ROBUSTNESS OF BLIND FRACTIONALLY-SPACED IDENTIFICATION/EQUALIZATION TO LOSS OF CHANNEL DISPARITY

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ABSTRACT

In this contribution, we address the comparison of Subspace (**SS**), Linear Prediction (**LP**) and Constant Modulus (**CM**) identificaton/equalization algorithms in terms of robustness to loss of Fractionally-Spaced **channel disparity**. We show that SS procedure leads to an inconsistent channel estimation. Investigating a left-inverse channel estimation, we show that LP results in the estimation of the so-called minimum-phase multivariate channel factorization. We show that CM criterion still perform reasonable channel estimation, even if proper algorithm initialization is still a critical subject.

Keywords: Fractionally spaced equalization. Channel diversity. Subspace method. Linear prediction. Constant Modulus Algorithm.

1. INTRODUCTION

Since the pioneer work of Tong et al. [1] many blind Fractionally Spaced (**FS**) channel estimation / equalization technics have been proposed for digital transmission systems ([2, 3, 6, 7],...). The basic idea motivating these approaches consists in introducing **channel diversity** generated by either oversampling the received data or using a multivariate data observed behind an array of sensors ([6]).

Under the so-called **identifiability** condition (no common zero in the multichannel transfer function) all algorithms have similar performances: they achieve perfect identification / equalization (in noise-free context). However, very few results are available on the performance in realistic operating conditions, namely, when additive noise is present and the channel is possibly affected by **lack of disparity** ([7]) (i.e., when the multichannel transfer function $h(z)$ have zeros numerically close). By introducing the borderline case when numerically close zeros are exactly equal (i.e., $h_0(z)$ in Figure 1), we compare, the robustness to lack of channel disparity of some algorithms which have been proposed recently. We evaluate, in particular, the best achievable performances that can be expected when one of the following Subspace like approach (**SS**), Linear Prediction (**LP**) and Constant Modulus (**CM**) criteria are used. For this class of blind identification / equalization procedures, we give a quantitative analysis of the loss of performance illustrated by numerical simulations. To the our best knowledge no such study exists.

2. FRACTIONALLY-SPACED SCHEME

The FS channel scheme consists in the l-dimensional FIR channel transfer function denoted $h(z) = h_0(z)\,\underline{h}(z)$ where $h_0(z)$ is a scalar FIR transfer function of degree Q_0 such that $h_0(z) = \sum_{p=0}^{Q_0} h_{0,p}\, z^{-p}$ and $\underline{h}(z) = (\underline{h}_1(z), ..., \underline{h}_l(z))^\top$. $h_0(z)$ corresponds to common zeros, i.e., the lack of disparity between the sub-channels $h_1(z), ..., h_l(z)$. Each function $\underline{h}_k(z)$ writes as $\underline{h}_k(z) = \sum_{p=0}^{Q-Q_0} h_{k,p}\, z^{-p}$, where $Q-Q_0$ denotes the degree of $\underline{h}(z)$ and Q the degree of $h(z)$ which is supposed known (**A-1**). Futhermore $\underline{h}(z) \neq 0$ for each z, i.e., there is no common zero between all components $\underline{h}_k(z)$ $k = 1, ..., l$ ($\underline{h}(z)$ is identifiable) (**A-2**). The additive noise is described by the l-variate vector $w(n) = (w_1(n), ..., w_l(n))^\top$. We consider that the input signal $s(n)$ is an i.i.d. sequence of finite symbols such that $E[s(n)] = 0$ and $E[s(n)^2] = 1$ (**A-3**). Finally, we suppose that the noise $w(n)$ is temporally and spatially white (i.e., $E[w(n)w^\top(m)] = \sigma^2 I_l$ if $m = n$, 0 otherwise) and independant of the sequence $s(n)$ (**A-4**).

