IDENTIFICATION OF TIME-VARYING LINEAR CHANNELS

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ABSTRACT

In this paper, we present a new method for on-line identification of time-varying FIR channels. Two conditionally coupled estimators are proposed. In both cases an augmented-state adaptive Kalman filter is employed for tracking the time-varying channel and estimating the mean channel response. Coupled to the Kalman filter is an algorithm for estimating the parameters of the underlying auto-regressive (AR) model which describes the time evolution of the channel. For the first coupled estimator, we propose a new recursive least squares algorithm for estimation of these AR parameters directly from the channel observations. An alternative algorithm based on estimation of the channel covariance is used in the second coupled estimator. A simulation example demonstrates the performance of the proposed estimators.

1. INTRODUCTION

Characterization and estimation of time-varying linear channels are important to a wide variety of applications, including digital communications, mobile radio, radar and sonar [1, 2, 3]. The time-varying nature of the channels can cause difficulties in the design of optimal receivers, since both time-selective and frequency-selective fading occur. On-line estimates of the channel are needed for equalization and detection.

The estimators described in this paper are suitable for providing channel estimates for an equalizer where periodic re-training is used. During training, the channel inputs are known at the receiver and are used with the observed channel output to form estimates of the underlying channel parameters. The equalizer uses these estimates to approximate the effect of the channel during reception of unknown inputs. Decision feedback equalization is also possible, where a digital input estimator (such as Viterbi algorithm or maximum a posteriori (MAP) decoder) can be used to condition the channel estimators presented here.

In this paper, we consider a discrete-time model for a randomly time-varying linear channel. The channel is modelled as a finite impulse response (FIR) tapped-delay line filter. An auto-regressive (AR) model is employed to describe the time evolution of the channel taps. This channel model is applicable to general fading channels, as will be discussed in Section 2. Using the AR model, we propose a Kalman filter for tracking the complex-valued channel taps. Importantly, we augment the state of the Kalman filter to estimate the mean value of the channel taps. This is presented in Section 3. In Section 4., we present our recursive least squares (RLS) algorithm for estimating the parameters of the AR process. In a similar manner to [4], we couple the RLS estimator with the Kalman filter to jointly estimate the channel tap means and the AR parameters (Section 5.). As an alternative to our RLS estimation of the AR parameters, we consider a method in Section 6. in which the AR parameters are derived from the channel tap covariance via the Yule–Walker equation. We reformulate the algorithm proposed by Tsatsanis *et al*. [5] for estimating the covariance to explicitly incorporate channels with non-zero-mean, and couple this with the augmented state Kalman filter for channels with unknown mean response. The performance of estimators presented in this paper is demonstrated with a simulation example in Section 7.

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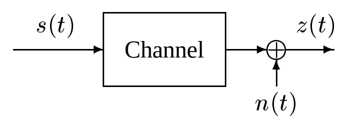

Figure 1. Observed Channel

2. CHANNEL MODEL

Consider the discrete-time transmission system shown in Figure 1, where the channel is modelled as a tapped-delay line filter which is finite in extent, and consequently has a finite impulse response. We choose the channel taps to have complex coefficients, and for convenience, the tap spacing is the same as the input signal sampling rate [6]. However, the algorithms presented in this paper also directly apply to channels modelled with fractional spacing.

By representing baseband signals in complex notation, the received signal is given by:

$$z(t) = \sum_{\epsilon=0}^{L-1} s(t-\epsilon)\, h(t,\epsilon) + n(t)\ , \quad t = 0, 1, 2\ldots \qquad (1)$$

where L is the number of channel taps, $s(t) \in \mathbb{C}$ (the set of complex numbers) is the input at time t, and $h(t,\epsilon) \in \mathbb{C}$ is the ϵ^{th} channel tap at time t. The observation noise, $n(t)$, is assumed to be zero-mean white Gaussian noise, with known variance σ_n^2 (i.e. the real and imaginary components each have variance $0.5\,\sigma_n^2$ [7]). In practice, there is some fixed delay, m, from transmitter to receiver. Without loss of generality, the delay is assumed to be zero in (1).

