IDENTIFICATION OF BILINEAR SYSTEMS USING BANDLIMITED REGRESSION

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ABSTRACT

This paper considers the identification of time-invariant bilinear models using observed input–output data. Bilinear models represent a parsimonious class of nonlinear parameterisations and have been used in a variety of applications. However the performance of the bilinear model can be limited in practice when standard least-squares techniques are used, as this leads to biased parameter estimates. Most existing solutions for this problem are restrictive, suboptimal, or computationally intensive. We propose an alternative approach to this identification task by utilising a robust regression technique, known as bandlimited regression, to obtain bilinear parameter estimates with reduced bias. The approach is numerically stable and computationally inexpensive. Simulations are given to demonstrate the usefulness of the technique for bilinear system identification.

1. INTRODUCTION

System identification is concerned with characterising an unknown system using measurements of the system's input–output signals. Although a linear parameterisation can be used as a system model, many real-life systems show nonlinear behaviour [1]. As a result, it is important to consider nonlinear models in order to accurately characterise real-life phenomena.

One particularly attractive model for nonlinear system identification is the bilinear model, which can economically characterise a wide class of nonlinear phenomena [1, 2]. However the performance of the bilinear model can be severely limited in practice when ordinary least-squares estimation techniques are used, as the recursive model leads to correlated residuals and thus biased parameter estimates.

A variety of approaches have been proposed in an attempt to overcome this problem, including high SNR assumptions [2], an assumed noise covariance structure [3], autoregressive noise models [1] and complicated variants of the recursive prediction error method using extended Kalman filters [4]. However these alternatives tend to be suboptimal, computationally intensive, and/or numerically sensitive. Hence there is a need for a simple procedure to accurately identify bilinear system in the noisy case

In an attempt to solve this identification problem, we consider applying a recently devised regression procedure, known as bandlimited regression [5], to estimate the parameters of the bilinear model. Unlike previous bilinear system identification procedures, the proposed approach is simple, not restricted to the high SNR case, and does not require detailed prior knowledge of the noise covariance matrix.

We introduce the bilinear model in Section 2. It should be noted, however, that the identification technique is general in that it can be applied to other nonlinear models such as the Volterra or Hammerstein series [6, 7]. In Section 3, we indicate how the concept of bandlimited regression can be used as a solution to the bilinear system identification problem. We discuss computational considerations in Section 4, and present simulation results in Section 5 to demonstrate the utility of the identification technique.

2. THE BILINEAR MODEL

The bilinear model is a member of the class of recursive polynomial filters [4, 2, 8]. Bilinear models represent a simple and powerful class of nonlinear models, and have subsequently found many applications in nonlinear system identification (e.g., see [1–4]). The bilinear model is also parsimonious: It has been shown that a finite order bilinear model can have an infinite number of non-vanishing terms when expanded as a Volterra series [8]. As a result, it can economically model a large class of nonlinear phenomena and is thus particularly attractive for nonlinear system identification.

We consider a discrete-time bilinear model given by

$$Y(t) = \sum_{k=0}^{M-1} a(k)X(t-k) + \sum_{k_1=1}^{N}\sum_{k_2=1}^{N} b(k_1,k_2)Y(t-k_1)X(t-k_2) + E(t)\,, \tag{1}$$

where $X(t)$ and $Y(t)$, $t \in \mathbb{Z}$, are the observed input and output signals for $t = 0, 1, \ldots, T-1$, and $a(k)$, $k = 0, 1, \ldots, M$, and $b(k_1, k_2)$, $k_1, k_2 = 0, 1, \ldots, N$, are called the linear and bilinear kernels, respectively. $E(t)$ is a zero-mean signal representing model and observation error, and it is assumed that $X(t)$ and $E(t)$ are independent. Figure 1 shows the basic configuration of the bilinear system in (1).

