BLIND PREDICTIVE DECISION-FEEDBACK EQUALIZATION VIA THE CONSTANT MODULUS ALGORITHM

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ABSTRACT

The noise predictive structure of DFE is attractive for the equalization of the coded modulation signals. In this paper, a blind predictive constant modulus (CM) decision feedback equalizer (PCM-DFE) is presented and analyzed. The PCM-DFE employs the CM linear equalizer as its forward filter and a feedback filter that optimizes the CM cost of the decision variable. It is shown that for any fixed forward filter with reasonable small residue intersymbol interference, the CM cost function for the feedback filter is approximately convex and its global minimum can be approximated in closed form. We demonstrate that the convergence rate of the feedback filter is similar to the least mean square (LMS) algorithm used in the nonblind design. We show that the PCM-DFE performs better than the nonblind linear MMSE equalizer in simulations.

1. INTRODUCTION

The importance of using the decision feedback equalization was highlighted in the seminal paper by Price [10] who showed that with the decision feedback equalization, the additional SNR required to achieve channel capacity is *independent* of the channel spectrum. Since the publication of [1] by Austin, the design of DFE with training signals or when the channel is known is well understood. However, there are applications where it is desirable that DFE can be designed "blindly", *i.e.,* without having the access to the training signal.

Two design criteria are commonly used in blind DFE. The decision directed (DD) approach minimizes the MSE between the decision variable and detected symbols. Unfortunately, DD is known to have undesired local minima [8, 6], and effects of error propagation may be catastrophic. The DFE can also be designed based on the constant modulus (CM) criterion. The CMA-DFE and its variations introduced in [5, 7, 9, 3, 2] are distinct alternatives to DD. Similar to CMA applied to linear equalization, the existing CMA-DFE illustrated in Figure 1 minimizes the CM cost of the decision variable:

$$\{F_c(z), B_c(z)\} = \arg \min_{F(z),B(z)} E(|y_k|^2 - r_p)^2. \qquad (1)$$

Although using the constant modulus criterion above to design both the forward filter $F(z)$ and the feedback filter $B(z)-1$ at the same time often gives satisfactory performance, such a design criterion has a flaw that may cause problem in its application. Specifically, one can achieve global minimum by setting $F(z)=0$ and $B(z)=1+z^{-k}$ for some integer k. Although this problem may be circumvented by imposing some forms of constraints on the forward filter as shown in [9], it is not clear whether such types of DFE will perform better than its linear counter parts.

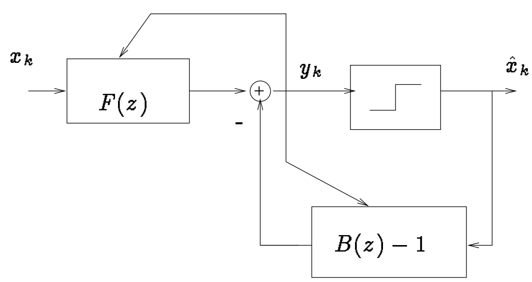

Figure 1: Schematic of CMA-DFE

In this paper, we investigate the *predictive constant modulus decision-feedback equalization* proposed in [12] where it was shown that the MMSE-DFE can be closely approximated by a linear CM equalizer as the forward filter and a feedback filter obtained by either the spectral factorization of the received signal, or the constant modulus algorithm. Derived from the MMSE (noise) predictive DFE, the new approach avoids the complications of using CM criterion (1). Perhaps more importantly, the predictive DFE structure is more suitable for trellis-coded signals [4].

We focus our attention to the design of PCM-DFE by using the constant modulus cost for both the forward and feedback filters. Such an approach has clear computational

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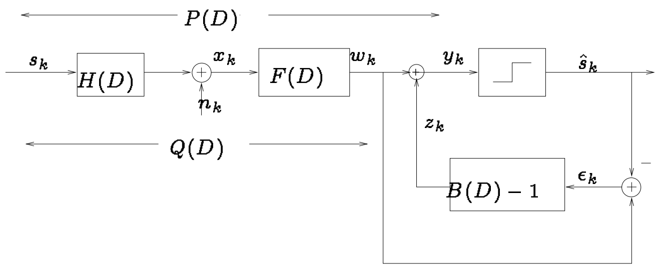

Figure 2: The Predictive structure of DFE

advantage over the spectral factorization approach. We prove that, if the forward filter (designed by any criterion) provides a reasonable compensation to ISI, the convergence of feedback filter is global and it can be obtained analytically. We demonstrate further that the convergence property for the feedback filter designed by CMA is almost the same as that using LMS algorithm with training. The benefit of using DFE is illustrated by simulation where the existing and the new blind DFE perform significantly better than the *non-blind* linear MMSE equalizer.