The time-varying impulse response of the channel, $h(t,\epsilon)$, consists of a mean response component, $\overline{h}(\epsilon)$, and zero-mean randomly time-varying component, $\tilde{h}(t,\epsilon)$:

$$h(t,\epsilon) = \overline{h}(\epsilon) + \tilde{h}(t,\epsilon) \tag{2}$$

A channel with non-zero-mean taps occurs when there are fixed scatterers or reflectors, giving rise to $\overline{h}(\epsilon)$. The random component results from the changing physical characteristics of the transmission medium. In this paper, we assume the random component, $\tilde{h}(t,\epsilon)$, has Gaussian real and imaginary components [8].

An auto-regressive process of order R models the time-varying components of the channel taps. The choice of R is a trade-off between the accuracy of the model and the difficulty in estimating its parameters. Now, consider the following equation:

$$\tilde{\mathbf{h}}(t) = \mathbf{F_1}\,\tilde{\mathbf{h}}(t-1) + \cdots + \mathbf{F_R}\,\tilde{\mathbf{h}}(t-R) + \mathbf{u}(t) \tag{3}$$

where $\tilde{\mathbf{h}}(t) = \left[\tilde{h}(t,0)\,\cdots\,\tilde{h}(t,L-1)\right]^T$, and ^{T} is the transpose operator. The generating noise for the random process, $\mathbf{u}(t) = [u(t,0)\,\cdots\,u(t,L-1)]^T$ is assumed to be zero-mean i.i.d. complex Gaussian, with known covariance σ_u^2. The AR parameters contained in the matrices $\mathbf{F_1}\ldots\mathbf{F_R}$ are complex in general. However, if it is assumed that the real and imaginary parts of $h(t,\epsilon)$ are independent, these AR parameters will be real. For stability and wide-sense stationarity of the AR process, the roots of the characteristic equation, $\mid \mathbf{I} - \sum_{\tau=1}^{\mathbf{R}} \mathbf{F}_\tau \mathbf{z}^{-\tau} \mid = \mathbf{0}$, must lie inside the unit circle of the z-plane [9]. Such a sub-class of channels is commonly known as wide-sense stationary (WSS) [2].

For some channels, the taps may be assumed independent. These channels are referred to as having uncorrelated scatterers (US) [2]. This assumption can be built into the AR model by appropriate choice of elements in the matrices, $\mathbf{F_1}\ldots\mathbf{F_R}$ (i.e. by setting cross-terms to zero). Note that in this paper, we consider a general WSS channel of which the commonly assumed WSS–US [1, 2], is only one possible subclass.

3. CHANNEL TAP COEFFICIENT ESTIMATION AND TRACKING

In this section, we note that when the AR parameters contained in the matrices $\mathbf{F_1}$ to $\mathbf{F_R}$ in (3) are known, a Kalman filter can be used to track the time-varying component of the channel taps. In addition, we augment the state to include the channel tap means $\overline{\mathbf{h}} = \left[\overline{h}(0)\,\cdots\,\overline{h}(L-1)\right]^T$. In doing so, we benefit from the fact that the Kalman filter takes into account the joint effect of the time varying component, $\tilde{\mathbf{h}}$, and the mean, $\overline{\mathbf{h}}$, of the channel response in the observations. This has advantages over alternative techniques which may estimate $\tilde{\mathbf{h}}$ and $\overline{\mathbf{h}}$ separately. Augmentation of the state vector also allows *adaptive* estimation of the channel mean, $\overline{\mathbf{h}}$, which may vary over time due to changes in the operating environment.

The augmented state vector for the Kalman filter is $\mathbf{x}(t) = \left[\tilde{\mathbf{h}}(t),\cdots,\tilde{\mathbf{h}}(t-R+1),\overline{\mathbf{h}}\right]^T$. Thus, we rewrite (3) in the following form:

$$\mathbf{x}(t) = \mathbf{A}\,\mathbf{x}(t-1) + \mathbf{v}(t)\;,\quad t=0,1,2\ldots \tag{4}$$

where $\mathbf{v}(t) = [\mathbf{u}(t),\mathbf{0},\cdots,\mathbf{0}]$, and:

$$\mathbf{A} = \begin{bmatrix} \mathbf{F_1} & \mathbf{F_2} & \cdots & \mathbf{F_R} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & & \vdots & \vdots \\ \vdots & & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & & \cdots & \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{5}$$

