BLIND ON-LINE DETECTION OF EQUALISATION ERRORS IN DIGITAL COMMUNICATIONS

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ABSTRACT

Equalisation errors result from discrepancies between transmitted symbols and their estimates at the channel equaliser output in a digital communication system. This paper presents an on-line test to detect the occurrence of equalisation errors without direct access to the channel input. The test draws on the observation that for linear time-invariant (LTI) channels the relationship between transmitted symbol estimates generated by the equaliser and the noisy channel output can be represented by an underlying linear time-invariant model if and only if no equalisation errors are present in the sequence of transmitted symbol estimates. The presence of equalisation errors renders this relationship time-varying, of which the occurrence is detected by the proposed on-line test using the recursive least squares (RLS) algorithm. Simulation studies corroborate the good detection performance of the test.

1. INTRODUCTION

We propose a blind on-line test for detection of equalisation errors arising from discrepancies between transmitted symbols and their estimates at the equaliser output in a data communication system. The test aims to detect jump changes in estimated channel impulse response derived from the noisy channel output and decision device output observations.

Off-line tests for detecting equalisation errors or convergence to open-eye local minima have been proposed in the literature [1, 2, 3, 4]. In this paper we depart from the usual off-line approach to testing for equalisation errors by constructing an on-line test with reduced complexity. The new test has relatively robust performance in the face of correlated channel inputs and coloured Gaussian channel noise with known variance but unknown covariance. The on-line test is predicated on the criterion developed in [4]. Rather than computing the least squares parameter estimates from observation blocks as in [4], we make use of the recursive least squares algorithm to construct our on-line test.

The paper is organised as follows. Section 2 gives a formal description of the testing problem along with a statement of the major assumptions made throughout the paper. In Section 3 we develop the on-line test criterion, implement it as an on-line test, and discuss its performance and the effect of uncertainties of the test parameters. Computer simulations are presented in Section 4.

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2. PROBLEM STATEMENT

Fig. 1 shows the set-up for the detection problem. The channel input sequence $\{u(k)\}$, $k \in \mathbb{Z}^+$, is drawn from a digital modulation signal constellation. In order to simplify the exposition we will restrict our attention to pulse amplitude modulated channel inputs, i.e.

$$u(k) \in \mathcal{S} = \{\pm 1, \ldots, \pm(M-3), \pm(M-1)\} \quad \forall k$$

where $M > 0$ is an even integer. The channel is assumed to be a finite impulse response (FIR) system with impulse response of length P

$$\boldsymbol{h} = [h_0, h_1, \ldots, h_{P-1}]^T$$

where ^{T} is the transpose operator. The channel will be assumed to be time-invariant during the test interval. The channel noise $n(k)$ is a stationary and possibly coloured Gaussian random process with zero mean and known variance σ_n^2. The knowledge of the noise autocorrelation is not required even if the channel noise is coloured. No assumptions are made about the type of equaliser used. In this sense, the on-line test is applicable to any equaliser structure, be it linear or nonlinear.

We will say that the channel input symbol estimates $\{\hat{u}(k)\} \in \mathcal{S}$ are in error if they cannot be related to $\{u(k)\}$ by means of a time-invariant delay and/or phase shift (sign reversal). This translates into the following objective for error-free equalisation:

$$\hat{u}(k) = \Gamma\, u(k-\Delta) \quad \forall k. \tag{1}$$

Here Γ is a real constant with $|\Gamma| = 1$ and Δ denotes the equalisation delay.

Our objective is to detect the channel input estimates violating the equalisation objective in (1) (which we will simply call *equalisation errors*) without resort to the channel input at any time. In this sense, the task of detecting equalisation errors needs to be carried out in a blindfolded manner.

