APPLYING ACOUSTIC ARRAY PROCESSING TO THE ESTIMATION OF THE PROPAGATION SPEED OF WAVES IN A CAR EXHAUST*

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ABSTRACT

A parametric method based on spatial filter techniques (beamforming) is proposed to estimate the propagation speed of acoustic waves. The propagation speed estimate is analyzed for the case of narrowband signals and compared to the maximum likelihood estimate (MLE) of the propagation speed. It is shown that for an array of 3 sensors our estimate coincides with the ML estimate but its performance analysis is simpler and its computational cost is much more reduced. The proposed estimate is also applied to the wideband waves propagating along a car exhaust. It is shown that the signal-to-noise ratio and the magnitude of the relative aperture (distance between the array sensors respect to the wavelength) for each frequency could limit the good performance of the speed estimator. Good results have been achieved when these limitations have been taken into account.

1. PROBLEM STATEMENT

There exist two low frequency wideband acoustic waves propagating along a car exhaust when the engine is working, the forward one generated by the engine, and the backward one originated from the misadjustment of the acoustic impedance at the muffler input. Both waves are cyclic and their characteristics depend basically on the number of revolutions-per-minute the car engine is rotating at. The main characteristics of the acoustic waves to be taken into account for a perfect separation of their waveforms are the direction of arrival and the propagation speed. The first one is perfectly known: -90° for the forward wave and 90° for the backward one respect to the exhaust pipe orthogonal direction. The second one is only approximately known because it depends on the uncertain temperature value, the stability of the car engine, and other factors.

As it is explained in [1], the separation of the

forward and the backward acoustic waves propagating along the exhaust is useful in several applications, and a good estimate of the speed propagation is needed. On the other hand, this parameter is fundamental for obtaining a better knowledge of the characteristics of the engine at work.

The estimation of the acoustic propagation speed is a relatively new problem but the solutions to it are well known because it is basically equivalent to the direction-of-arrival finding. Both problems imply to estimate the steering vector of the corresponding plane waves noting that in our case the signals are fully correlated. There exists also in our application a severe limitation on the number of sensors, mainly due to the non-linear propagation of the waves if the array aperture is large. For this reason, four sensors are used as a practical maximum and the distance between them is under 0.1 m.

2. PROPOSED TECHNIQUE

The technique which is proposed is a *parametric method* to estimate the propagation speed in the case that a limitation in the number of sensors and high correlation between signals is given. Suppose there are two narrowband acoustic waves propagating along the pipe, one in the forward sense and the other in the backward one. If we use 3 sensors in order to estimate the propagation speed, we have, at the sensor outputs, the vector

$$\mathbf{x}(t) = \mathbf{A}(c_0)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), x_3(t)]^T$, $\mathbf{A}(c_0) = [\mathbf{a}(c_0), \mathbf{a}(-c_0)]$ is the steering matrix whose columns are defined below, and $\mathbf{s}(t) = [s_f(t), s_b(t)]^T$ is the signal vector, being $s_f(t)$ the forward wave and $s_b(t)$ the backward one. The vector $\mathbf{n}(t)$ represents the noise at the sensor outputs and its components are zero-mean, i.i.d. random variables.

The steering vector of the forward signal is

$$\mathbf{a}(c_0) = \left[1 \quad e^{j\omega \frac{d}{c_0}} \quad e^{j2\omega \frac{d}{c_0}} \right]^H \tag{2}$$

where $(\cdot)^H$ means conjugate transpose, ω is the wave pulsation, c_0 its propagation speed and d is the distance between sensors. Its direction of arrival is $-90°$, and the steering vector of the backward wave is $\mathbf{a}(-c_0)$, being its angle of arrival $90°$. Due to this similarity between the steering vectors - and the limitations pointed above - we can think in a parametric method in order to estimate the propagation speed: there are two waves but a unique parameter to find.

The proposed estimate of the propagation speed is

$$\hat{c}_0 = \arg\left\{\min_c\left\{\sigma_y^2(c)\right\}\right\} \qquad (3)$$

where $\sigma_y^2(c)$ is the output power of a Delay-and-Sum (DS) beamformer [2] which has been designed to cancel the forward and the backward waves supposing that these waves are propagating at c m/s.