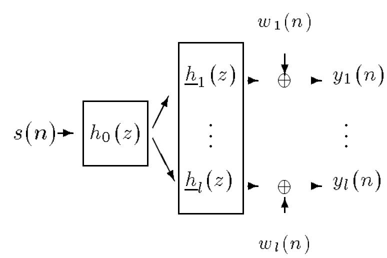

Figure 1. FSE Scheme Under Lack of Disparity

The output multivariate signal can be written:

$$y(n) = [h(z)]\, s(n) + w(n) \tag{1}$$

$$\Updownarrow$$

$$Y_N(n) = \mathcal{T}_N(h) S_{N+Q}(n) + W_N(n) \tag{2}$$

$$\text{with,} \quad \mathcal{T}_N(h) = \mathcal{T}_N(\underline{h})\mathcal{T}_{N_0}(h_0) \tag{3}$$

where $Y_N(n)$ is the Nl-long observation vector $(y(n), ..., y(n-N-1))^\top$. Let $N_0 = N+Q-Q_0$ then $\mathcal{T}_N(\underline{h})$ denotes the $Nl \times N_0$ Sylvester channel convolution matrix associated to $\underline{h}(z)$ and $\mathcal{T}_{N_0}(h_0)$ the $N_0 \times (N+Q)$ Sylvester channel convolution matrix associated to $h_0(z)$. Throughout the paper, we assume that the condition $Nl > N_0$ holds (**A-5**). Note that, under the fondamental condition $N > Q$ (**A-6**), $\mathcal{T}_N(\underline{h})$ is a full-column rank matrix and $\mathcal{T}_{N_0}(h_0)$ is full-row rank. $S(n)$ contains the input sequence at $n, n{-}1, ..., n{-}N{-}Q{+}1$

and $W_N(n)$ is the Nl multivariate noise regression vector $(w(n), ..., w(n-N-1))^{\top}$.

The identification problem consist in the estimation of the multichannel $h(z)$ (or equivalently the convolution matrix $\mathcal{T}_N(h)$), whereas the equalization problem addresses an estimation of a left inverse $g(z)$, such as $g(z)^{\top} h(z) = z^{-\nu}$, where ν is an arbitray delay.

3. SUBSPACE IDENTIFICATION

In this section, we investigate the robustness with respect to the channel disparity for the subspace (**SS**) identification method which was first introduced in ([6]). The analysis is performed with exact statistics.

According to assumptions A1-4, the covariance matrix of $Y_N(n)$ is,

$$\mathcal{R}_N^y = \mathcal{T}_N(\underline{h}) \mathcal{R}_{N_0}^0 \mathcal{T}_N(\underline{h})^{\top} + \sigma^2 I_{lN} \tag{4}$$

where $\mathcal{R}_{N_0}^0 = \mathcal{T}_{N_0}(h_0)\, \mathcal{T}_{N_0}(h_0)^{\top}$. In particular the rank of $\mathcal{R}_{N_0}^0$ is the same than the row-rank of $\mathcal{T}_{N_0}(h_0)$, i.e, N_0. This ensures that $\mathcal{R}_{N_0}^0$ is a full rank definite positive matrix. Consequently, the signal subspace $\mathcal{E}_S$, i.e, the subspace spanned by the row vectors of $\mathcal{T}_N(\underline{h}) \mathcal{R}_{N_0}^0 \mathcal{T}_N(\underline{h})^{\top}$ is the range of $\mathcal{T}_N(\underline{h})$. In other words, the signal subspace is spanned by the N_0 eigenvectors associated to the $\mu_i > \sigma^2$ eigenvalues of $\mathcal{R}_N^y$, whereas the noise subspace $\mathcal{E}_B$ is spanned by the $lN - N_0$ eigenvectors associated to the smallest eigenvalues $\mu_i = \sigma^2$. Thanks to the hermitian properties of $\mathcal{R}_N^y$, $\mathcal{E}_S$ and $\mathcal{E}_B$ are orthogonal, so for each $g \in \mathcal{E}_B$ of dimension Nl, we have $g^{\top} \mathcal{T}_N(\underline{h}) = 0$.

For a given N, one effect of the lack of channel disparity is to increase the noise subspace dimension by Q_0 with respect to the case of an irreducible $h(z)$. However, the lack of channel disparity phenomenon do not appear here as an handicap for the noise subspace estimation, which corresponds to the first step for of the SS method (see [6]). In particular, if the degree Q_0 of the scalar transfer function $h_0(z)$ is known, we are able to identify (up to a constant) the irreducible contribution $\underline{h}(z)$.

However when Q_0 is unknown, which is supposed here, the analysis of the subspace method performance is connected to the knowledge of the set minimizing the quadratic criterion:

$$Q(f) = \sum_{k}^{dim\,\mathcal{E}_B} |g_k^{\top} \mathcal{T}_N(f)|^2 = f^{\top} \mathcal{Q}_N \; f \tag{5}$$

This is the second step of SS method. Herein, g_k are the eigenvectors associated to $\mathcal{E}_B$. Hence, $\mathcal{Q}_N$ denotes the positive semi-definite matrix of dimension $Nl \times Nl$ writes as $\sum_k^{dim\,\mathcal{E}_B} \mathcal{T}_N(g_k) \mathcal{T}_N(g_k)^{\top}$, Note that, in order to avoid the non trivial solution $f = 0$, the minimization is generally made under a constraint such that $||f||^2 = 1$.