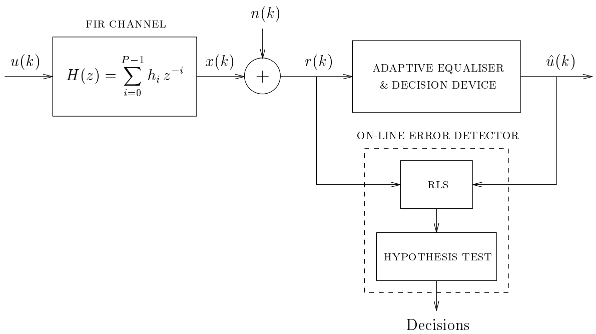

Figure 1: Set-up for on-line detection of equalisation errors

3. ON-LINE TEST FOR EQUALISATION ERRORS

3.1. Overview of the Test Criterion

To test for the presence of equalisation errors we will use the following blind test criterion [4]:

Property. *Provided that the channel $H(z)$ is linear time-invariant (LTI) and all finite length subsequences of the channel input sequence occur with nonzero probability, the equaliser and decision device combination has an underlying LTI model with the following response from $\hat{u}(k)$ to $r(k)$*

$$r(k) = \sum_{i=0}^{P-1} \frac{h_i}{\Gamma} \hat{u}(k+\Delta-i) + n(k) \quad \forall k \tag{2}$$

if and only if $\{\hat{u}(k)\}$ is error-free. If, on the other hand, $\{\hat{u}(k)\}$ contains equalisation errors, then (2) takes the form

$$r(k) = \sum_{i=0}^{P-1} v_i(k)\, \hat{u}(k+\Delta-i) + n(k) \quad \forall k \tag{3}$$

where $v_i(k) = h_i/\gamma(k-i)$, $i = 0, 1, \ldots, P-1$ is time-varying with $\gamma(k) \triangleq \hat{u}(k+\Delta)/u(k)$.

In the light of the above property an obvious approach to testing for the presence of equalisation errors is to test for time variations in the parameters of the underlying linear model from $\hat{u}(k)$ to $r(k)$. This approach is further pursued in the next subsection.

3.2. On-line Test

The on-line test constructed here will be based on RLS estimation of the underlying linear model parameters from $\hat{u}(k)$ to $r(k)$. In addition to satisfying the conditions stipulated by the property in Section 3.1, the channel input will be assumed to be *persistently exciting* in order that the RLS algorithm does not suffer from numerical instability [5].

Least Squares Formulation:

Let us assume that no equalisation errors occur at the decision device output and that the channel length P and the equalisation delay Δ are known *a priori*. The noisy channel output $r(k)$ can then be written in terms of $\hat{u}(k)$ as

$$\boldsymbol{r} = \boldsymbol{A}\boldsymbol{v} + \boldsymbol{n} \tag{4}$$

where $\boldsymbol{r}$ denotes the channel output regressor $\boldsymbol{r} = [r(k-\Delta), r(k-\Delta-1), \ldots, r(1)]^T$, $\boldsymbol{A}$ is the Hankel data matrix $\boldsymbol{A} = [\hat{\boldsymbol{u}}(k), \hat{\boldsymbol{u}}(k-1), \ldots, \hat{\boldsymbol{u}}(1+\Delta)]^T$ with $\hat{\boldsymbol{u}}(k) \triangleq [\hat{u}(k), \hat{u}(k-1), \ldots, \hat{u}(k-P+1)]^T$, $\boldsymbol{v}$ is the channel parameter vector scaled by $1/\Gamma$ $\boldsymbol{v} = [h_0/\Gamma, h_1/\Gamma, \ldots, h_{P-1}/\Gamma]^T$, and $\boldsymbol{n}$ is the channel noise vector $\boldsymbol{n} = [n(k-\Delta), n(k-\Delta-1), \ldots, n(1)]^T$.