By defining a vector of input signal samples $\mathbf{s}(t) = [\,s(t),\; s(t-1),\;\cdots,\; s(t-L+1)]^T$ and the observation matrix $\mathbf{C}(t) = \left[\mathbf{s}^T(t),\mathbf{0},\cdots,\mathbf{0},\mathbf{s}^T(t)\right]$, the observation equation (1) for the system can be written:

$$z(t) = \mathbf{C}(t)\,\mathbf{x}(t) + n(t)\;,\quad t=0,1,2\ldots \tag{6}$$

With this formulation, a complex Kalman filter [9, pg. 321] can be used to provide on-line estimates of the channel tap offsets, $\tilde{h}(t,\epsilon)$, and the channel tap means, $\overline{h}(\epsilon)$, which define the channel impulse response at each time instant.

4. AR PARAMETER ESTIMATION

In the previous section the AR parameters incorporated into the Kalman filter were assumed known. Here, we detail our algorithm for estimating the matrices of AR parameters, $\mathbf{F_1}$ to $\mathbf{F_R}$. Assuming knowledge of the channel taps, $h(t,\epsilon)$, we use the observations, $z(t)$, and the inputs $s(t)$, in a recursive least squares algorithm to estimate the AR parameters directly.

Consider the observation equation (6). By substituting (4), and rewriting the AR parameters in $\mathbf{A}$ into a column vector, $\mathbf{f} = [(\mathrm{vec}\,\mathbf{F_1})^{\mathrm{T}},\;\cdots,\;(\mathrm{vec}\,\mathbf{F_R})^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{\mathrm{RL}^2\times 1}$, we can rewrite equation (6) as a linear function of $\mathbf{f}$:

$$\begin{aligned} z(t) &= \mathbf{s}^T(t)\,\mathbf{F_1}\,\tilde{\mathbf{h}}(t-1) + \cdots + \mathbf{s}^T(t)\,\mathbf{F_R}\,\tilde{\mathbf{h}}(t-R) \\ &\quad + \mathbf{s}^T(t)\,\overline{\mathbf{h}} + \mathbf{s}^T(t)\,\mathbf{u}(t) + n(t) \\ &= \mathbf{T}(t)\,\mathbf{f} + \mathbf{s}^T(t)\,\overline{\mathbf{h}} + \mathbf{s}^T(t)\,\mathbf{u}(t) + n(t) \end{aligned} \tag{7}$$

where

$$\begin{aligned} \mathbf{T}(t) \stackrel{\triangle}{=} [&\mathbf{s}^T(t)\,\tilde{h}(t-1,0),\;\mathbf{s}^T(t)\,\tilde{h}(t-1,1),\;\cdots, \\ &\mathbf{s}^T(t)\,\tilde{h}(t-1,L-1),\;\cdots,\cdots,\mathbf{s}^T(t)\tilde{h}(t-R,L-1)] \end{aligned}$$

Thus:

$$\begin{aligned} e(t) = z(t) - \mathbf{s}^T(t)\,\overline{\mathbf{h}} &= \mathbf{T}(t)\,\mathbf{f} + \mathbf{s}^T(t)\,\mathbf{u}(t) + n(t) \\ &= \mathbf{T}(t)\,\mathbf{f} + \text{filtered noise} \end{aligned} \tag{8}$$

This way of manipulating the observation equation is the key to our estimation of the AR parameters. We can now apply a complex recursive least squares (RLS) algorithm [9] to (8) in order to estimate $\mathbf{f}$.

5. COUPLED ESTIMATOR

In Section 3., we presented an augmented state Kalman filter to track channel tap offsets, $\tilde{\mathbf{h}}(t)$, and to estimate the mean value of the channel taps, $\overline{\mathbf{h}}$. The AR parameters contained in the $\mathbf{F}$ matrices were assumed known. In Section 4., we presented a new RLS method for estimating the AR parameters, assuming the values of the channel taps were known. The estimators are combined in the proposed coupled estimator structure of Figure 2.