The DS beamformer tries to cancel the signals, so the weight vector must follow

$$\mathbf{A}^H(c)\mathbf{w} = \mathbf{0} \qquad (4)$$

where $\mathbf{0}$ is a (2x1) vector of zeros. We have to impose another condition in order to obtain a non-trivial solution. This condition is $\|\mathbf{w}\|_2 = 1$; in this way we assure that the noise gain is independent of the value of c used in the design. Under these assumptions we obtain the following expression for the weight vector

$$\mathbf{w}(c) = \frac{1}{\sqrt{2(2 + \cos(2\omega d / c))}}\begin{bmatrix} 1 \\ -2\cos(\omega d / c) \\ 1 \end{bmatrix} \qquad (5)$$

with ω, d and c defined above. The beamformer output is

$$y(c) = \mathbf{w}^H(c)\mathbf{x} \qquad (6)$$

The output vector $\mathbf{y}(t) = [y(t_0), y(t_1),..., y(t_{M-1})]$ has contributions from the noise input filtered by the beamformer and from a residue of the signals $s_f(t)$ and $s_b(t)$. When c, the propagation speed used in the beamformer design, and c_0, the true speed, are the same, the signal term is totally canceled. If we work with the output power, under the assumption of signals uncorrelated with noise, the mean power of the beamformer output can be expressed as

$$\sigma_y^2(c) = E\left[\mathbf{y}\mathbf{y}^H\right] = \frac{2(\cos(\omega d / c_0) - \cos(\omega d / c))^2}{2 + \cos(2\omega d / c)} \cdot$$
$$(\sigma_f^2 + \sigma_b^2 + 2 \cdot \Re\{\rho \exp\{-j2\omega d / c_0\}\}) + \sigma_n^2 \qquad (7)$$

where σ_f^2 and σ_b^2 are the power of the forward and the backward waves respectively, and $\rho = E[s_f(t)s_b^*(t)]$ is the cross-correlation coefficient between signals. An example of this kind of curves is displayed in figure 1, where the
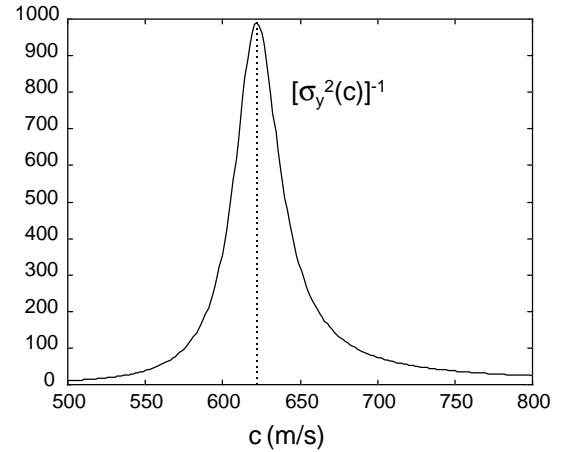

Figure 1. Inverse of the mean output power $\sigma_y^2(c)$ for $d/\lambda=1/5$, $c = 623$ m/s and $(S/N) = 30$ dB. Units are linear.

inverse of $\sigma_y^2(c)$ is represented and c_0 is estimated as its maximum value.

The practical procedure to build up the propagation speed estimator stated in (3) would be simple: we would design K beamformers in a range of values where we knew it was probable to be c_0, and would choose the $\hat{c}_0$ as the one that minimized the total output power of the beamformer.

However, this procedure can be improved by means of a Newton method [3] for searching the minimum. The convergence of the method is achieved in 5 or 6 steps as maximum if the derivatives of $\sigma_y^2(c)$ respect to c are used. An example of the search for the minimum procedure is shown in figure 2 in comparison with the whole function $\sigma_y^2(c)$ in the range of 400 to 750 m/s.

3. COMPARISON WITH OTHER ESTIMATORS

The maximum likelihood estimate (MLE) of the propagation speed under the assumption of stationary Gaussian white noise and signal and noise uncorrelated is given by [4]

$$\hat{c}_{ML} = \arg\left\{\min_c \mathrm{Tr}\left[\mathbf{P}_A^\perp(c)\hat{\mathbf{R}}_x\right]\right\} \qquad (8)$$

where $\hat{\mathbf{R}}_x$ is the sample covariance matrix and $\mathbf{P}_A^\perp(c)$ is the orthogonal projection matrix onto the null space of $\mathbf{A}^H(c)$, i.e.,