In particular, the main question is to know if it is possible to identify the channel $h(z) = h_0(z)\, \underline{h}(z)$ as an unique solution of Argmin $Q(f)$.

Lemma 1 *The global minimum f_* of $Q(f)$ is,*

$$f_* = \mathcal{T}_{N_0}(h_0')^{\top} \underline{h} \tag{6}$$

$$\Updownarrow$$

$$f_*(z) = h_0'(z)\, \underline{h}(z) \tag{7}$$

where $\mathcal{T}_{N_0}(h_0')$ denotes the Sylvester matrix associated to an arbitrary scalar transfer function $h_0'(z)$ of the same degree as $h_0(z)$, i.e., Q_0 and $\underline{h}$ is associated to the multivariate transfer function $\underline{h}(z)$ of degree $Q - Q_0$.

Proof: We referred here to the concept of minimal polynomial basis of rational subspace introduced in [4] (see also [2]). Under the condition $N > Q - Q_0$, the rational subspace of dimension $l - 1$ associated to the orthogonal complement of the l-variate polynomial function $f(z)$ of degree Q is characterized by a basis of polynomials functions of degree at most N. In particular there is a bijection between the l-variate function $g_k(z)$, such as $g_k(z)^{\top} f(z) = 0$ and the Nl dimensional vector which lives in the left null space of $\mathcal{T}_N(f)$. Moreover, according to the fundamental assumption A-2, all polynomials of the one-dimensional signal subspace are l-variate function colinear to $f(z)$ of the form $f(z) = f_0(z)\underline{h}(z)$. $f_0(z)$ denotes a polynomial scalar function of degree Q_0 and $\underline{h}(z)$ the minimal basis of the subspace signal of degree $Q - Q_0$.

Consequently, the orthogonality condition $g_k(z)^{\top} f(z) = 0$ can conveniently be rewritten as:

$$g_k^{\top} \mathcal{T}_N(\underline{h}^{\top} \mathcal{T}_{N_0}(f_0)) = 0 \Leftrightarrow \underline{h}^{\top} \mathcal{T}_{N_0}(f_0)\, \mathcal{T}_N(g_k) = 0 \tag{8}$$

where $g_k \in \mathcal{E}_B$.
Since the criterion $Q(f)$ is of the form,

$$Q(f) = \sum_{k}^{dim\,\mathcal{E}_B} |g_k^{\top} \mathcal{T}_N(f)|^2 = \sum_{k}^{dim\,\mathcal{E}_B} |g_k^{\top} \mathcal{T}_N(\underline{h}^{\top} \mathcal{T}_{N_0}(f_0))|^2$$

it turns to the quadratic expression:

$$Q(f) = \underline{h}^{\top} (\mathcal{T}_{N_0}(f_0)\, \mathcal{Q}_N\, \mathcal{T}_{N_0}(f_0)^{\top})\, \underline{h}$$

As $\mathcal{T}_N(f_0)\, \mathcal{Q}_N\, \mathcal{T}_N(f_0)^{\top}$ is a positive semi definite matrix, the minimum of $Q(f)$ corresponding to a vector f orthogonal to all g_k, is of the form $f_* = \mathcal{T}_{N_0}(f_0)^{\top} \underline{h}$. □□□

It is interesting to know that this result can be recast in a more general framework.In particular, the robustness analysis can be extended in the case of multiples sources.

According to the previous lemma, the nullspace of $\mathcal{Q}$ is reduced to one dimentional subspace spanned by the vector $\underline{h}$. In this case the subspace method results in a non consistent estimator. Indeed, the minimization problem (5) admits infinitely many solutions of the form $h(z) = h_0'(z)\, \underline{h}(z)$, where $h_0'(z)$ is any scalar polynomial with degree Q_0. As a consequence of this result, the channel is well estimated if and only if $h(z)$ is irreducible, i.e, in the case where $\mathcal{T}_{N_0}(h_0') = I_{N_0}$ corresponding to $h_0(z) = 1$. Futhermore, it is interesting to see that lack of channel disparity and overestimation of the channel degree (see [2]) lead for the subspace method to the same conclusion.

