  $\mathbf{R}_{hc}(0)$  in figure 4.

As shown in figure 5, the losses corresponding to the loaded sample matrix inverse algorithm using  $M = 152$  are on average the same as those expected from the standard (unloaded) sample matrix inverse method with  $M = 512$ , resulting in a significant saving in the number of hot clutter samples required.

### 4. CONCLUSION

The simulation results reflect the efficiency of the proposed SC STAP algorithm and demonstrate the potential ability of this approach to cancel highly nonstationary multimode interference without degrading backscattered sub-clutter visibility.

It may be noticed that, in its presented form, the new algorithm is not strictly operational, since pure backscattered clutter samples are not always directly available, often being masked by hot clutter interference. Different approaches for the design of operational algorithms implementing the

SC STAP approach might be proposed, one of them is described in [1] and [2].

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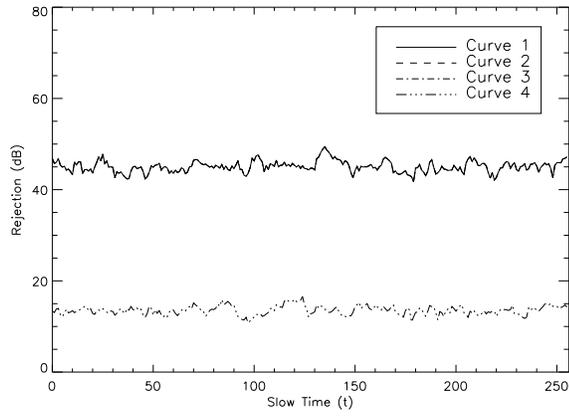


Figure 1. Hot Clutter Cancellation

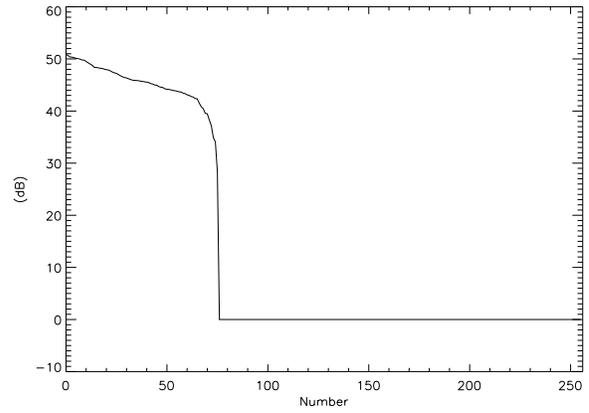


Figure 4. Eigenvalue Spectrum of  $R_{hc}(0)$

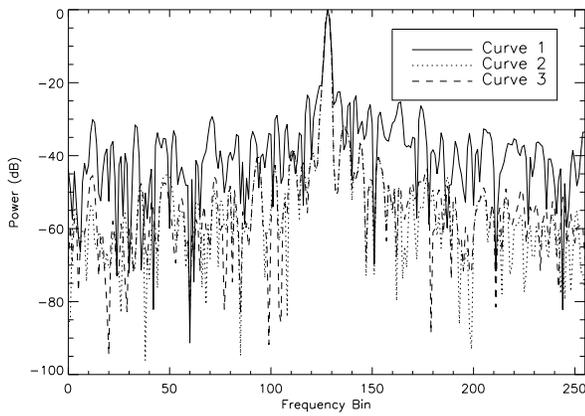


Figure 2. First Order Clutter Model

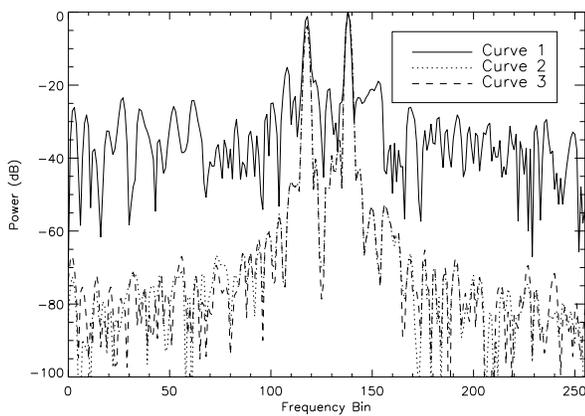


Figure 3. Second Order Clutter Model

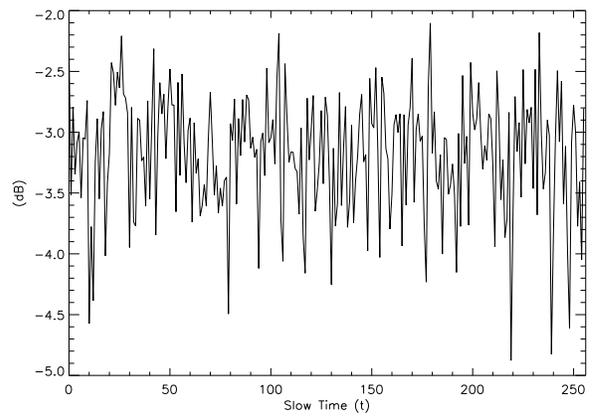


Figure 5. Average Losses