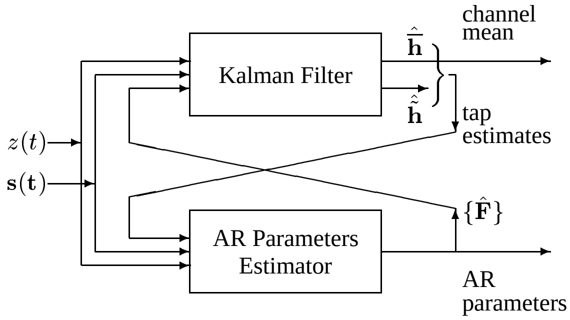

Figure 2. Coupled Kalman Filter and AR Parameter Estimator

The coupled Kalman filter and the RLS estimator jointly estimate the channel taps, $h(t, \epsilon)$, of (2), and the AR parameters contained in the $\mathbf{F}$ matrices of (3). At each sample, the channel output, $z(t)$, and the input, $s(t)$, are used by the Kalman filter to estimate the time-varying channel offsets, $\hat{\tilde{h}}(t, \epsilon)$, as well as the mean values of the channel taps, $\hat{\overline{h}}(\epsilon)$. These estimates are then used in the RLS estimation of the AR parameters, and hence the $\hat{\mathbf{F}}$ matrices. A new estimate of $\mathbf{A}$ is then formed for use by the Kalman filter at the next sample.

In general, the issue of convergence for coupled estimators remains open. It is clear, however, for the coupled estimator of Figure 2, that if the Kalman filter provides the correct values for the coupled parameters to the AR parameter estimator, then the AR parameter estimator will converge to the correct estimates, and *vice versa*. Simulation results in Section 7. (and [4]) indicate that the coupled estimates converge in practice if appropriately initialized.

6. CHANNEL COVARIANCE ESTIMATION

For a wide-sense stationary (WSS) channel, the channel covariance is $k_{\tilde{h}}(t, s, \epsilon, \eta) = E[\,\tilde{h}(t, \epsilon)\,\tilde{h}^*(s, \eta)\,] = k_{\tilde{h}}(\tau, \epsilon, \eta)$, where * denotes complex conjugate and $\tau = t - s$. This covariance is directly related to the AR parameters via the Yule–Walker equation [9]. The Yule–Walker equation is:

$$[\mathbf{K_1} \cdots \mathbf{K_R}] = [\mathbf{F_1} \cdots \mathbf{F_R}] \begin{bmatrix} \mathbf{K_0} & \cdots & \mathbf{K_{R-1}} \\ \vdots & \ddots & \vdots \\ \mathbf{K_{-R+1}} & \cdots & \mathbf{K_0} \end{bmatrix} \qquad (9)$$

where $\mathbf{K}_\tau = E[\,\tilde{\mathbf{h}}(t+\tau)\tilde{\mathbf{h}}^+(t)\,]$ ($^+$ is the Hermitian operator). Note that $k_{\tilde{h}}(\tau, \epsilon, \eta) = k^*_{\tilde{h}}(-\tau, \eta, \epsilon)$, and therefore $\mathbf{K}_{-\tau} = \mathbf{K}_\tau{}^+$.

If the channel tap covariance matrices $\mathbf{K_0} \ldots \mathbf{K_R}$ are known, (9) can be used to determine the AR parameters. In this section we consider an algorithm recently proposed by Tsatsanis *et al*. for estimating the channel tap covariance. The method, as described in [5], separately estimates the channel mean and the channel covariance. In this paper, we reformulate the estimation of the channel covariance to explicitly incorporate the channel mean. When the channel mean is unknown, it can be estimated with a Kalman filter, as in Section 3..

Consider the conditional expectation of the output covariance, $E\,[\,[z(t+\tau) - \overline{z}(t+\tau)]\,[z(t) - \overline{z}(t)\,]^* \;\, | \; \mathbf{s}(t)]$, where the output from the mean channel response at time t is $\overline{z}(t) = \sum_{\epsilon=0}^{L-1} s(t-\epsilon)\,\overline{h}(\epsilon)$. From (1), and using the fact that $\tilde{h}(t, \epsilon)$ is independent of $n(t)$, we obtain:

