AN EFFICIENT RADAR TRACKING ALGORITHM USING MULTIDIMENSIONAL GAUSS-HERMITE QUADRATURES

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ABSTRACT

In radar tracking target motion is best modeled in Cartesian coordinates. Its position is however measured in polar coordinates (range and azimuth). Tracking in Cartesian coordinates with noisy polar measurements requires either converting the measurements to a Cartesian frame of reference and then applying the linear Kalman filter to the converted measurement [1] or using the extended Kalman filter (EKF) [2] in mixed coordinates. The first approach is accurate only for moderate cross-range errors; the second approach is consistent only for small errors. A new efficient tracking algorithm using the multidimensional Gauss-Hermite quadratures [3] to propagate the mean and the covariance of the conditional probability density function is presented. This method is compared with the EKF and the converted measurement Kalman filter (CMKF) and it is shown to be more accurate.

1. INTRODUCTION

In target tracking the motion of a moving target is best described in Cartesian coordinates by the following state-space model [4].

$$\mathbf{x}_{n+1} = \mathbf{F} \cdot \mathbf{x}_n + \mathbf{G} \cdot \mathbf{v}_n \tag{1}$$

where $\mathbf{x}_n$ is the vector of Cartesian coordinates target states $[x_n \; v_{x,n} \; y_n \; v_{y,n}]$: x_n and y_n are the position of the target in x and y directions; $v_{x,n}$ and $v_{y,n}$ are the velocities of the target in x and y directions. $\mathbf{F}$ is the state transition matrix; $\mathbf{G}$ is the noise gain matrix. $\mathbf{v}_n$ is the system noise process which is modeled as a zero-mean white Gaussian random process with covariance matrix $\mathbf{Q}_n$.

The polar coordinate measurement of the target position is related to the Cartesian coordinate target state as follows:

$$\mathbf{z}_n = \mathbf{h}(\mathbf{x}_n) + \mathbf{w}_n \tag{2}$$

where $\mathbf{z}_n$ is the vector of polar coordinates measurement $[r_n \; \theta_n]$: r_n is the range and θ_n is the azimuth of the target. $\mathbf{h}(\cdot)$ is the Cartesian-to-polar coordinate transformation. $\mathbf{w}_n$ is the observation noise process which is assumed to be zero-mean white Gaussian noise process with covariance matrix $\mathbf{R}_n$. Target tracking becomes the problem of estimating the target states $\mathbf{x}_n$ from the noisy polar measurements $\mathbf{z}_n$.

Target tracking in Cartesian coordinates using polar measurements can be handled in two ways. One method is called the converted measurement Kalman filter (CMKF) [1], which uses a Kalman filter with polar measurements converted to a Cartesian frame of reference. In this case the Cartesian components of the errors in the converted measurements become correlated and non-Gaussian, which can seriously degrade the performance of the Kalman filter. An improved method using debiased converted measurements [5] is showed to be more accurate and consistent for all practical situations. The other method is the extended Kalman filter (EKF) which employs the first-order Taylor series approximation to adapt the linear Kalman filter to the nonlinear system described by equations (1) and (2). Error is introduced because higher-order terms in the series are ignored and linearization is done about the predicted state, not the actual state. There exists alternative approaches to the EKF, such as the "quasi-extended" Kalman filter [6], which shows improvements when tracking maneuvering targets at close range.

To improve the performance of the existing approaches, a new tracking algorithm based on multidimensional Gauss-Hermite quadrature is presented. Instead of approximating the nonlinear measurement equation (2) with a linear one with white Gaussian noise, our approach uses multidimensional Gauss-Hermite quadrature to evaluate the optimal estimate of the target states at each iteration directly from the Bayesian equations [7]. This quadrature technique approximates the integrals in the Bayesian equations as summations and this approximation can be very accurate when the integrand is smooth. To reduce the computation of applying these quadratures analytic results are employed in the prediction stage because the system dynamics are linear; quadrature techniques are applied only to the measurement update stage. Simulation results show that this method is more accurate than the EKF and the converted measurement Kalman filter (CMKF).