4. LINEAR PREDICTION BASED METHODS

In this section, we are interested in Linear Prediction (**LP**) robustness in noise-free conditions and with exact statistics.

When the FIR model (1) is also a finite order AR model (in noise-free case), it is possible to find with a linear prediction method, a l-variate FIR transfer function $g(z) = (g_1(z), ..., g_l(z))^\top$ of degree N with $[g(z)^\top]\, y(n) = s(n)$. This is indeed satisfied under the identifiability condition ([2]). Now, the question is to understand the behavior of LP for a channel of the form $h(z) = h_0(z)\underline{h}(z)$.

The key result is given by the following lemma which connect the scalar innovation of the input sequence $[h_0(z)]\, s(n)$ to the l-variate innovation process of the observation vector $y(n)$.

Lemma 2 *Under the hypotheses A-1,3, the innovation of the process* y(n) *is given by,*

$$i_{y,N}(n) = \underline{h}(0)\; i_{v,N_0}(n) \tag{9}$$

where $\underline{\mathrm{h}}(0)$ *is a* $l-$*length vector and where* $i_{v,N_0}(n)$ *is the innovation of the process* $v(n) = [h_0(z)]\, s(n)$.

Proof: Let $\mathcal{H}_{n-1,N}(y) = span\{y(n-l) \in R^q; 1 \leq l \leq N\}$. The l-variate innovation process over a past of dimension N is defined as,

$$i_{y,N}(n) = y(n) - \hat{y}_N(n) = y(n) - y(n)/\mathcal{H}_{n-1,N}(y)$$

Since $\mathcal{T}_N(\underline{h})$ is a full column rank matrix, there is a matrix G of dimension $N_0 \times Nl$ such that $G\,\mathcal{T}_N(\underline{h}) = I_{N_0}$, consequently if we note $V_{N_0}(n) = GY_N(n) = \mathcal{T}_{N_0}(h_0)\, S_{N+Q}(n)$ which is a N_0-variate process then,

$$\mathcal{H}_{n,N}(y) = \mathcal{H}_{n,N_0}(v) \tag{10}$$

Moreover, we can split $\mathcal{H}_{n,N_0}(v)$ in two subspaces such that $\mathcal{H}_{n,N_0}(v) = \{v(n)\} \oplus \mathcal{H}_{n-1,N_0-1}(v)$. Note, however, that the subspaces are not orthogonal. So, the projection of $y(n)$ on $\mathcal{H}_{n,N}(y)$ leads to the estimation,

$$\hat{y}_N(n) = \underline{h}(0)\, \hat{v}_{N_0}(n) + y(n)/\mathcal{H}_{n-1,N_0-1}(v)$$

Herein, $\hat{v}_{N_0}(n)$ denotes a scalar process estimation. Consequently, one can writes the l-variate innovation process of the observation as,

$$i_{y,N}(n) = y(n) - \underline{h}(0)\, \hat{v}_{N_0}(n) - y(n)/\mathcal{H}_{n-1,N_0-1}(v)$$

We know on the other hand that,

$$y(n) = \underline{h}(0)v(n) + \sum_{k=1}^{Q-Q_0} h(k)v(n-k)$$

Futhermore, we may noticed that $y(n)/\mathcal{H}_{n-1,N_0-1}(v)$ is reduce to the subspace $y(n)/span\{v(n-k); 1 \leq k \leq Q-Q_0\}$. In other words,

$$y(n)/\mathcal{H}_{n-1,N_0-1}(v) = \sum_{k=1}^{Q-Q_0} h(k)v(n-k)$$

So we get,

$$i_{y,N}(n) = \underline{h}(0)\, v(n) - \underline{h}(0)\, \hat{v}_{N_0}(n)$$
$$i_{y,N}(n) = \underline{h}(0)\, i_{v,N_0}(n)$$

□□□

According to the previous lemma, now, we are able to connect the l-variate innovation formulation to the linear filter prediction.