$$\begin{aligned} E & \left[[z(t+\tau) - \overline{z}(t+\tau)]\,[z(t) - \overline{z}(t)\,]^* \;\, | \; \mathbf{s(t)} \right] \\ = & \; E \Bigg[[\sum_{\epsilon=0}^{L-1} s(t+\tau-\epsilon)\,h(t+\tau, \epsilon) - \\ & \qquad \sum_{\epsilon=0}^{L-1} s(t+\tau-\epsilon)\,\overline{h}(\epsilon) + n(t+\tau)\,] \\ & \qquad [\sum_{\eta=0}^{L-1} s(t-\eta)\,h(t, \eta) - \\ & \qquad \sum_{\eta=0}^{L-1} s(t-\eta)\,\overline{h}(\eta) + n(t)\,]^* \Bigg] \\ = & \sum_{\epsilon,\eta=0}^{L-1} s(t+\tau-\epsilon)\,s^*(t-\eta)\,k_{\tilde{h}}(\tau, \epsilon, \eta) + \sigma_n^2 \delta(\tau) \end{aligned} \qquad (10)$$

where $\delta(\tau)$ is the Kronecker delta function. Note that the assumption of a WSS channel has been used (i.e. $E[\,\tilde{h}(t+\tau, \epsilon)\tilde{h}^*(t, \eta)\,] = k_{\tilde{h}}(\tau, \epsilon, \eta)\,)$. Thus for each time t, we have:

$$\begin{aligned} E[\,[\,z(t+\tau) - \overline{z}(t+\tau)\,][\,z(t) - \overline{z}(t)\,]^* \;\, | \; \mathbf{s}(t)\,] = \\ \phi^+(t, \tau)\,\theta(\tau) + \sigma_n^2 \delta(\tau) \end{aligned} \qquad (11)$$

where

$$\begin{aligned} \theta(\tau) &= [k_{\tilde{h}}(\tau, 0, 0), \cdots, k_{\tilde{h}}(\tau, 0, L-1), \\ & \qquad \cdots, \cdots, k_{\tilde{h}}(\tau, L-1, L-1)]^T \\ \phi^+(t, \tau) &= [s(t+\tau)\,s^*(t), \cdots, s(t+\tau)\,s^*(t-(L-1)) \\ & \qquad \cdots, \cdots, s(t+\tau-(L-1))\,s^*(t-(L-1))] \end{aligned}$$

Following [5], we use the instantaneous value for the output covariance as an approximation to the expected value. However, here we include the mean response. Therefore, (11) becomes:

$$[\,z(t+\tau) - \overline{z}(t+\tau)\,][\,z(t) - \overline{z}(t)\,]^* - \sigma_n^2 \delta(\tau) \simeq \phi^+(t, \tau)\,\theta(\tau) \qquad (12)$$

Now, (12) provides a basis for estimating the channel tap covariance, $\theta(\tau)$. We note that using the instantaneous approximation for the output covariance actually ensures a stationary point on the likelihood surface [10].

Now, at the arrival of each data sample, the mean responses $\overline{z}(t)$ and $\overline{z}(t+\tau)$ are calculated using the mean values of the channel taps. Several RLS algorithms are employed to update the estimates of the covariance, $\hat{\theta}(\tau)$, one for each $\tau = 0 \ldots R$. These covariance estimates are then used in the Yule–Walker equation (9) to derive estimates for the AR parameters.

When the mean values of the channel taps are not known, we propose to couple this estimator with the Kalman filter of Section 3. in the same way as the new RLS estimator for the AR parameters (Figure 2).

7. SIMULATION EXAMPLE

In this section, we use the example in [5], to demonstrate the performance of the coupled estimators. A channel with $L = 2$ taps is considered. An AR model of order $R = 1$ is used to model the zero-mean time-varying components. Therefore $\tilde{\mathbf{h}}(t) = \mathbf{F_1}\tilde{\mathbf{h}}(t-1) + \mathbf{u}(t)$. The process noise is chosen to be $\sigma_u^2 = 0.005$, and the regression matrix is:

$$\mathbf{F_1} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.8 \\ -0.5 & 0.3 \end{bmatrix} \qquad (13)$$

