

MULTI-CHANNEL MLSE EQUALIZER FOR GSM USING A PARAMETRIC CHANNEL MODEL

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ABSTRACT

In this paper, we propose a novel algorithm for the MLSE equalizer for the GSM system. Specifically, we use a parametric model for the channel, to obtain a modified Viterbi Equalizer which we refer to as the Parametric Channel-Viterbi Equalizer (PC-VE). In contrast to the conventional Viterbi Equalizer with a FIR channel description, the PC-VE avoids the linear approximation error and has a lower computational complexity. The proposed algorithm is applicable to both single and multi-antenna receivers. Some simulation results that illustrate the performance of the proposed algorithm are presented.

1. INTRODUCTION

Large variety of receiver structures have been proposed for the GSM system. The Limiter-Discriminator-Integrator(LDI) detector [1, 2] and the differential detector [3, 4] are simple, but they may not work effectively in the severe channel environment of multi-path propagation and Doppler spread. A MMSE (+DFE) coherent detector [2, 5] has also been suggested in the literature. However, this is considered sub-optimal due to the nonlinear nature of the GMSK modulation and its noise enhancement problem. The MLSE detector described in [6, 7] assumes linearity in the modulation scheme, and estimates an FIR approximation of the channel before applying the Viterbi Algorithm. An ambiguity function is involved in the channel estimation [6] which causes an increase in the length of the equivalent FIR channel. A 16-state Viterbi Equalizer is usually sufficient to implement the MLSE detectors in the mobile channels [7]. It can be simplified by using the M-algorithm [7]. In order to further simplify the MLSE detectors, several reduced-state MLSE [6, 8, 9] approaches have been proposed. By ignoring the phase states of the GMSK signal, the number of states can be reduced to one fourth [6]. By introducing a linear equalizer or a decision feedback equalizer before the MLSE detector, the channel length can be shortened [8, 9]. These are all, however, sub-optimal algorithms with acceptable degradation.

GMSK is a non-linear modulation scheme that does not have a pulse-shaping function for the complex baseband signal. Hence, for a discrete channel transmitting GMSK modulated signals (as suggested by [10]), an exact FIR equivalent model can not be constructed. On the other hand, if we assume a multi-ray propagation model and use path

delays, path directions of arrival (DOA's) and complex path amplitudes to describe the channel, we can avoid the linear approximation, which is normally used in the FIR channel description, and proceed with an exact MLSE computation.

We describe the signal model in Section 2. In Section 3, we present the proposed PC-VE algorithm. We present some simulation results in Section 4, and Section 5 concludes the paper.

2. SIGNAL AND CHANNEL MODEL

In this section, we describe the signal and channel model. We will first introduce a multi-path channel model and GMSK modulation. Next, we will describe the GMSK states that we obtain with the approximated phase pulse-shaping function, state-transition diagram and the inherited trellis structure. Finally, we present a sampled signal model.

2.1. Multi-path Model and GMSK Modulation

The radio channel in a wireless communication system is often characterized by multi-path propagation. Thus, the signals received at the output of the antenna array can be expressed as

$$\mathbf{x}(t) = \sum_{k=1}^p \mathbf{a}(\theta_k) \beta_k(t) s(t - \tau_k) + v(t) \quad (1)$$

where $\mathbf{x}(t)$ is the received signal vector, $\mathbf{a}(\theta_k)$ is the array response vector to a signal arriving from direction θ_k , β_k is the time-varying path amplitude which includes both the propagation loss and the signal fading due to the Doppler spread, s is the transmitted complex baseband signal and τ_k is the propagation delay of k^{th} path.

A GMSK modulated signal can be represented as

$$s = e^{j\theta(t)}, \quad \theta(t) = \theta_0 + \sum_i d_i \phi(t - iT) \quad (2)$$

where θ_0 is an unknown initial phase and $d_i \in \{1, -1\}$ are the differentially encoded data bits. In terms of the raw data bits $b_i \in \{0, 1\}$, $d_i = 1 - 2(b_i \oplus b_{i-1})$, where \oplus denotes modulo 2 addition. The phase pulse-shaping function $\phi(t)$ is given by

$$\phi(t) = \frac{\pi}{2} [G(\frac{t}{T}) - G(\frac{t}{T} - 1)] \quad (3)$$

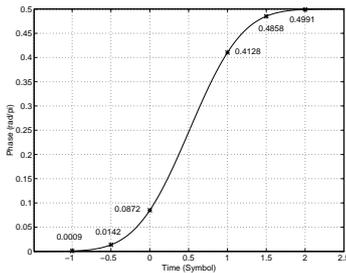


Figure 1. Phase pulse-shaping function.

where $G(x)$ is defined as

$$G(x) = x \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-t^2/2\sigma^2} dt + \frac{\sigma}{\sqrt{2\pi}} e^{-x^2/2\sigma^2} \quad (4)$$

with $\sigma = 0.441624$ [10]. For convenience, we absorb θ_0 into $\{\beta_k\}$ and set its value in (2) equal to zero. This will not affect the statistical properties of $\{\beta_k\}$.