$$\mathbf{P}_A^\perp(c) = \mathbf{I} - \mathbf{A}(c)\mathbf{A}^+(c) = \mathbf{I} - \mathbf{A}\left[\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H\right] \qquad (9)$$
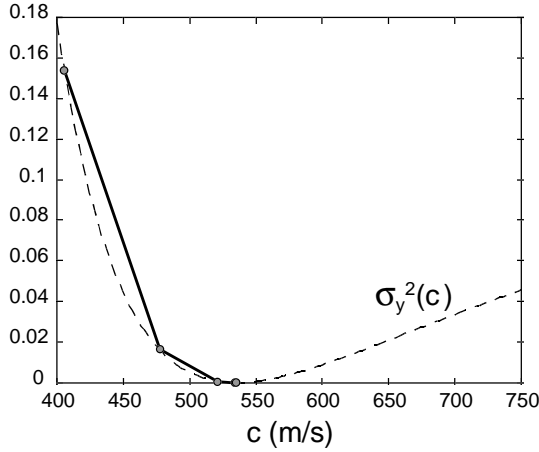
Figure 2. Example of the estimation of the propagation speed applying Newton method to $\sigma_y^2(c)$. The convergence is achieved in 5 steps for c=534 m/s with an initial guess of 408 m/s.
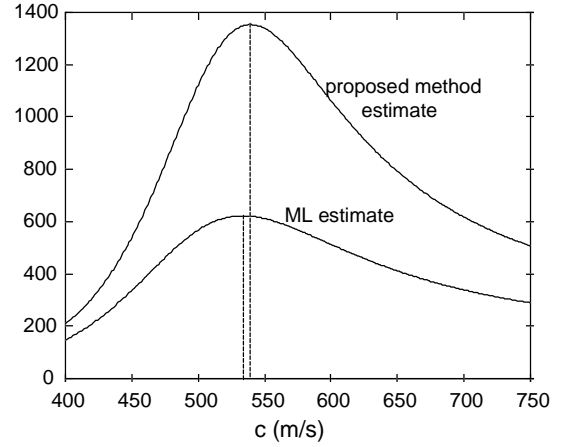

Figure 3. Comparison between the MLE and our estimate for 4 sensors and 64 samples. The true propagation speed is 535 m/s and the _(S/N)_ = 30 dB. The ML estimates 533 m/s and our method does 539 m/s. The inverse of the output power is represented and units are linear.

where $\mathbf{A}^+(c)$ is the Moore-Penrose pseudo-inverse of $\mathbf{A}(c)$. The interpretation of the MLE is that $\mathbf{x}$(t) is projected onto a subspace orthogonal to the signals supposed to be arriving to the array. Then a power measurement, the trace of $(\mathbf{P}_A^\perp \hat{\mathbf{R}}_x)$, is evaluated. The energy should be smallest when the projector removes all the true signal components, i.e., when $c = c_0$.

If the array is composed by 3 sensors, the orthogonal subspace to the forward and backward waves of the pipe is formed by a unique vector. This vector builds up the projector $\mathbf{P}_A^\perp(c)$ and its main characteristics are to be orthogonal to the steering matrix, $\mathbf{A}(c)$, and to have unit norm. These conditions are the ones we impose to the weight vector $\mathbf{w}(c)$ of the DS beamformer we use to estimate the propagation speed of the waves by the proposed method. In fact, we estimate c_0 as the value that minimizes the beamformer output power but it could also be seen as the value that minimizes the projection of the covariance matrix $\hat{\mathbf{R}}_x$ onto the unitary vector $\mathbf{w}$. When $\mathbf{w}(c)$ belongs to the orthogonal subspace of the true waveforms, the signal components are suppressed and only the noise component remains.

Therefore we can conclude that the estimator proposed in (3) is the ML estimate of the propagation speed when the number of sensors of the array is 3. If the number of sensors is 4 or 5 - in our application we have stated that the aperture of the array should be small so supposing much more sensors is nonsense - the MLE makes the projection of $\hat{\mathbf{R}}_x$ onto the whole orthogonal subspace to the signal components, meanwhile our estimate projects the covariance matrix onto a subspace formed by a unique vector, $\mathbf{w}(c)$, being $\mathbf{w}$ a vector

belonging to the same orthogonal subspace. The difference between both estimates is null in theory, but in practice the effect of a finite number of samples and the addition of noise slightly worsens the performance of our estimate in comparison to the ML estimator.

An example of the influence of the number of samples is shown in figure 3 where the number of sensors is 4.