Indeed, since $v(n) = [h_0(z)]\, s(n)$ its innovation process is given by,

$$i_v(n) = \left[\frac{1}{h_0(z)^{(-)}}\right] v(n)$$

where $h_0(z)^{(-)}$ denotes le minimum-phase factorization of $h_0(z)$. In particular, if we introduce $g_0(z)$ the FIR scalar predictor associated to the innovation $i_{v,N_0}(n)$, one may see $g_0(z)$ as an truncature of the transfer function $1/h_0^{(-)}(z)$. Thus,

$$i_{v,N_0}(n) = [g_0(z)]\, v(n) = \sum_{k=0}^{N_0-1} g_0(k)\, z^{-k}$$

where $g_0(0), ..., g_0(N_0-1)$ denotes the coefficient of the scalar prediction filter $g_0(z)$. Accordingly, the l-variate innovation process of $y(n)$ is deduced from the $l \times l$ prediction filter $G(z)$ such as,

$$i_{y,N}(n) = [G(z)]\, y(n) = [\, I_l + \sum_{k=1}^{N-1} G(k)\, z^{-k}\,]\, y(n)$$

where $G(1), ..., G(N-1)$ denotes the $l \times l$ coefficient of the prediction filter $G(z)$. According lemma 2, if we multiply the expression (9) at left by $\underline{h}(0)^\top$, we get:

$$\frac{\underline{h}(0)^\top\, [G(z)]}{||\underline{h}(0)||^2}\, \underline{h}(z) = g_0(z) \tag{11}$$

if we denotes $\underline{g}(z) = h(0)^\top\, [G(z)]/||h(0)||$ the left inverse of $\underline{h}(z)$, the linear prediction formulation turns to the expression,

$$\underline{g}(z)^\top\, \underline{h}(z) = g_0(z) \tag{12}$$

Actually, the interesting conclusion is that in all cases the linear prediction method leads to a minimum phase factorization ! When $h_0(z) = 1$ the condition $h(z) \neq 0$ for all z imply that $h(z)$ is a minimum-phase transfer function leading $g(z)$ to be also minimum-phase. The expression (12) generalizes in some sense this result when $h(z) = h_0(z)\underline{h}(z)$ (with $h_0(z)$ a scalar transfer function). In particular, if $h_0(z)$ is minimum-phase, for N large enough, the "left inverse" estimation of $h(z)$ turns to the expression $g(z) = \underline{g}(z)/g_0(z) \simeq \underline{g}(z)h_0(z)^{-1}$.

Futhermore, under the minimum phase assumption for $h_0(z)$, one can recover approximatively the taps of h with the estimation procedure ([2]):

$$h(k) = E[y(n)\,\big([\underline{g}(z)^\top]\, y(n-k)\big)] \tag{13}$$

Indeed, a straigthforward calculus, gives:

$$E[y(n)\,\big([\underline{g}(z)^\top] y(n-k)\big)] = h(z)\, \underline{g}(z)^\top\, h(z)\, h_0(z)\, E[s(n)s(n-k)]$$

According to the relation (12), we have $\underline{g}(z)^\top h(z) h_0(z) \approx 1$ and from A-3, $[h(z)]\, E[s(n)\, s(n-k)] = h(k)\, \delta(k)$.

Simulations: Next, we give an example of the linear prediction approach based on the algorithm proposed in ([2]) for a 2-dimentional channel $h(z) = h_0(z)\, \underline{h}(z)$ where $\underline{h}(z)$ is of degree $Q = 2$ and $h_0(z)$ of degree $Q = 1$. The input sequence $s(n)$ is defined by a BPSK sequence. The simulations were performed in noise-free condition with exact statistics and $N = 4$. In Table I, we give the zeros locations of $\underline{h}(z)$.

zeros locations of $\underline{h}(z)$		
$\underline{h}_1(z)$	-1.10	0.60
$\underline{h}_2(z)$	0.80	0.30

Table I

We investiagte the LP approach for tree differents transfer function $h_0(z)$ defined in the Table II (first row).

zeros locations			
case:	(a)	(b)	(c)
$h_0(z)$	0.20	-0.90	1.50
$\hat{h}_0(z)$	**0.20**	**-0.84**	**0.66**

Table II

For the tree channels (a,b,c) the irreducible part $\underline{h}(z)$ is exactly estimated. That is why, we give only the estimation zeros locations estimation of the scalar fonction h_0. The results given in the Table II (second row), shown that the channel estimation is all the more accurate that the common zeros are inside and "far" from the unit circle.

5. CONSTANT MODULUS CRITERION

For BPSK sequence, the Fractionally-Spaced Constant Modulus (**CM**) equalization is based on the minimization of the criterion,

$$\mathcal{C}(g(z)) = E[\left(([g(z)^{\top}]\, y(n)\,)^2 - 1\right)^2] \qquad (14)$$

In the case of lack of disparity it was shown in [9] that $\underline{h}(z)$ is perfectly equalized and that what remains in the equalization of the non-fractional $h_0(z)$. Thus, the proper initialization of the corresponding CMA which leads to an appropriate delay ν and to avoid eventual local minima is still a critical problem that has not be solved.