2.2. Approximated GMSK States

To implement the Viterbi Algorithm, we need to have finite number of states for the GMSK modulated signal. This is possible if we truncate the phase pulse-shaping function $\phi(t)$ to a smaller number of symbol intervals. We note from Fig. 1 that $\phi(t)$ is close to zero for $t < -\frac{T}{2}$ and to $\frac{\pi}{2}$ for $t > \frac{3T}{2}$. We, therefore, approximate $\phi(t)$ as

$$\hat{\phi}(t) = \begin{cases} 0 & \text{for } t < -\frac{T}{2} \\ \phi(t) & \text{for } -\frac{T}{2} < t < \frac{3T}{2} \\ \frac{\pi}{2} & \text{for } t > \frac{3T}{2} \end{cases} \quad (5)$$

To show that this approximation is not severe, consider the eye diagrams shown in Fig. 2 for $\theta_0 = 0$. Observe that the difference between the eye with exact $\phi(t)$ and that with the approximation ($\hat{\phi}(t)$) is almost zero over most of the time interval excepting in the regions highlighted by rectangular boxes. Even in these regions, the difference is negligible.

Following the above approximation and noting that the GMSK modulated signals are of unit modulus, we see from Fig. 2(b) that there are only 8 possible transmitted complex signals during the symbol period $[(k-0.5)T, (k+0.5)T]$ (one set of 8 complex signals for even k and another set of 8 for odd k), where k is an arbitrary integer. We define these complex signals (i.e., amplitudes of the in-phase and quadrature pairs) as states, each of which is a function of the path delay τ . In the following, we refer to the states corresponding to k even as even states and those corresponding to k odd as odd states. These states, as a function of τ , form a temporal manifold. For the sake of compactness, we denote these states as $A, A', B, B', C, C', D, D', E, F, G, H, I, J, K, L$. [11] These states and their transition paths will constitute a trellis diagram, as shown in Fig. 3.

2.3. Sampled Signal Model

We oversample the received signals by a factor of two, and process the received data in the digital domain rather than using continuous-time matched filtering, thereby reducing the complexity substantially. Assume an antenna array with

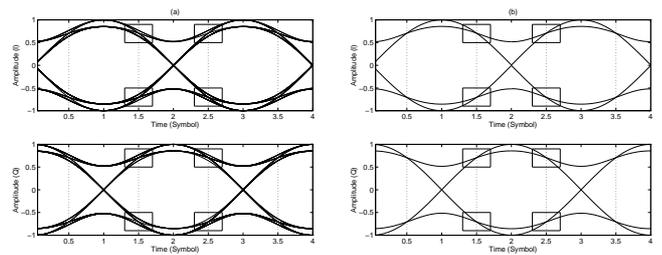


Figure 2. Eye diagram of the GMSK modulated signal: (a) with exact $\phi(t)$, (b) with approximation $\hat{\phi}(t)$.

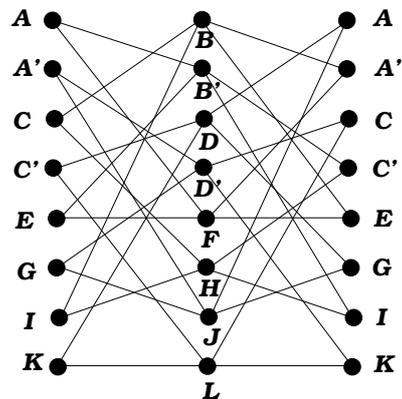


Figure 3. Trellis diagram of the GMSK modulated signal.

m elements and a multi-path channel with p independently-fading paths. We also assume that the fading amplitudes do not change significantly during the a data burst of the GSM system. Thus, we can express N snapshots of $x(t)$ (see (1)) in a matrix form as

$$\mathbf{X} = \mathbf{A} \text{diag}(\boldsymbol{\beta}) \mathbf{S} + \mathbf{V} \quad (6)$$

$m \times N \quad m \times p \quad p \times p \quad p \times N \quad m \times N$

where \mathbf{X} is the sampled received signal and $\mathbf{A} = [\mathbf{a}(\theta_1)\mathbf{a}(\theta_2)\dots\mathbf{a}(\theta_p)]$. The k^{th} row of \mathbf{S} represents the sampled values of $s(t - \tau_k)$ and β_i denotes the complex fading amplitude of the i^{th} path. Note that $s(t - \tau_k)$ is fully described by the path delay and the data sequence.