2. THE OPTIMAL NONLINEAR FILTER

The optimal nonlinear filter computes the minimum-variance estimate of the state at each discrete time n which is just the mean of the state density function conditioned on the measurement history $\mathbf{Z}^n : \mathbf{z}_0, \ldots, \mathbf{z}_n$.

$$\hat{\mathbf{x}}_{opt,n|n} = E[\mathbf{x}_n|\mathbf{Z}^n] = \int \mathbf{x}_n p(\mathbf{x}_n|\mathbf{Z}^n) d\mathbf{x}_n \tag{3}$$

This requires the *a posteriori* density function $p(\mathbf{x}_n|\mathbf{Z}^n)$ be known at each iteration. This density function can be determined recursively by the following Bayesian equations [8]:

$$p(\mathbf{x}_n|\mathbf{Z}^n) = \frac{p(\mathbf{x}_n|\mathbf{Z}^{n-1})p(\mathbf{z}_n|\mathbf{x}_n)}{p(\mathbf{z}_n|\mathbf{Z}^{n-1})} \quad (4)$$

$$p(\mathbf{x}_n|\mathbf{Z}^{n-1}) = \int p(\mathbf{x}_{n-1}|\mathbf{Z}^{n-1})p(\mathbf{x}_n|\mathbf{x}_{n-1})d\mathbf{x}_{n-1} \quad (5)$$

where the normalizing constant $p(\mathbf{z}_n|\mathbf{Z}^{n-1})$ in equation (4) is given by

$$p(\mathbf{z}_n|\mathbf{Z}^{n-1}) = \int p(\mathbf{x}_n|\mathbf{Z}^{n-1})p(\mathbf{z}_n|\mathbf{x}_n)d\mathbf{x}_n \quad (6)$$

By assumption, the density function $p(\mathbf{z}_n|\mathbf{x}_n)$ in equation (4) has a Gaussian distribution with mean $\mathbf{h}(\mathbf{x}_n)$ and covariance matrix $\mathbf{R}_n$.

$$p(\mathbf{z}_n|\mathbf{x}_n) = \frac{1}{2\pi|\mathbf{R_n}|^{1/2}}e^{-\frac{1}{2}(\mathbf{z}_n-\mathbf{h}(\mathbf{x}_n))^T\mathbf{R}_n^{-1}(\mathbf{z}_n-\mathbf{h}(\mathbf{x}_n))} \quad (7)$$

Similarly, the density $p(\mathbf{x}_n|\mathbf{x}_{n-1})$ in equation (5) also has a Gaussian distribution with mean $\mathbf{F}\mathbf{x}_{n-1}$ and covariance matrix $\mathbf{G}\mathbf{Q}_{n-1}\mathbf{G}^T$.

The initial *a posteriori* density $p(\mathbf{x}_0|\mathbf{Z}^0)$ is given by

$$p(\mathbf{x}_0|\mathbf{Z}^0) = p(\mathbf{x}_0|\mathbf{z}_0) = \frac{p(\mathbf{x}_0)p(\mathbf{z}_0|\mathbf{x}_0)}{p(\mathbf{z}_0)} \quad (8)$$

where $p(\mathbf{x}_0)$ is usually assumed to be white Gaussian.

It is however generally impossible to accomplish the integration indicated in equations (5) and (6) in closed form because of the presence of the nonlinear function $\mathbf{h}(\mathbf{x}_n)$. When the measurement equations are linear and the initial state and the noise sequences are Gaussian. Then, the equations (5) and (6) can be evaluated in closed form and the posterior density $p(\mathbf{x}_n|\mathbf{Z}^n)$ is Gaussian for all n. The mean and the covariance matrix of the *a posteriori* density $p(\mathbf{x}_n|\mathbf{Z}^n)$ are known as the Kalman filter equations. Most of the sub-optimal nonlinear filter are based on the linear Kalman filter equations by transforming the nonlinear measurement equation into a linear equation with white additive Gaussian noise, i.e. forcing the requirements of the Kalman filter equations satisfied. The next section presents two sub-optimal filters used extensively in target tracking in Cartesian coordinates with noisy polar measurements.