Model and Assumptions

We consider a discrete-time baseband model

$$x_t = \sum_k h_k s_{t-k} + n_t, \tag{2}$$

where $\{s_t\}$, $\{n_t\}$ and $\{x_t\}$ are sequences of the source, the noise and the received signal. The impulse response of the composite channel $\{h_k\}$ includes the propagation channel, the transmitter and receiver front-end filters.

We make the usual assumptions: (A1) $\{s_k\}$ is BPSK signal with zero mean, i.i.d., and unit variance; (A2) $\{n_k\}$ is zero mean, i.i.d., Gaussian with variance σ^2 , and is independent of $\{s_k\}$; (A3) $\{h_k\}$ has a stable inverse. (A4) The detected symbols are correct in all the DFE schemes.

2. THE PREDICTIVE CM DFE

The predictive DFE shown in Figure 2 consists of a linear equalizer as its forward filter. The estimation error ϵ_k of the linear equalizer output is obtained using the detected symbols $\hat{s}_k$, and is filtered by the feedback filter to provide increase of SNR at the decision variable y_k. A geometrical explanation of this structure is given in [12].

It can be shown that when the forward filter is implemented using IIR filters, the MMSE-DFE implemented by the predictive structure is equivalent to that using the standard structure shown in Figure 1. The forward filter $F_m(D)$ is a MMSE linear equalizer and $B_m(D) - 1$ is an optimal linear predictor that can be obtained using the spectral factorization of the received process.

The predictive MMSE-DFE immediately suggests the use of CM cost. Shown in Figure 3, the predictive CM-DFE has essentially the same structure of the predictive MMSE-DFE. Similar to the predictive MMSE-DFE, the forward and feedback filters are designed separately. Note that Constant Modulus equalization has an arbitrary constant phase ambiguity, *i.e.*, the output of the CM equalizer is an estimate of the source symbol with an unknown rotation. In order to use detected symbols in the estimation, it is necessary to compensate the phase ambiguity such that the output CM equalizer has the same constellation as the source symbols*.

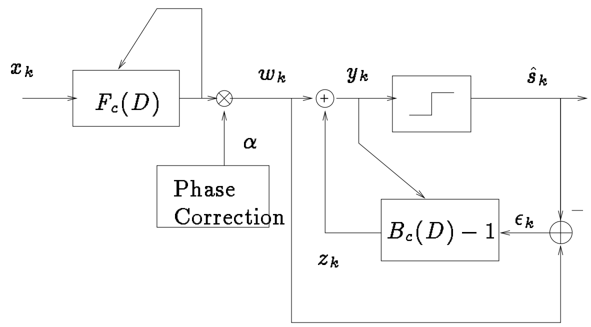

Figure 3: The Predictive CM-DFE

The CM Forward Filter

The design of the forward filter is based on a recent result [13] that establishes the connection between the constant modulus equalizer and the MMSE equalizer. It is shown that in the immediate neighborhood of the MMSE equalizer, there is a constant modulus equalizer, a fact that has been observed in simulations by many including Godard in his original paper. In light of this result, it was shown that the constant modulus $\mathbf{f}_c$ equalizer is approximately a scaled version of the MMSE equalizer $\mathbf{f}_m$ [12]:

$$\mathbf{f}_c = (1 - \frac{3}{2}\mathcal{E})\mathbf{f}_m + O(\mathcal{E}^3), \tag{3}$$

where $\mathcal{E}$ is the MSE of the optimal MMSE linear equalizer. The extra MSE of the CM equalizer over the MMSE equalizer is given by $\frac{9}{4}\mathcal{E}^2 + O(\mathcal{E}^6)$.

The CM Feedback Filter

There are two basic approaches to designing the feedback filter. We shall consider the more practical one that uses the constant modulus criterion:

$$B_c(D) = \arg\min_{B(D)} E(|y_k|^2 - 1)^2. \tag{4}$$

While it is not immediately clear why such a criterion is legitimate, it turns out that not only this criterion is easy to implement, at high SNR, it is also optimal in the sense of minimizing the MSE at y_k. We show in the following analysis that, at high SNR, the design of the feedback filter based on the constant modulus cost function does not have the problem of local minima. Further, the stochastic gradient implementation of (4) converges as fast as the

*Note that this does not mean that the phase ambiguity has to be eliminated completely. For example, if QPSK is used, the phase ambiguity is reduced to multiples of $\frac{\pi}{2}$

(nonblind) LMS update of the feedback filter when training is available. At low SNR, however, this approach may be affected by detection error propagation and the existence of local minima.