For a long enough equalizer N and for a small enough σ^2 the global minima of CM are close to the optimal Wiener solution. This result was shown in [7].

To get a better insight of the CM behavior for large N let us consider the comparison with Wiener equalizer. Although that the CM and Wiener receiver seems very different, it is interesting to notice that there is a connection between the both approches. Indeed, the optimal Wiener receiver solutions is an IIR multi-variate transfer function of the form,

$$g(z) = z^{-\nu}\frac{h(z)}{||h(z)||^2 + \sigma^2} \qquad (15)$$

here, $||h(z)||^2$ denotes the scalar transfer function $h(z)^{\top} h(z^{-1})$. In noise-free conditions, according to the model (2), the solution (15) is exactly a global minimum of the CM criterion. Consequently, for a large enough degree of equalizer N, and a small enough σ^2 one can see that the CM criterion consists in some sense in pseudo-inverting the Bezout idendity (see [7] for more details),

$$\underline{h}(z)^{\top}\, g(z) = \frac{z^{-\nu}}{h_0(z)} \qquad (16)$$

If we compare the expression (16) to le LP approach (12) one can see that using implicit high order statistics CM criterion minimization still performs reasonable equalization for any $h_0(z)$ (even if it is not a minimum-phase transfer function).

Finally, we may remark that a possible estimation of $h(z)$ can then be done by using a "delayed" version of (13) or by solving at "best" the linear system $\mathcal{T}(g)^{\top} h = f$. Because of the CM robstness to lack of disparity, the resulting estimate may be better than a direct estimate such as by SS approach (see simulations).

6. COMPARATIVES SIMULATIONS

Next we give a comparison of the different methods in terms of channel estimation (under exact statistics). The simulations were performed with a 2-dimensional channel $h(z)$ of degree $Q=4$ driven by a BPSK sequence. Herein, $h_0(z) = z + 3.2$ is a non-minimum phase transfer function. Consequently, $h_0(z)$ is recovered only by the CM criterion minimization (with here $\nu = 9$) (see Table III). We set $N=4$ at SNR= $30\,dB$.

Zeros locations of the channel				
$h_1(z)$	4.00	1.40	-0.80	**-3.20**
$h_2(z)$	2.30	0.30	1.90	**-3.20**

Zeros location estimation					
SS	$h_1(z)$	4.00	1.40	0.80	**-1.92**
	$h_2(z)$	2.30	1.90	0.30	**-1.92**
PL	$h_1(z)$	4.00	1.40	-0.80	**-0.31**
	$h_2(z)$	2.30	0.30	1.90	**-0.31**
CM	$h_1(z)$	3.98	1.40	-0.80	**-3.19**
	$h_2(z)$	2.28	0.30	1.91	**-3.20**

Table III

REFERENCES

[1] L. Tong, G. Xu, and T. Kailath., "A new approach to blind identification and equalization of multipath channels". In 25th Asilomar Conf., CA, 1991.

[2] K. Abed Meraim, et al., "Prediction error methods for time-domain blind identification of multichannel FIR filters", in Proc. ICASSP'95, 1995.

[3] D.T.M. Slock, and C.B. Papadias, "Further results on blind identification and equalization of multiple FIR channels", in Proc. ICASSP'95, pp. 1964-1967, 1995.

[4] G. Forney, " Minimal bases of rational vector spaces, with applications to multivariate linear systems", SIAM J. on Control, vol. 13, pp. 493-529, 1975.

[5] Z. Ding, Y. Li, "On channel identification based on second-order cyclic spectra", IEEE Tr. on SP, 1994.

[6] E. Moulines, et al., "Subspace methods for the blind identification of multichannel FIR filters", IEEE Tr. on SP, 1995.

[7] I. Fijalkow, A. Touzni, and J. Treichler, "Fractionally-Spaced Equalization using CMA: Robustness to channel noise and lack of disparity", to appear in IEEE Trans. on SP, January 1997.

[8] H. Zeng, L. Tong, and C.R. Johnson Jr., "Relationships between the constant modulus and Wiener receivers", submitted to IEEE Trans. Info. Theory, 1996.

[9] I. Fijalkow, J.R. Treichler, C.R. Johnson Jr., "Fractionally Spaced Blind Equalization: Loss of Channel Disparity", in Proc. ICASSP 95, vol. 4, pp.1988-1991 , 1995.