3. SUB-OPTIMAL NONLINEAR FILTERS FOR RADAR TRACKING

3.1. Extended Kalman Filter (EKF)

In the "mixed coordinate" EKF [2] the state is in Cartesian coordinates and the measurements are in polar coordinates. Therefore, there is a nonlinear measurement function $\mathbf{h}(\mathbf{x}_n)$. Denote the first order Taylor series approximation of this nonlinear function about the predicted state $\hat{\mathbf{x}}_{n|n-1}$ as $\bar{\mathbf{h}}(\mathbf{x}_n)$; it is given by the following equation:

$$\bar{\mathbf{h}}(\mathbf{x}_n) = \mathbf{h}(\hat{\mathbf{x}}_{n|n-1}) + \mathbf{H}_n \cdot (\mathbf{x}_n - \hat{\mathbf{x}}_{n|n-1}) \quad (9)$$

where $\mathbf{H}_n$ is the Jacobian of the nonlinear function $\mathbf{h}(\mathbf{x}_n)$

$$\mathbf{H}_n = \left[\frac{\partial \mathbf{h}_n(\mathbf{x}_n)}{\partial \mathbf{x}_n}\right]_{\mathbf{x}_n = \hat{\mathbf{x}}_{n|n-1}}$$

When the above approximation is substituted into the derivations of the standard Kalman filter equations, the Extended Kalman filter equations are obtained. Its accuracy however depends heavily on the stability of the Jacobian matrix. In practice, the Jacobian matrix is often numerically unstable resulting in filter divergence.

3.2. Converted Measurement Kalman Filter (CMKF)

With the converted measurement Kalman filter [1], the polar coordinate measurement $\mathbf{z}_n^p$ is first converted to the Cartesian coordinate measurements $\mathbf{z}_n^c$ using an inverse nonlinear transformation $\mathbf{h}^{-1}(\mathbf{z}_n^p)$ The original noise process $\mathbf{w}_n$ acting on the converted measurement $\mathbf{z}_n^c$ no longer behaves rigorously as an additive term, but in some complicated fashion. However, at least when the covariance of the noise $\mathbf{w}_n$ is small, the new Cartesian coordinate measurement equation can be written as follows:

$$\mathbf{z}_n^c = \mathbf{D}\,\mathbf{x}_n + \tilde{\mathbf{w}}_n \quad (10)$$

where $\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $\tilde{\mathbf{w}}_n$ is approximated as a white Gaussian noise process on the converted measurement $\mathbf{z}_n^c$ with zero mean and covariance matrix $\mathbf{M}_n$

$$\mathbf{M}_n = E[\tilde{\mathbf{w}}_n\tilde{\mathbf{w}}_n^T] = \left[\frac{\partial \mathbf{h}^{-1}(\mathbf{z}_n^p)}{\partial \mathbf{z}_n^p}\right]\mathbf{R}_n\left[\frac{\partial \mathbf{h}^{-1}(\mathbf{z}_n^p)}{\partial \mathbf{z}_n^p}\right]^T\Bigg|_{z_n^p = \hat{z}_{n|n-1}} \quad (11)$$

where $\hat{\mathbf{z}}_{n|n-1}$ is the predicted measurement. As a result, the new measurement equation in Cartesian coordinates becomes linear and the noise process is Gaussian, the standard Kalman filter can be applied. This method however is an acceptable approximation only for moderate cross-range errors.

4. PROPOSED FILTER

4.1. Basic Principles

Instead of transforming the measurement equation linear and the observation noise Gaussian, our method evaluates the optimal estimate of the target states from the Bayesian equations (4) and (5) directly using multidimensional Gauss-Hermite quadratures [7]. Multidimensional Gauss-Hermite quadrature is an approximation of an multidimensional integral of a function of the following form with a weighted sum of the functional values evaluated at a set of pre-defined grid points [3].

$$\int_{\mathbf{R}_n} f(\mathbf{x})e^{-(\mathbf{x}-\hat{\mathbf{x}})^T\mathbf{P}^{-1}(\mathbf{x}-\hat{\mathbf{x}})}d\mathbf{x} \approx \sum_{i=1}^{N}\mathbf{w}_i f(\mathbf{x}_i) \quad (12)$$

where $\mathbf{x}$ and $\hat{\mathbf{x}}$ are n-dimensional vector and $\mathbf{P}$ is a $n \times n$ nonsingular matrix; $\mathbf{x}_i$ is the i^{th} n-dimensional grid points and $\mathbf{w}_i$ is the corresponding weight. N is the total number of grid points. The complexity of this quadrature technique is of order of N. This approximation is very accurate especially when the function $f(\mathbf{x})$ is algebraic.