4. BROADBAND APPLICATION

The application of the speed estimator to the broadband case implies to make the Fourier Transform of the signals and to apply the narrowband estimator for each frequency [4]. The propagation speed estimate is

$$\hat{c}_0 = \arg\left\{ \underset{c}{min}\left\{\sigma_y^2(c)\right\}\right\} =$$
$$\arg\left\{ \underset{c}{min}\left\{\frac{1}{L}\sum_{l=0}^{L-1}\left[\sigma_y^2(c,f_l)\right]\right\}\right\} \quad (10)$$

In fact, this speed estimator is not valid for all the frequencies because the equation expressed above supposes the signal and the noise to be uncorrelated. This is true if we consider the theoretic mean output power (7), but in practice, there is a finite number of samples and we have to work with an estimate of the mean output power, $\hat{\sigma}_y^2$.

For a finite number of samples, when the DS beamformer is used to detect a signal, the output signal power is much larger than the covariance between signal and noise and this last term is supposed to be zero.
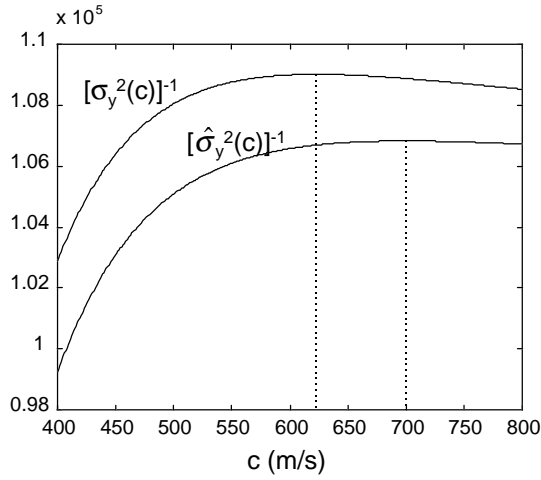
Figure 4. Inverse of the estimated mean output power $\hat{\sigma}_y^2$ and inverse of the theoretic mean output power $\sigma_y^2(c)$ for a frequency whose $d/\lambda \in$ (0.004, 0.008) and *(S/N)* = 50 dB. The estimated value of c_0 is 699 m/s and the true value is 623 m/s. Units are linear.

However when we pretend to cancel the signal, this term is comparable to the noise power and can introduce an error in the speed estimator shifting the minimum of $\hat{\sigma}_y^2$ to another c different from c_0. The complete expression of the estimated mean output power of the beamformer is

$$\hat{\sigma}_y^2(c) = \mathbf{yy}^H = \mathbf{w}^H\left\{(\mathbf{As}+\mathbf{n})(\mathbf{As}+\mathbf{n})^H\right\}\mathbf{w} \cong \quad (11)$$
$$\sigma_y^2(c) + \mathbf{w}^H(c)\left(\mathbf{A}(c_0)\mathbf{sn}^H + \mathbf{ns}^H\mathbf{A}^H(c_0)\right)\mathbf{w}(c)$$

It can be shown that the perturbation introduced in $\hat{\sigma}_y^2$ by the cross-covariance between signal and noise is not negligible depending on the range of the relative aperture, d/λ, for each frequency, and on the signal-to-noise ratio (*SNR*) at the beamformer input. For example, when the relative aperture is very small, the curve of the mean output power $\sigma_y^2(c)$ is nearly flat around c_0 and the cross-covariance term has a large influence as it can be seen in figure 4. On the other hand, when the *SNR* is very small, the signal power of both waves is comparable to the noise power, the mean output power σ_y^2 is comparable to the second term of (11) and consequently the minimum of $\hat{\sigma}_y^2$ is not achieved at c_0.

Therefore, the application of the propagation speed estimate to broadband acoustic waves must take into account the expression of the mean output power estimate in (11). For an appropriate use of our method it is necessary to establish some criteria to eliminate those low frequencies whose relative aperture is very small, and those frequencies where the *SNR* is not large enough. After this, we assure that $\hat{\sigma}_y^2$ is a good estimate of σ_y^2 and consequently $\hat{c}_0$ is a good estimate of c_0. Good results have been achieved when the broadband propagation speed estimate (10) has been applied to acoustic waves generated by car engines.

5. CONCLUSIONS

We have formulated a new parametric estimate of the propagation speed of acoustic waves based on a spatial filtering technique (DS beamformer). We have demonstrated that for a small number of sensors scenario the performance of the new estimator is similar to the MLE, and it coincides with the MLE when the array has only 3 sensors.

We have also extended the proposed method to the case of wideband signals. Under certain conditions of relative aperture and signal-to-noise ratio for every frequency which contributes to the estimate, we can use the new method in broadband applications as it is the case of the forward and backward wave propagating along a car exhaust.

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