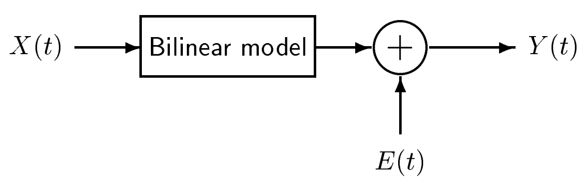


Figure 1: A schematic of the bilinear system considered.

Stability issues are of special relevance here as the bilinear model is recursive. We rely on the conditions for bilinear model stability given in [1,2]. It is therefore assumed that the bilinear model in (1) is stable in that it produces a sufficiently well behaved output for any bounded input. Note also that it is not our intention to solve the problem of optimal model order selection, i.e., the choice of M and N in (1) (see [9]), but rather to introduce the use of bandlimited regression for bilinear system identification.

3. PARAMETER ESTIMATION

For simplicity, let the bilinear model in (1) be expressed in matrix form as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\alpha} + \mathbf{e}\,, \tag{2}$$

where $\mathbf{y}$ is the $[T \times 1]$ output vector, $\mathbf{X}$ is the $[T \times (M+N^2)]$ augmented "input" matrix whose columns span the model space, $\boldsymbol{\alpha}$ is the $[(M+N^2)\times 1]$ parameter vector, and $\mathbf{e}$ is the $[T \times 1]$ residual (or error) vector.

The underlying problem is that the use of ordinary least-squares techniques lead to *biased* parameter estimates in the presence of observation noise[1]. If the $[T \times T]$ noise covariance matrix of $\mathbf{e}$ was known and invertible, then a minimum variance unbiased estimate of $\boldsymbol{\alpha}$ could be readily obtained.

[1]This is also known as the equation-error approach.

However, the noise covariance matrix is usually not known in practice, and is frequently difficult to estimate. Unfortunately, the limitations and complications associated with existing solutions to this problem tend to detract from the underlying benefits of the bilinear model as a system parameterisation.

3.1. Bandlimited Regression

In an attempt to solve this bilinear system identification problem, we consider applying a *bandlimited* regression technique [5] to estimate the parameters of the bilinear model. Bandlimited regression is a method of performing regression that is robust against *correlation* in the errors, and can provide superior performance over ordinary linear regression schemes. Bandlimited regression also leads to less biased estimates of the coefficient variances, which is particularly important for hypothesis testing and model validation.

In bandlimited regression, the bilinear model parameters are chosen so as to minimise

$$(\mathbf{y} - \mathbf{X}\boldsymbol{\alpha})'\,\mathbf{A}\,(\mathbf{y} - \mathbf{X}\boldsymbol{\alpha}) \tag{3}$$

with respect to $\boldsymbol{\alpha}$. Here $\mathbf{A}$ is a $[T \times T]$ truncated spectral expansion matrix with elements

$$A(t_1, t_2) = \sum_{l=0}^{L-1} a_l\, v_l^{(T)}(t_1) v_l^{(T)}(t_2)\,, \tag{4}$$

for $t_1, t_2 = 0, 1, \ldots, T-1$, where $v_l^{(T)}(t_1)$ are the discrete prolate spheroidal (or Slepian) sequences, and a_l, $l = 0, 1, \ldots, L-1$ are spectral weighting parameters. The Slepian sequences and their associated eigenvalues, λ_l, are solutions of the eigenvector equation [10]

$$D^{(T)}(t_1, t_2; W) = \frac{\sin\big(2\pi W(t_1 - t_2)\big)}{\pi(t_1 - t_2)} \tag{5}$$

where $0 < W < 1/2$ is the local bandwidth parameter. The eigenvalues of (5) are ordered such that $1 > \lambda_0 > \lambda_1 > \cdots > \lambda_{L-1} > 0$. The Slepian sequences provide the best approximants to bandlimited functions on a given time and frequency support. Specifically, the truncated Slepian sequences on $[0, T-1]$ have the lowest energy *outside* the spectral band $[-W, W]$ Hz from among all bandlimited functions. Figure 2 shows the first five Slepian sequences for $W = 0.005$ and $L = 1024$.