To evaluate the state prediction density $p(\mathbf{x}_n|\mathbf{Z}^{n-1})$ efficiently from equation (5), our method collapses the *a posteriori* density $p(\mathbf{x}_{n-1}|\mathbf{Z}^{n-1})$ which is non-Gaussian in general at each iteration into a single Gaussian density function with mean $\hat{\mathbf{x}}_{n-1|n-1}$ and covariance matrix $\mathbf{P}_{n-1|n-1}$. This approximation is closely satisfied in the radar tracking applications because the *a posteriori* density is unimodal and the system equation is linear. By using this approximation, the equation (5) can be evaluated in closed form and the resulting state prediction density $p(\mathbf{x}_n|\mathbf{Z}^{n-1})$ is a Gaussian density function with mean $\hat{\mathbf{x}}_{n|n-1}$ and covariance matrix $\mathbf{P}_{n|n-1}$.

$$\hat{\mathbf{x}}_{n|n-1} = \mathbf{F}\,\hat{\mathbf{x}}_{n-1|n-1} \tag{13}$$
$$\mathbf{P}_{n|n-1} = \mathbf{F}\mathbf{P}_{n-1|n-1}\mathbf{F}^T + \mathbf{G}\mathbf{Q}_n\mathbf{G}^T \tag{14}$$

The calculation of the *a posteriori* density $p(\mathbf{x}_n|\mathbf{Z}^n)$ from equation (4) requires the evaluation of the normalizing constant $p(\mathbf{z}_n|\mathbf{Z}^{n-1})$ of the form

$$p(\mathbf{z}_n|\mathbf{Z}^{n-1}) = \int C_1 e^{-\frac{1}{2}(\mathbf{z}_n-\mathbf{h}(\mathbf{x}_n))^T\mathbf{R}_n^{-1}(\mathbf{z}_n-\mathbf{h}(\mathbf{x}_n))} e^{-\frac{1}{2}(\mathbf{x}_n-\hat{\mathbf{x}}_{n|n-1})^T\mathbf{P}_{n|n-1}^{-1}(\mathbf{x}_n-\hat{\mathbf{x}}_{n|n-1})} d\mathbf{x}_n \tag{15}$$

where $C_1 = \frac{1}{2\pi|\mathbf{R_n}|^{1/2}}\frac{1}{2\pi|\mathbf{P}_{n|n-1}|^{1/2}}$. One may use the expression

$$F_1(\mathbf{x}_n) = e^{-\frac{1}{2}(\mathbf{x}_n-\hat{\mathbf{x}}_{n|n-1})^T\mathbf{P}_{n|n-1}^{-1}(\mathbf{x}_n-\hat{\mathbf{x}}_{n|n-1})}$$

as the Gaussian weighting function, but the remaining expression

$$F_2(\mathbf{x}_n) = e^{-\frac{1}{2}(\mathbf{z}_n-\mathbf{h}(\mathbf{x}_n))^T\mathbf{R}_n^{-1}(\mathbf{z}_n-\mathbf{h}(\mathbf{x}_n))}$$

is not algebraic and the result may not be accurate. The expression $F_2(\mathbf{x}_n)$ is thus factored into two expressions: one is Gaussian, $F_2^1(\mathbf{x}_n)$; the other is nearly algebraic within the desired region, $F_2^2(\mathbf{x}_n)$ as follows:

$$F_2^1(\mathbf{x}_n) = e^{-\frac{1}{2}(\mathbf{z}_n^c-\mathbf{D}\mathbf{x}_n)^T\mathbf{M}_n^{-1}(\mathbf{z}_n^c-\mathbf{D}\mathbf{x}_n)}$$
$$F_2^2(\mathbf{x}_n) = e^{\left[-\frac{1}{2}(\mathbf{z}_n^p-\mathbf{h}(\mathbf{x}_n))^T\mathbf{R}_n^{-1}(\mathbf{z}_n^p-\mathbf{h}(\mathbf{x}_n)) + \frac{1}{2}(\mathbf{z}_n^c-\mathbf{D}\mathbf{x}_n)^T\mathbf{M}_n^{-1}(\mathbf{z}_n^c-\mathbf{D}\mathbf{x}_n)\right]}$$