The least squares estimate of $\boldsymbol{v}$ is obtained by solving the *normal equations* $\boldsymbol{A}^T\boldsymbol{A}\hat{\boldsymbol{v}} = \boldsymbol{A}^T\boldsymbol{r}$ for $\hat{\boldsymbol{v}}$. In RLS, a weighted version of the normal equations is solved recursively using the matrix inversion lemma [6]. Upon settlement of the initial parameters, the RLS estimate of $\boldsymbol{v}$ is given by $\hat{\boldsymbol{v}}(k) = \boldsymbol{P}(k)\boldsymbol{\theta}(k)$ where $\boldsymbol{P}(k)$ is the inverse of the $P \times P$ autocorrelation matrix of the decision device output

$$\boldsymbol{P}(k) = \left(\sum_{i=1+\Delta}^{k} \lambda^{k-i} \hat{\boldsymbol{u}}(i)\hat{\boldsymbol{u}}^T(i) + \epsilon\lambda^k \boldsymbol{I} \right)^{-1}$$

and $\boldsymbol{\theta}(k) = \sum_{i=1+\Delta}^{k} \lambda^{k-i}\hat{\boldsymbol{u}}(i) r(i-\Delta)$ is the crosscorrelation vector with $0 < \lambda < 1$ denoting the forgetting factor and ϵ the soft-constrained initialisation constant [6], which is a small positive number. We will subsequently assume that k is sufficiently large and ϵ is very small so that the term $\epsilon\lambda^k\boldsymbol{I}$ in $\boldsymbol{P}(k)$ can be neglected safely.

Our primary interest lies in the statistical properties of the RLS update term $\boldsymbol{\delta}(k) = \hat{\boldsymbol{v}}(k) - \hat{\boldsymbol{v}}(k-1)$. In terms of $\boldsymbol{v}$, the RLS parameter estimates can be written

$$\hat{\boldsymbol{v}}(k) = \boldsymbol{v} + \boldsymbol{P}(k) \sum_{i=1+\Delta}^{k} \lambda^{k-i} \hat{\boldsymbol{u}}(i) n(i-\Delta).$$

RLS Update with No Equalisation Errors:

Under the assumption of no equalisation errors, the RLS update term becomes

$$\boldsymbol{\delta}(k) = \big(\lambda\boldsymbol{P}(k) - \boldsymbol{P}(k-1)\big) \sum_{i=1+\Delta}^{k-1} \lambda^{k-1-i} \hat{\boldsymbol{u}}(i) n(i-\Delta) + \boldsymbol{P}(k)\hat{\boldsymbol{u}}(k) n(k-\Delta) \tag{5}$$

where the summation in the first term is the only quantity that cannot be readily evaluated. If, however, λ is chosen very close to 1 so that $\boldsymbol{P}^{-1}(k-1) \gg \hat{\boldsymbol{u}}(k)\hat{\boldsymbol{u}}^T(k)/\lambda$, then we can write to a good approximation $\lambda\boldsymbol{P}(k) - \boldsymbol{P}(k-1) \approx \boldsymbol{0}$ for large k, which results in $\boldsymbol{\delta}(k) \approx \boldsymbol{P}(k)\hat{\boldsymbol{u}}(k)n(k-\Delta)$. Then $\|\boldsymbol{\delta}(k)\|_2^2$ can be approximately written as $\|\boldsymbol{\delta}(k)\|_2^2 \approx n^2(k-\Delta)\sum_{i=1}^{P}\big(\boldsymbol{p}_i^T\hat{\boldsymbol{u}}(k)\big)^2$ where $\boldsymbol{p}_i^T$ denotes the ith row of $\boldsymbol{P}(k)$.

Normalisation of $\|\boldsymbol{\delta}(k)\|_2^2$ by its approximate mean value yields

$$T(k) = \frac{\|\boldsymbol{\delta}(k)\|_2^2}{\sigma_n^2 \sum_{i=1}^{P}\big(\boldsymbol{p}_i^T\hat{\boldsymbol{u}}(k)\big)^2} \approx \frac{n^2(k-\Delta)}{\sigma_n^2} \tag{6}$$

which is distributed approximately according to central chi-square with one degree of freedom (χ_1^2). Note that in (6) no knowledge of the channel noise autocorrelation is required even though the channel noise may be coloured.