3. CONVERGENCE ANALYSIS

For the predictive DFE, the design of forward and feedback filters are done separately. The convergence analysis of the forward filter is difficult, and few results are available. The analysis of the feedback filter, which is the focus of this section, turns out to be much simpler and relatively strong results can be obtained. In what follows, we assume that the forward filter has been obtained in some way not necessarily through the constant modulus criterion.

Consider the predictive constant modulus DFE shown in Figure 2 with the fixed forward filter $F_c(D)$, *e.g.,*. a CMA linear equalizer. Let $\mathbf{q} = [q_k] \leftrightarrow Q(D) \triangleq F_c(D)H(D)$ be the equalized channel, and $\bar{\mathbf{q}} = [\bar{q}_k] \leftrightarrow \bar{Q}(D) \triangleq Q(D) - 1$ be the residue interference. With the standard assumption that detected symbols are correct, we have the following system equations

$$\begin{aligned} w_k &= s_k + \underbrace{(Q(D)-1)}_{\bar{Q}(D)} s_k + F_c(D) n_k & (5) \\ \epsilon_k &= w_k - \hat{s}_k = \bar{Q}(D) s_k + F_c(D) n_k & (6) \\ y_k &= w_k + (B(D)-1)\epsilon_k & (7) \\ &= s_k + B(D)(F(D) n_k + \bar{Q}(D) s_k). & (8) \end{aligned}$$

The design of the feedback filter $B(D)-1$ should be such that the power of $B(D)(F(D)n_k + \bar{Q}(D)s_k)$ is minimized. We investigate next the relationship between the CM feedback filter $B_c(D) - 1$ obtained from (4) and the MMSE feedback filter $B_m(D) - 1$. Unlike in [12], we do not assume that the forward filter has completely eliminated ISI.

It can be shown that the CM cost function for the design of the feedback filter $\mathbf{b}$ is given by

$$\begin{aligned} J_c(\mathbf{b}) &= 3||\mathbf{b}||^4_{\mathbf{R}} + 4||\mathbf{b}||^2_{\mathbf{R}} + 12\mathbf{b}^t\mathbf{g}||\mathbf{b}||^2_{\mathbf{R}} \\ &\quad + 8\mathbf{b}^t\mathbf{g} + 12(\mathbf{b}^t\mathbf{g})^2 - 2||\mathbf{p}||^4_4 + 2, \end{aligned} \quad (9)$$

where $\mathbf{R}$ is the covariance matrix of ϵ_k, is the impulse response of the transfer function $P(D)$ between the source and the decision variable, $\mathbf{g}$ is the anti-causal part of $\bar{q}_k$, *i.e.*, $g_k = \bar{q}_k u_{-k}$ and u_k is the unit step function. For a reasonable design of the forward filter, $\bar{q}_k$ should be small. It can be shown that

$$J_c(\mathbf{b}) = 3||\mathbf{b}||^4_{\mathbf{R}} + 4||\mathbf{b}||^2_{\mathbf{R}} + 12\mathbf{b}^t\mathbf{g}||\mathbf{b}||^2_{\mathbf{R}} + O(||\bar{\mathbf{q}}||^3) \quad (10)$$

In the following analysis, we shall ignore $O(||\bar{\mathbf{q}}||^3)$.

Define

$$\mathbf{b} = \begin{pmatrix} 1 \\ \mathbf{b}_1 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} r_0 & \mathbf{r}_1^t \\ \mathbf{r}_1 & \mathbf{R}_1 \end{pmatrix}, \mathbf{g} = \begin{pmatrix} g_0 & \mathbf{g}_1 \end{pmatrix} \quad (11)$$

we have

$$\mathbf{b}^t\mathbf{R}\mathbf{b} = r_0 + 2\mathbf{b}_1^t\mathbf{r}_1 + \mathbf{b}_1^t\mathbf{R}_1\mathbf{b}_1. \quad (12)$$

Let $\mathbf{b}_1 = \underbrace{-\mathbf{R}^{-1}\mathbf{r}_1}_{\mathbf{b}_{1o}} + \Delta\mathbf{b}_{1o}$, we have

$$\mathbf{b}^t\mathbf{R}\mathbf{b} = \underbrace{r_0 - \mathbf{r}_1^t\mathbf{R}_1^{-1}\mathbf{r}_1}_{\mathcal{E}_m} + \Delta\mathbf{b}_{1o}^t\mathbf{R}_1\Delta\mathbf{b}_{1o} \quad (13)$$