3. PARAMETRIC CHANNEL VITERBI EQUALIZER (PC-VE)

In this section, we develop the PC-VE algorithm. This algorithm needs the estimates of the path DOA's, path delays and complex path amplitudes. We obtain these estimates using a recently-developed subspace-based method, TSTMUSIC [12]. Once these estimates are available, the complex path amplitudes can be determined following a least squares approach given below.

3.1. Estimation of the Complex Path Amplitudes

During the training period, (6) can be expressed as

$$\mathbf{X}_{tr} = \mathbf{A} \text{diag}(\boldsymbol{\beta}) \mathbf{S}_{tr} + \mathbf{V} \quad (7)$$

where the subscript '*tr*' refers to the signals during the training. We use the least squares (LS) criterion to solve for $\boldsymbol{\beta}$, under the assumption that the noise is white both temporally and spatially. The LS estimate is given as

$$\begin{aligned} \hat{\boldsymbol{\beta}} &= [((\mathbf{A} \text{diag}(\mathbf{s}_1))^T \dots (\mathbf{A} \text{diag}(\mathbf{s}_N))^T)^T]^\dagger \text{vec}(\mathbf{X}_{tr}) \\ &= \mathcal{A}^\dagger \text{vec}(\mathbf{X}_{tr}) \end{aligned} \quad (8)$$

where \mathbf{s}_k is the k^{th} column of \mathbf{S}_{tr} , ' \dagger ' denotes the pseudo-inverse.

3.2. Viterbi Algorithm for Data Retrieval

After the channel is estimated, we apply the Viterbi Algorithm to demodulate the data bits. We oversample the received signal by a factor of two, and order the paths according to their delays such that $0 \leq \tau_1 \leq \tau_2 \leq \dots \leq \tau_p$. In the Viterbi Algorithm, we search for a trellis path h , corresponding to a possible transmitted signal $\bar{\mathbf{S}}_h$, which minimizes the metric $\|\mathbf{X} - \bar{\mathbf{X}}_h\|_F^2$, where $\bar{\mathbf{X}}_h = \mathbf{A} \text{diag}(\hat{\boldsymbol{\beta}}) \bar{\mathbf{S}}_h$, $\bar{\mathbf{S}}_h$ is the reconstructed GMSK-modulated signal, and $\|\cdot\|_F$ denotes the Frobenius norm. Please refer to [11] for details.

We modified the conventional vector Viterbi Equalizer associated with FIR channel description (FIR-VE) in two ways. First, we used a much smaller number of parameters, $p + \frac{2p}{B}$, to describe the channel (B is the burst update rate of θ_k and τ_k , and p is the number of paths), in contrast to the FIR channel description which needs mMr parameters where M is the length of the equivalent FIR channel measured in the symbol intervals and, m and r are as defined before. Second, the branch metric used in the PC-VE is based on the finite number of states according to the temporal manifold, instead of running convolutions between the FIR channel and the data bits.

3.3. Algorithm Summary

Here, we summarize the proposed PC-VE algorithm:

- step 1** Use a subspace-based algorithm to estimate the path delays and the path DOA's.
- step 2** Construct a table consisting of all $16r$ possible received complex signals ($8r$ for even and $8r$ for odd states) for each path.
- step 3** For each burst, apply (8) to estimate the complex path amplitudes $\hat{\boldsymbol{\beta}}$ during the training period.
- step 4** Data estimation using the Viterbi Algorithm —
 - (a) Begin at the last bit of the training sequence.
 - (b) Initialize the node metric, $n_{h,t}$, as zero for the trellis path starting from the last bit (first bit in the case of backward search) of the training sequence and infinity otherwise.
 - (c) Use the table obtained from **step 2** and $\hat{\boldsymbol{\beta}}$ from **step 3** to reconstruct possible received signals as $\bar{\mathbf{X}}_{h,t} = \mathbf{A} \text{diag}(\hat{\boldsymbol{\beta}}) \bar{\mathbf{S}}_{h,t}$ for both branches of each node.

- (d) Calculate the branch metric $m_{h,t} = \|\mathbf{X}_t - \bar{\mathbf{X}}_{h,t}\|_F^2$.
- (e) Pick one survival branch for each node which minimizes the node metric $n_{h,t} = n_{h,t-1} + m_{h,t}$.
- (f) Go back to **step 4(c)** to continue on to the next state unless the end of the burst has been reached.
- (g) Pick the survival path with the lowest node metric and retrieve the data bits.
- (h) Repeat the same procedure as in **step 4(b) - step 4(g)**, but do it backward starting from the first bit of the training sequence.

step 5 Update the path DOA's and the path delays if necessary.

step 6 Go back to **step 3** and repeat the procedure for the next burst.