RLS Update with Equalisation Errors:

If $\hat{u}(k)$ is in error, the RLS update term becomes

$$\boldsymbol{\delta}(k) \approx \boldsymbol{P}(k)\hat{\boldsymbol{u}}(k)n(k-\Delta) + h_0 \begin{bmatrix} \frac{u(k-\Delta)}{\hat{u}(k)} - \Gamma \\ \boldsymbol{0} \end{bmatrix}_{P\times 1}. \tag{7}$$

The normalised squared ℓ_2 norm of $\boldsymbol{\delta}(k)$ now takes the form

$$T(k) \approx \frac{n^2(k-\Delta)}{\sigma_n^2} + \frac{h_0^2\left(\frac{u(k-\Delta)}{\hat{u}(k)} - \Gamma\right)^2}{\sigma_n^2 \sum_{i=1}^{P} \left(\boldsymbol{p}_i^T \hat{\boldsymbol{u}}(k)\right)^2} + \frac{2h_0 \boldsymbol{p}_1^T \hat{\boldsymbol{u}}(k) n(k-\Delta)\left(\frac{u(k-\Delta)}{\hat{u}(k)} - \Gamma\right)}{\sigma_n^2 \sum_{i=1}^{P} \left(\boldsymbol{p}_i^T \hat{\boldsymbol{u}}(k)\right)^2}.$$

On-Line Hypothesis Test:

At this stage it is instructive to make a comparison between the (approximate) distributions of $T(k)$ under no equalisation errors and in the presence of an equalisation error. In the former case, $T(k)$ has unit mean, while in the latter $T(k)$ has the following mean conditioned on $\boldsymbol{P}(k)$ and $\hat{\boldsymbol{u}}(k)$

$$E\{T(k) \mid \boldsymbol{P}(k), \hat{\boldsymbol{u}}(k)\} = 1 + \frac{h_0^2\left(\frac{u(k-\Delta)}{\hat{u}(k)} - \Gamma\right)^2}{\sigma_n^2 \sum_{i=1}^{P} \left(\boldsymbol{p}_i^T \hat{\boldsymbol{u}}(k)\right)^2} \geq 1. \tag{8}$$

In the hypothesis testing framework we can construct the following on-line threshold test to distinguish between the null hypothesis (H_0: $\hat{u}(k)$ is correct) and the alternative hypothesis (H_1: $\hat{u}(k)$ is in error)

$$T(k) \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{9}$$

where the test threshold η is determined by $P_{\mathrm{FA}} = \Pr\{\chi_1^2 > \eta\} = \alpha$ where α is the significance level.

3.3. Unknown Channel Length and Equalisation Delay

Let P' denote the assumed channel length and Δ' the assumed equalisation delay (as opposed to their true values). As shown in [4, Proposition 2], the following inequalities are required to hold in order for the on-line test to work properly: (i) $\Delta' \geq \Delta$ and (ii) $P' \geq P - \Delta + \Delta'$. If $\Delta' > \Delta$ and (ii) holds, equalisation errors can be detected with a minimum delay of $\Delta' - \Delta$ [7]. Blind estimation of a lower bound on Δ is addressed in [2, 4].

3.4. Detection Performance and Goodness of Approximation for the Test Statistic Distribution

The detection performance of the on-line test is affected *inter alia* by the magnitude of h_0 and the channel noise variance σ_n^2 (see (8)). After the first incidence of an equalisation error, the error propagates through the entries of $\boldsymbol{v}$. Thus even if the error goes unheeded at its first occurrence, say, because of a small $|h_0|$, the chances of its detection may improve at consecutive time instants as it goes through larger components of $\boldsymbol{h}$.

	True	Estimated $\lambda = 0.970$	Estimated $\lambda = 0.999$
$E\{T(k) \mid H_0\}$	1	1.1574	1.0191
$\mathrm{Var}\{T(k) \mid H_0\}$	2	2.7528	2.0629
P_{FA}	0.050	0.0679	0.0498
	0.025	0.0370	0.0255
	0.010	0.0176	0.0106
	0.005	0.0084	0.0057
	0.001	0.0025	0.0012

Table 1: True and estimated mean, variance and P_{FA} of $T(k)$ under H_0.