There are several other approaches using linear approximation techniques to the nonlinear function $\mathbf{h}(\mathbf{x}_n)$ to factor $F_2(\mathbf{x}_n)$ [7], but we employ the approximation technique in the converted measurement Kalman filter (CMKF), because the converted measurement Kalman filter (CMKF) has the correct covariance and it yields smaller errors than the EKF in radar tracking applications. The new weighting function determined by $F(\mathbf{x}_n) = F_1(\mathbf{x}_n)\cdot F_2^1(\mathbf{x}_n)$ becomes

$$F(\mathbf{x}_n) = C_2 e^{-\frac{1}{2}(\mathbf{x}_n-\tilde{\mathbf{x}}_{n|n})^T\tilde{\mathbf{P}}_{n|n}^{-1}(\mathbf{x}_n-\tilde{\mathbf{x}}_{n|n})} \tag{16}$$

where

$$\tilde{\mathbf{x}}_{n|n} = \hat{\mathbf{x}}_{n|n-1} + \mathbf{K}_n(\mathbf{z}_n^c - \mathbf{D}\mathbf{x}_n) \tag{17}$$
$$\mathbf{P}_{n|n} = \mathbf{P}_{n|n-1} - \mathbf{K}_n\mathbf{D}_n\mathbf{P}_{n|n-1} \tag{18}$$
$$\mathbf{K}_n = \mathbf{P}_{n+1|n}\mathbf{D}_n^T(\mathbf{D}_n\mathbf{P}_{n|n-1}\mathbf{D}_n^T + \mathbf{M}_n)^{-1} \tag{19}$$

These equations are nothing else but the converted measurement Kalman filter (CMKF) equations, and the constant C_2 is not necessary to be known as it will be canceled later. The normalizing constant $p(\mathbf{z}_n|\mathbf{Z}^{n-1})$ can be evaluated using multidimensional Gauss-Hermite quadrature as follows:

$$p(\mathbf{z}_n|\mathbf{Z}^{n-1}) = C_1C_2|\mathbf{W}_n|^{-1}\sum_{i=1}^{K}\mathbf{B}_iF_2^2(\mathbf{x}_{n,i}) \tag{20}$$

where

$$\mathbf{W}_n\mathbf{W}_n^T = \mathbf{P}_{n|n} \tag{21}$$
$$\mathbf{x}_{n,i} = \mathbf{W}_n\mathbf{u}_i + \tilde{\mathbf{x}}_{n|n} \tag{22}$$
$$\mathbf{u}_i = [u_{i_1},\ldots,u_{i_N}]^T \tag{23}$$
$$\mathbf{B}_i = B_{i_1}\cdots B_{i_N} \tag{24}$$

where $\mathbf{W}_n$ is the square root of $\mathbf{P}_{n|n}$ from the Cholesky algorithm; $\mathbf{x}_{n,i}$ is the i^{th} N-dimensional grid points and $\mathbf{B}_i$ is the corresponding weight. u_{i_j} and B_{i_j} are the grid points and the weights for one dimensional Gauss-Hermite quadrature. K is the total number of grid points. Finally multidimensional Gauss-Hermite quadrature is used to compensate the error introduced from the approximation and the estimate becomes

$$\hat{\mathbf{x}}_{n|n} = \frac{\sum_{i=1}^K\mathbf{x}_{n,i}\mathbf{B}_iF_2^2(\mathbf{x}_{n,i})}{\sum_{i=1}^K\mathbf{B}_iF_2^2(\mathbf{x}_{n,i})} \tag{25}$$

4.2. Filter Structure

The block diagram of the proposed filter is presented in Figure 1. It consists of the following four stages:

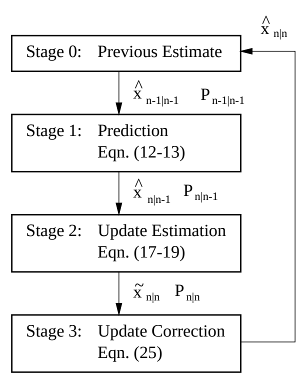

Figure 1: Flow Diagram of the Proposed Filter

Stage 0: Previous estimates:
Assume at step n the mean $\hat{\mathbf{x}}_{n-1|n-1}$ and the covariance $\mathbf{P}_{n-1|n-1}$ of the *a posteriori* density $p(\mathbf{x}_{n-1}|\mathbf{Z}^{n-1})$ are known. The initial values for the mean and the covariance are estimated from the first two measurements.

Stage 1: Prediction:
The mean $\hat{\mathbf{x}}_{n|n-1}$ and the covariance $\mathbf{P}_{n|n-1}$ of the state prediction density $p(\mathbf{x}_n|\mathbf{Z}^{n-1})$ are predicted by equations (13) and (14).

Stage 2: Update Estimation:

The mean $\check{\mathbf{x}}_{n|n}$ and the covariance $\mathbf{P}_{n|n}$ of the *a posteriori* density $p(\mathbf{x}_n|\mathbf{Z}^n)$ are first estimated by the converted measurement Kalman filter (CMKF) equations (17), (18) and (19).