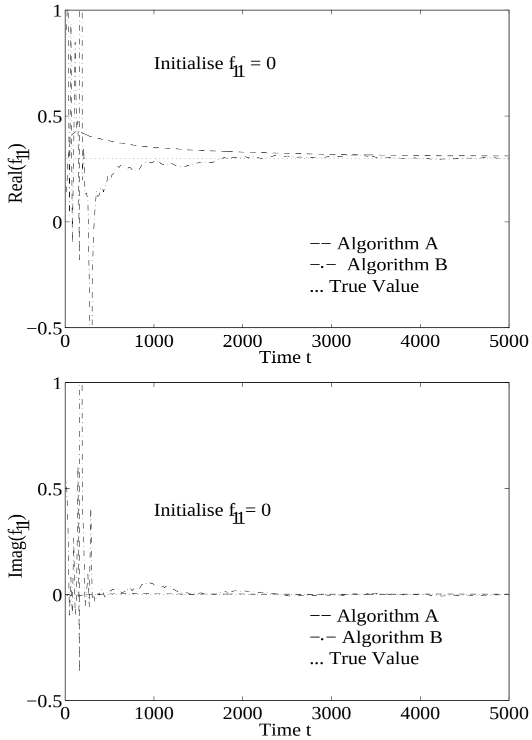

Figure 3. Estimation of $\mathrm{Real}\{\mathrm{f}_{11}\}$ **and** $\mathrm{Imag}\{\mathrm{f}_{11}\}$**. Average performance, excluding failures**

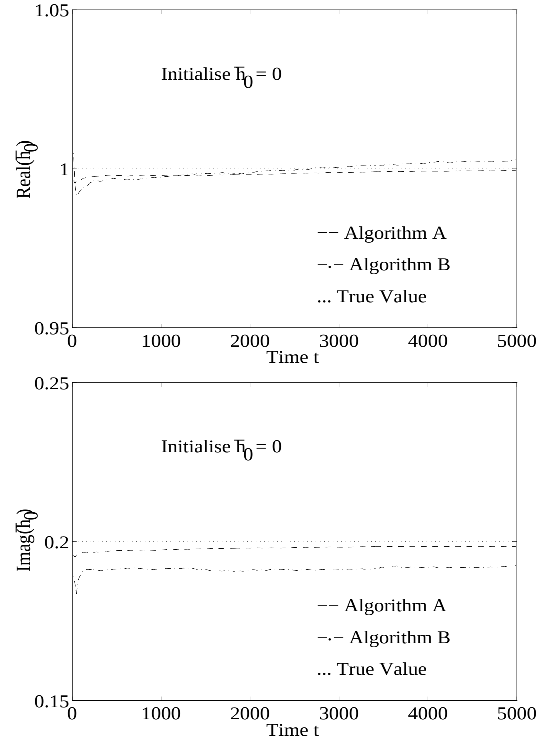

Figure 4. Estimation of $\mathrm{Real}\{\overline{\mathrm{h}}(0)\}$ **and** $\mathrm{Imag}\{\overline{\mathrm{h}}(0)\}$**. Average performance, excluding failures.**

The mean impulse response is chosen to be $\overline{\mathbf{h}} = [\,1{+}0.2j,\,-0.5{+}0.5j\,]^T$. The input $s(t)$ is drawn from a 16-QAM constellation at random, thereby effectively assuming an i.i.d. digital source with no channel coding. The channel is observed in additive white Gaussian noise, $n(t)$, with variance $\sigma_n^2 = 1 \times 10^{-2}$.

In the following, the coupled estimator in which the AR parameters are estimated directly from the observations using an RLS algorithm is labelled Algorithm A. Algorithm B is the coupled estimator in which the AR parameters are derived from estimates of the channel covariance.

The performance of Algorithms A and B was characterized over 500 data sets. A failure was declared if any of the final parameter estimates were not within $\pm 25\%$ of their true value. Failures were observed to occur in three ways: i) no final estimates of the parameters obtained i.e. catastrophic failure (Algorithm B only); ii) the estimates converge to a local minima and not the true parameters; and iii) estimates converge extremely slowly and therefore are not within the $\pm 25\%$ bound at the cessation of the run (5000 samples). With all parameter estimates initialized to zero, Algorithm A was successful on 84.4% of the data sets, and Algorithm B was successful on only 6.4%. The average performance of the estimators over the successful data sets is shown in Figures 3 and 4. The estimates of the first AR parameter and first channel tap mean are shown. These results highlight the benefit of estimating the AR parameters directly from the observations (Algorithm A), rather than via the channel tap covariances (Algorithm B).

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