If $n(k)$ is white Gaussian, then the conditional mean of $\|\boldsymbol{\delta}(k)\|_2^2$ given $\boldsymbol{P}(k)$, $\boldsymbol{D}(k) = \lambda\boldsymbol{P}(k) - \boldsymbol{P}(k-1)$ and $\boldsymbol{A}$ is

$$\begin{aligned} E\{\|\boldsymbol{\delta}(k)\|_2^2 \mid \boldsymbol{P}(k), \boldsymbol{D}(k), \boldsymbol{A}\} = \sigma_n^2 \sum_{i=1}^{P} \Big(& \left(\boldsymbol{p}_i^T \hat{\boldsymbol{u}}(k)\right)^2 \\ & + \left(\boldsymbol{d}_i^T \hat{\boldsymbol{u}}(k-1)\right)^2 + \lambda^2 \left(\boldsymbol{d}_i^T \hat{\boldsymbol{u}}(k-2)\right)^2 + \cdots \\ & + \lambda^{2(k-\Delta)} \left(\boldsymbol{d}_i^T \hat{\boldsymbol{u}}(1+\Delta)\right)^2 \Big) \end{aligned} \tag{10}$$

where $\boldsymbol{d}_i^T$ denotes the ith row of $\boldsymbol{D}(k)$. The true value of the conditional mean is clearly greater than the approximation $\sigma_n^2 \sum_{i=1}^{P} \left(\boldsymbol{p}_i^T \hat{\boldsymbol{u}}(k)\right)^2$. If $n(k)$ is coloured, the conditional mean in (10) will include additional crossterms, which could either increase or decrease the true mean with respect to the approximation depending on the noise autocorrelation. Thus, the $\boldsymbol{d}_i^T \hat{\boldsymbol{u}}(j)$, $j = k-1, k-2, \ldots, 1+\Delta$, should be negligible compared to $\boldsymbol{p}_i^T \hat{\boldsymbol{u}}(k)$ to ensure a good approximation.

4. COMPUTER SIMULATIONS

In the simulations we have assumed a 4-level PAM channel input which is generated by a Markov chain with transition probability matrix

$$\boldsymbol{\Pi} = \begin{bmatrix} 0.3 & 0.2 & 0.4 & 0.1 \\ 0.2 & 0.4 & 0.2 & 0.2 \\ 0.4 & 0.2 & 0.3 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.6 \end{bmatrix}$$

where the (i,j)th entry is $\Pr\{\hat{u}(k) = s_j \mid \hat{u}(k-1) = s_i\}$, $i,j = 1,\ldots,4$ with $[s_1, s_2, s_3, s_4] = [-3, -1, 1, 3]$. The Markov chain has equiprobable initial states.

The channel is a nonminimum-phase FIR system

$$H(z) = 0.1 - 0.6z^{-1} + 0.4z^{-2} + 0.2z^{-3}$$

whose output $x(k)$ is corrupted by additive coloured Gaussian noise $n(k)$ with $\sigma_n^2 = R_n(0) = 0.0120$, $R_n(\pm 1) = -0.0048$, $R_n(\pm 2) = 0.0020$, and $R_n(\tau) = 0$, $|\tau| > 2$.

The (approximate) null hypothesis distribution of $T(k)$ has been simulated using $P' = 10$ and $\Delta' = 0$. Table 1 lists the estimated mean, variance and probability of false alarm values of the test statistic $T(k)$ along with their true values under the assumption $T(k) \sim \chi_1^2$ for 10,000 observations and two values of the RLS weighting factor. Note that