3.2. Choice of regression parameters

The spectral expansion matrix $\mathbf{A}$ is defined by the local bandwidth parameter W (or equivalently L) and the spectral weighting parameters a_l, $l = 0, 1, \ldots, L-1$.

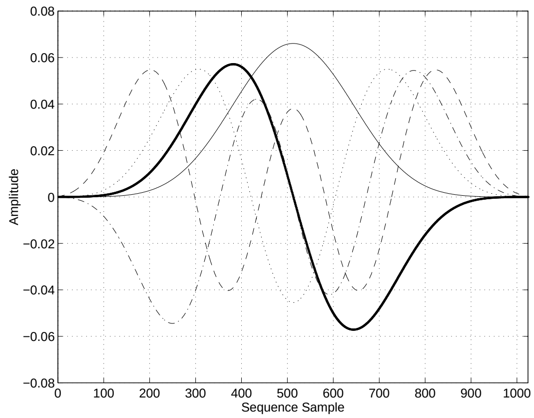


Figure 2: The first five Slepian sequences for $W = 0.005$ and $L = 1024$.

Given that the residual spectral density is bandlimited on $[-W, W]$ Hz, it is then sufficient to consider (at most) the first $L = \lfloor 2WT \rfloor$ Slepian sequences; conversely, the first L Slepian sequences can be used to express any bandlimited residual spectral density on $[-L/(2T), L/(2T)]$ Hz [10]. An estimate of the optimal bandwidth may also be obtained by examining the variance of the residual as a function of W [5].

The spectral weighting parameters can be chosen so that each observation has approximately the same statistical leverage on each estimate. This type of estimate is robust against correlated residuals and outliers. It can however lead to increased computational costs and some interpretability complications [5]. Alternately, a simpler scheme involves setting $a_l = \lambda_l$, which does not substantially degrade the estimates.

The linear and bilinear kernels in (1) are subsequently extracted from the estimated parameter vector $\hat{\boldsymbol{\alpha}}$ (from (3)), which is a linear least-squares problem given values of a_l and L.

4. DISCUSSION

The bandlimited regression technique can provide parameter estimates which are robust against correlated errors and outliers. It is clear from (5) where the notion of "bandlimited" regression arises, as the regression is performed over regions where the columns of $\mathbf{X}$ (i.e., the model space) have their spectral energy most concentrated. In particular, bandlimited regression produces less biased estimates of the bilinear parameter variances [5].

Note also that the formulation in (3) bears strong similarity to the minimum variance unbiased estimator of $\boldsymbol{\alpha}$, i.e., when the covariance matrix of the error is known and invertible. However the error covariance matrix is *not* usually known in practice and is frequently difficult to estimate, and thus the bandlimited regression approach is well suited for the bilinear system identification problem.

4.1. Computation of the Slepian sequences

The direct approach for obtaining Slepian sequences requires a complete eigen-analysis of (5), which can be computationally expensive for large T. In addition, only a few eigenvectors of $D^{(L)}(m, n; W)$ may be needed.

A more computationally efficient approach lies in the use Slepian's equation [10, pp. 1376], which involves solving for the eigenvectors of the tri-diagonal matrix equation, $M^{(T)}(t_1, t_2; W)$,

$$M^{(T)}(t_1, t_2; W) = \begin{cases} \frac{1}{2}(N - t_1)t_1, & t_2 = t_1 - 1 \\ \left(\frac{T-1}{2} - t_1\right)^2 \cdot \cos(2\pi W), & t_2 = t_1 \\ \frac{1}{2}(t_1 + 1)(T - 1 - t_1), & t_2 = t_1 + 1 \\ 0, & |t_2 - t_1| > 1, \end{cases}$$

for $t_1, t_2 = 0, 1, \ldots, T - 1$. Computationally efficient methods exist for the inversion of tri-diagonal matrices, and thus this approach is favoured over a direct eigen-analysis of (5).