4. SIMULATION RESULTS

In this section, we present some simulation results to compare the performance of two Viterbi Equalizers, FIR-VE [6] and PC-VE (proposed algorithm). We also compare these results with the theoretical lower bound [11].

Our simulations were done as follows. For the FIR-VE algorithm, we used 16 states as suggested in [7]. The channel models considered were those of TU and HT as given in [10], assuming the corresponding DOA's as $-25^\circ, -15^\circ, -5^\circ, 5^\circ, 15^\circ$, and 25° . We used a uniform linear array with 6 isotropic elements, spaced half wavelength apart. The complex path fading amplitudes were modeled as a zero mean complex circularly distributed Gaussian random variables that are mutually independent. They were held constant over each burst, but they were varied independently from burst to burst. The additive noise was modeled as white complex circularly distributed Gaussian with zero mean and real and imaginary parts each having variance σ^2 , where σ^2 was varied to give the desired SNR. Also, we assumed perfect carrier synchronization. We oversampled the received signals by a factor of two (except when otherwise specified) and averaged the BER over 500 independent trials.

Fig. 4 shows BER vs. SNR for the TU channel model with one antenna. The figure compares the BER performance of the PC-VE with that of the FIR-VE for both baud-rate sampling and oversampling by a factor of two. Fig. 5 gives the above results for the HT channel. In order to test the robustness of the proposed algorithm, we evaluated its performance for three different cases: i) with true DOA's and path delays, ii) with true path delays perturbed by $\pm \frac{T}{20}$ randomly, keeping the DOA's unchanged, and iii) with paths 1, 3, and 5 ignored in the TU case, and with paths 2, 4, 5 and 6 ignored in the HT case. The case iii) was chosen to mimic a diffused multi-path channel with a few dominant paths.

We make the following observations from the results.

1. The FIR-VE with baud rate sampling always causes BER flooring, which is clearly due to the channel mismatch. On the other hand, the PC-VE performs well with baud rate sampling. With an oversampling factor of two, the PC-VE always performs better than the

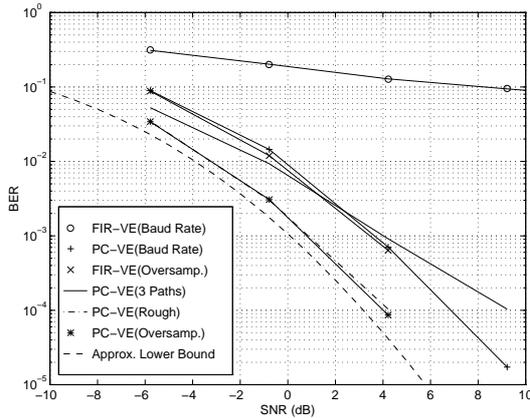


Figure 4. BER vs. SNR for the TU channel model

FIR-VE by approximately 2-4 dB, even-though the PC-VE uses only half the number of states as compared to the FIR-VE.

2. We studied the sensitivity of the algorithm to errors in parameter values. For the case ii) where an error of $\pm \frac{T}{20}$ was introduced in the delay values, the degradation is almost invisible. For the case iii) where some paths were totally ignored, the degradation of the BER turns out to be far less severe than expected; the PC-VE still performs better than the FIR-VE. This observation confirms the robustness of the proposed algorithm to errors in the delay estimates and the channel modeling.

5. CONCLUSION

In this paper, we presents a parametric channel Viterbi Equalizer (PC-VE) for the GSM system. The computational complexity of this algorithm is less than that of conventional Viterbi Equalizer with FIR channel description. Further, its performance has been shown to be better by about 2 to 4 dB, in general. The results also show that the proposed algorithm is robust to errors in the parameter estimates and the channel modeling.

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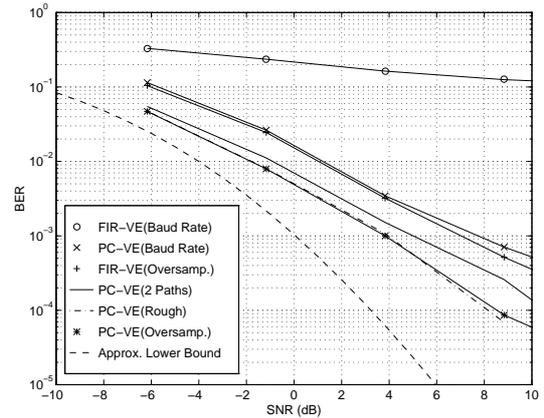


Figure 5. BER vs. SNR for the HT channel model

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