Now let $\mathbf{R}_1$ have SVD $\mathbf{R}_1 = \mathbf{U}\Sigma^2\mathbf{U}^t$, and define

$$\beta = \Sigma\mathbf{U}^t\Delta\mathbf{b}_{1o}, \ \gamma_0 = g_0 + \mathbf{b}_{1o}^t\mathbf{g}_1, \ \gamma = \Sigma^{-1}\mathbf{U}^t\mathbf{g}, \quad (14)$$

we have

$$\mathbf{b}^t\mathbf{R}\mathbf{b} = \mathcal{E}_m + ||\beta||^2, \quad \mathbf{b}^t\mathbf{g} = \gamma_0 + \gamma^t\beta \quad (15)$$

Substituting the above into (10), we have

$$\begin{aligned} J_c(\beta) &= 3(\mathcal{E}_m + ||\beta||^2)^2 + 4(\mathcal{E}_m + ||\beta||^2) \\ &\quad + 12(\gamma_0 + \gamma^t\beta)(\mathcal{E}_m + ||\beta||^2) \end{aligned} \quad (16)$$

We show next that under mild conditions on the forward filter, the convergence of the feedback filter is global.

Theorem 1 *For a given forward filter, if*

$$||\gamma||^2 \le \frac{\mathcal{E}_m + 2\gamma_0 + \frac{2}{3}}{3}, \quad (17)$$

then the CM cost function is convex with global minimum $\beta_c = \mu\gamma$ *where* μ *is the real root of*

$$||\gamma||^2\mu^3 + 3||\gamma||^2\mu^2 + (\mathcal{E}_m + 2\gamma_0 + \frac{2}{3})\mu + \mathcal{E}_m = 0. \quad (18)$$

The extra mean square error of the decision variable y_k *is given by* $\mu^2||\gamma||^2$.

4. SIMULATIONS

In this simulation, we examine two issues. First, can the predictive CMA-DFE perform better than the linear CMA and linear MMSE? Second, what is the convergence property of the feedback filter?

The channel considered in our simulation is the channel b from [11, page 616], which has a spectral null. The DFE has 32 forward taps and 2 feedback taps.

4.1. BER Performance

In applying the PCM-DFE, 100,000 symbols are used in which 50,000 data samples are used to update the forward CMA filter first and the rest of data are used to update both filters. Figure 4 shows the BER performance comparison with nonblind MMSE, MMSE-DFE and blind linear equalizer CMA. We observe that while the PCM-DFE performs better than the two linear equalizers, it has a considerable gap between the optimal MMSE-DFE. One reason is that the convergence rate of CMA for the forward filter is poor for this channel. We have verified that when the true CM cost function is used, with considerable iterations, a center spike initialization will lead to the minimum very close to the MMSE equalizer, in which case the gap between the PCM-DFE and the MMSE-DFE will vanish. We also observed that, for this channel, the direct implementation of CMA-DFE presented in [9] appear to have better performance over the PCM-DFE.

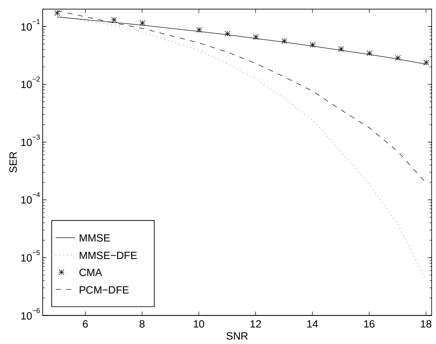

Figure 4: BER Performance

4.2. Convergence of the Feedback Filter

It is interesting to examine the convergence of the feedback filter. Analysis suggests that, at high SNR, CMA updates should perform in a similar way as the non-blind LMS algorithm. This is confirmed at SNR=30dB in Figure 5 where both CMA and non-blind LMS converge to the optimal feedback filter. The contour of the true CM cost function indicates the convexity of the cost as proved in Theorem 1 for the approximated cost function. The optimal feedback filter $b_o = [0.7487, -0.1391]$ is marked by an across that appears to be close to the minimum of the true CM cost function. Marked as a circle is the solution obtained from equation (11) and (12) $b_c = [0.7097, -0.1316]$. The extra MSE of the PCM-DFE is 2.3×10^{-4}.

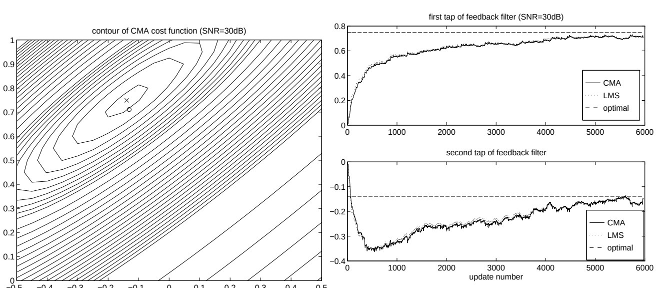

Figure 5: SNR=30dB.
$\mu_{CMA} = 0.005 \mu_{LMS} = 0.014$

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