5. SIMULATION

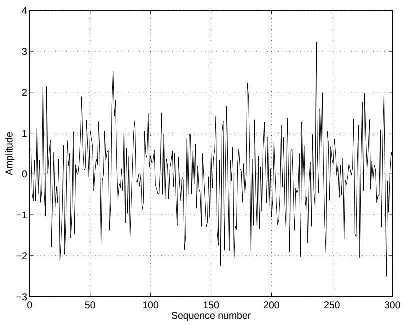
We applied the proposed bilinear system identification procedure to a simulated input–output system following the examples in [1, p.52]. The time-invariant bilinear system is given by

$$\begin{aligned} Y(t) = {} & 1.5X(t) + 1.2X(t-1) - 0.2X(t-1) \\ & + 0.7X(t-1)Y(t-1) - 0.1X(t-2)Y(t-2) \\ & + E(t), \end{aligned} \quad (6)$$

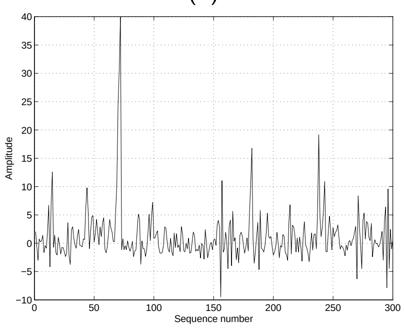
where $M = 3$ and $N = 3$. Zero initial conditions were assumed. A white, zero-mean Gaussian noise process was used as the input signal for $T = 300$. The ordinary least-squares technique was compared with the bandlimited regression technique over a range of SNRs using $a_l = \lambda_l$ for $L = 30$. The normalised prediction error was used for comparison and validation, i.e., $\left(\sum_{t=0}^{T-1}(Y(t) - \hat{Y}(t))^2\right) / \left(\sum_{t=0}^{T-1} Y(t)^2\right)$, where $\hat{Y}(t)$ is the predicted output signal using the estimated bilinear model.

Figures 3(a) and 3(b) show typical input and output signals for the bilinear system in (6). Figure 3(c) shows the normalised mean-square prediction errors for the two regression methods versus SNR (dB), averaged over 50 realisations.

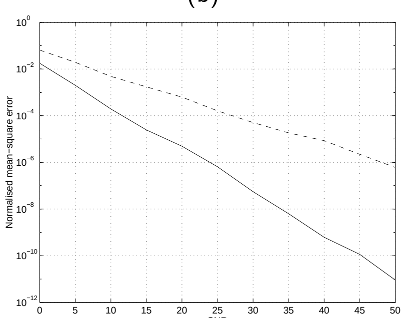
As would be expected the improvement gained with the bandlimited regression technique over the ordinary least-squares technique is most noticeable at lower values of SNR. The bilinear system is recursive, and thus additional care must be taken to ensure that the parameter estimates lead to a stable model. Stability should therefore be checked to ascertain that the estimated parameters lead to a stable bilinear model [1,2].



(a)



(b)



(c)

Figure 3: Typical (a) input and (b) output signals, and (c) a comparison of the normalised prediction errors of the ordinary least-squares method (dashed) and bandlimited regression method (solid) verses SNR (dB).

6. CONCLUSIONS

We have developed a new procedure for identifying time-invariant bilinear systems from measured input–output data. By exploiting the concept of bandlimited regression, we can obtain improved bilinear parameter estimates over the use of ordinary least-squares methods. The technique can also be applied to other nonlinear models (e.g., the Volterra series) when correlated residuals are evident. The overall solution represents computationally inexpensive and numerically stable approach to an otherwise difficult nonlinear system identification problem.

7. REFERENCES

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