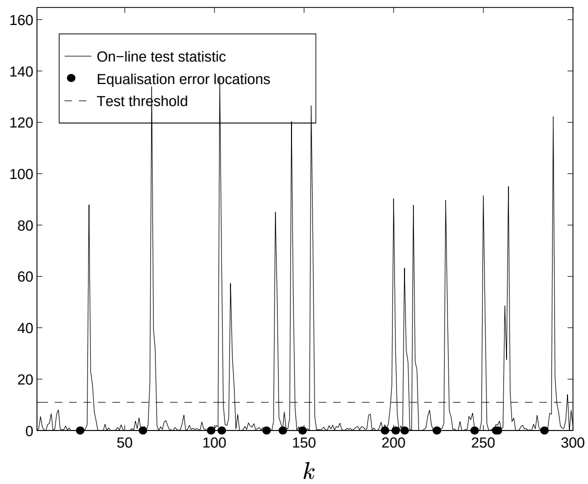

Figure 2: Plot of on-line test statistic $T(k)$ versus k and equalisation errors

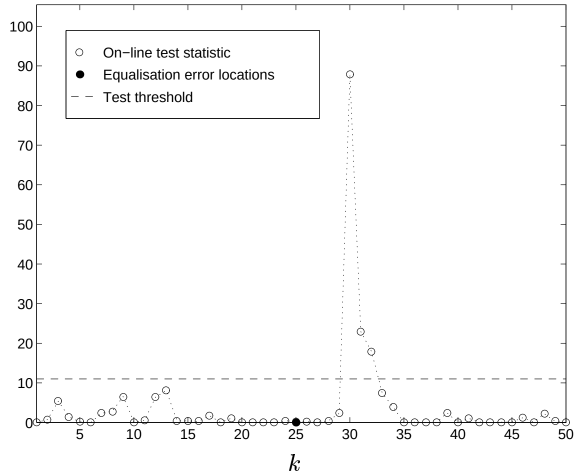

Figure 3: Close-up of test statistic $T(k)$ for the first 50 symbols

$\lambda = 0.999$, being very close to 1, yields a more accurate approximation than $\lambda = 0.970$ as expected.

The detection performance of the on-line test has been simulated using $H(z)$ and a 30-tap linear equaliser $\{\theta_i\}$ followed by a 4-level quantiser, yielding CLEM=0.8777 (i.e. the eye is open). The equalisation delay is $\Delta = 11$, and the test parameters were chosen as $P' = 10$, $\Delta' = 15$ and $\eta = 11$ ($\alpha = 9 \times 10^{-4}$). The RLS parameters were $\epsilon = 10^{-5}$ and $\lambda = 0.995$. Fig. 2 shows a plot of the on-line test statistic for 400 transmitted symbol estimates. The test statistic for the first 50 symbols are shown in Fig. 3 where it is seen that equalisation errors are detected with a delay of 5 symbols which is larger than $\Delta' - \Delta$. The reason for this observed extra delay is that $|h_0|^2 \ll |h_1|^2$ (see (8)). Fig. 4 shows the tested equaliser output sequence $y(k) = \sum_{i=0}^{L-1} \theta_i r(k-i)$. It is clear that the eye pattern test would fail to detect the individual equalisation errors.

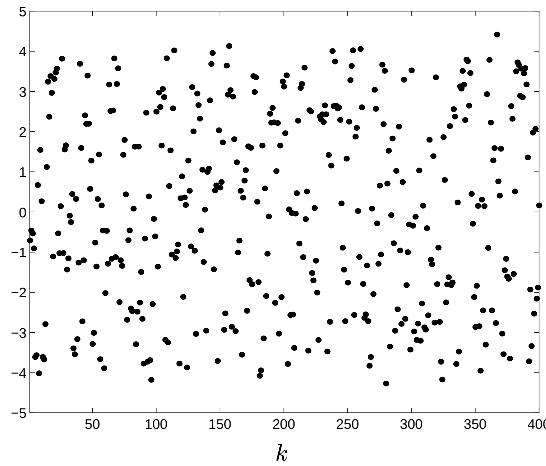

Figure 4: Plot of equaliser outputs during the testing interval

5. CONCLUSION

A blind on-line test for equalisation errors has been developed, which has a low computational complexity of $O(P'^2)$. The detection performance of the test was analysed and demonstrated in simulation studies. While the well-known eye pattern test may fail to detect equalisation errors in scenarios we dealt with, the proposed on-line test exhibits high detection probability for isolated equalisation errors with a small delay depending on the channel impulse response. One possible improvement would be to employ parameter resetting after the detection of an equaliser error to avoid false alarms due to the exponential decay of $T(k)$.

6. REFERENCES

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