Stage 3: Update Correction:

To compensate for the errors introduced in the approximation from Stage 2 multidimensional Gauss-Hermite quadrature is used to evaluate the optimal estimate of the target state $\hat{\mathbf{x}}_n$ directly from the Bayesian equations and the final estimate is given by equation (25).

4.3. Comments

This algorithm basically consists of two parts: one is the converted measurement Kalman filter (CMKF) and the other is an error-compensation unit using multidimensional Gauss-Hermite quadrature. The complexity of this algorithm is of an order of N where N is the total number of grid points. The more the total number of grid points is, the more accurate the result we will get. Simulation results show that this algorithm is accurate compared with other methods even when the number of the grid points is as small as five for each dimension. This algorithm is also efficient because only simple additions and multiplications are required in the computation; the computation of the grid points $\mathbf{u}_i$ and the corresponding weights $\mathbf{B}_i$ can be done offline.

5. SIMULATION RESULTS

To compare the performance of our proposed filter with that of currently popular approximate filters a two-dimensional target tracking application described by the system equation (1) and the measurement equation (2) with the following parameters is simulated.

$$\mathbf{F} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{G} = \begin{bmatrix} 1/2 & 0 \\ 1 & 0 \\ 0 & 1/2 \\ 0 & 1 \end{bmatrix};$$

$$\mathbf{h}(\mathbf{x}_n) = \begin{bmatrix} \sqrt{x_n^2 + y_n^2} \\ \tan^{-1} y_n/x_n \end{bmatrix};$$

$$\mathbf{Q} = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}; \quad \mathbf{R} = \begin{bmatrix} 100 & 0 \\ 0 & 0.01 \end{bmatrix};$$

$$\mathbf{x}_0 = \begin{bmatrix} 50km & -10m/s & 10km & 20m/s \end{bmatrix}^T$$

Tracks are initiated with two point differencing to obtain the initial velocity estimate. The results presented in Figures 2 and 3 are based on 100 measurements averaged over 500 independent realizations of the experiment with the sampling interval of one second.

The proposed filter is compared with the well-known classical filters, e.g. the EKF and the converted measurement Kalman filter (CMKF). The position errors and the velocity errors for each filters are shown in Figs. 2 and 3 where the error is defined as the root mean square of the difference between the actual value and the estimated value. Our proposed method converges faster and yields results of smaller error than the EKF and the converted measurement Kalman filter (CMKF) does whereas the EKF diverges due to the instability of the Jacobian matrix.

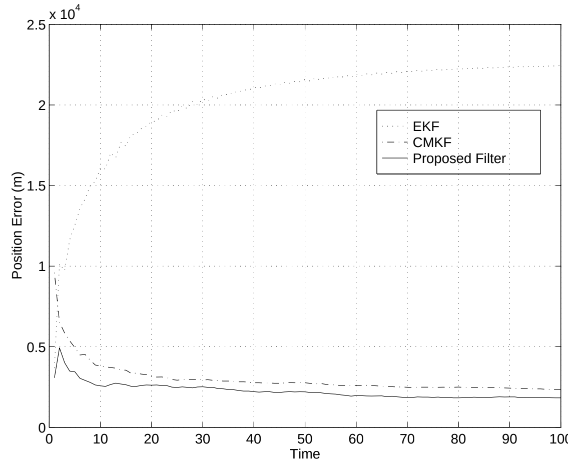

Figure 2: Comparison of the position errors

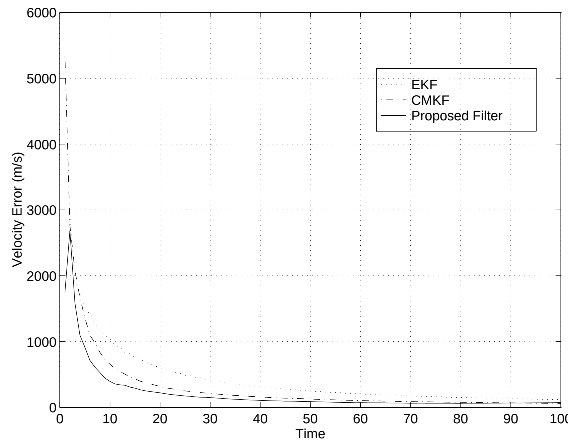

Figure 3: Comparison of the velocity errors

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