IMPROVED ITERATIVE LEAST SQUARES RAKE-COMBINER USING BEAMSPACE TRANSFORMATIONS

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ABSTRACT

Antenna arrays can be used in mobile communication systems to increase capacity and performance. However, as the number of antenna elements grows, the computational burden increases significantly. To reduce the total computational cost, a beamspace transformation is derived, based on the statistical model of the array signal. Although the transformation derived here is applicable to many algorithms, it is matched to the decoupled weighted iterative least squares with projections RAKE-combiner algorithm. Not only is the computational cost reduced, the overall performance is improved by using an appropriate beamspace transformation.

1. INTRODUCTION

Several algorithms have been proposed to increase coverage and spectrum efficiency in a mobile communication system by using an antenna array at the base station. In [2], the decoupled weighted iterative least squares with projections (DWILSP) RAKE-combiner algorithm was introduced. It constitutes an extension to the DWILSP algorithm for time-dispersive fading channels [1,11]. The main advantage of the DWILSP approach, compared to conventional beamforming techniques, is its superior exploitation of the signal structure, as well as the spatial structure of the channel. For a brief overview of the DWILSP and DWILSP RAKE-combiner, see the Appendices.

An increase in the number of array elements improves the potential performance, at the expense of an increased computational burden. To reduce the complexity, while keeping the performance advantages of the larger array, the dimension of the data can be reduced with a suitably chosen transformation. In a multi-user environment, each user signal belongs to a subspace. These subspaces are usually not perfectly orthogonal. The objective is to extract the subspace of the signal of interest, while simultaneously rejecting interference, without affecting the spatial adaptivity of the DWILSP algorithm.

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2. DATA MODEL

In a scenario with d cochannel users, the signal measured at the m-element base station antenna array can be modelled as

$$\boldsymbol{x}(t) = \sum_{i=1}^{d} \boldsymbol{H}_i(t,q)\, s_i(t) + \boldsymbol{n}(t) \tag{1}$$

where $s_i(t)$ is the scalar signal from the i:th user (belonging to a finite alphabet), $\boldsymbol{n}(t)$ is the m-dimensional measurement noise, possibly colored, and $\boldsymbol{H}_i(t,q)$ is the channel response from user i. The channel in a mobile communication scenario is a time-varying random filter. Assuming that the channel is constant during N array snapshots, it can be modelled as an $(L+1)$ tap FIR-filter

$$\boldsymbol{H}(q) = \boldsymbol{h}_0 + \boldsymbol{h}_1 q^{-1} + \ldots + \boldsymbol{h}_L q^{-L} \tag{2}$$

where each filter tap $\boldsymbol{h}_k$ is a random vector due to a local scatter distribution [6]. The data model (1) can thus be rewritten as

$$\boldsymbol{x}(t) = \sum_{i=1}^{d} \sum_{k=0}^{L} \boldsymbol{h}_{ik} s_i(t - kT_s) + \boldsymbol{n}(t) \tag{3}$$

where $s_i(t\text{-}kT_s)$ are delayed versions of the signal from user i. These source signals are assumed to be temporally white and independent between users. Averaging over different realizations of the channels $\boldsymbol{H}_i(q)$ and source signals (for example bursts in a TDMA system), the array covariance can be formulated as

$$\mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)] = \boldsymbol{R}_x = \sum_{i=1}^{d} \boldsymbol{R}_i + \boldsymbol{Q}_n \tag{4}$$

where $\boldsymbol{Q}_n$ is the covariance of noise and unknown interference, and $\boldsymbol{R}_i$ is the covariance of user i:

$$\boldsymbol{R}_i = \mathrm{E}\left[\sum_{k=0}^{L} \boldsymbol{h}_{ik} \boldsymbol{h}_{ik}^H\right] \tag{5}$$

In (5), the received source signal power is included in the channel vectors. Numerically, $\boldsymbol{R}_i$ is of full rank. However, the effective rank is in most cases low.

3. BEAMSPACE TRANSFORMATION

A beamspace transformation is a linear mapping, from the element space (ESP) m-dimensional data $\boldsymbol{x}(t)$ to the reduced r-dimensional beamspace (BSP) data $\boldsymbol{y}(t)$:

$$\boldsymbol{y}(t) = \boldsymbol{T}^H \boldsymbol{x}(t) \qquad (6)$$

The $\langle m|r\rangle$ matrix $\boldsymbol{T}$ is derived, so as to reduce the ESP data dimension as much as possible (i.e. choosing minimum r), without affecting the bit error rate (BER) performance. In [3-6], Cramér-Rao bound (CRB) preserving BSP transformations are derived in the case of certain parametrizations of the signal model. Here, the problem is somewhat different, as the signal model does not rely on any strict parametrization or array structure. In fact, the goal is to be independent of the array response, so as to enable the use of uncalibrated arrays. Therefore, the derivation of $\boldsymbol{T}$ is based on the statistical model (4), which also allows a slow updating of $\boldsymbol{T}$. Applying (6), the BSP array covariance becomes

$$\mathrm{E}[\boldsymbol{y}(t)\boldsymbol{y}^H(t)] = \boldsymbol{R}_y = \sum_{i=1}^{d} \boldsymbol{T}^H \boldsymbol{R}_i \boldsymbol{T} + \boldsymbol{T}^H \boldsymbol{Q}_n \boldsymbol{T} \qquad (7)$$

Regarding user j as the signal of interest, the BSP signal to interference plus noise ratio (SINR) can be defined as

$$\mathrm{SINR}_j = \frac{\mathrm{Tr}(\boldsymbol{T}^H \boldsymbol{R}_j \boldsymbol{T})}{\mathrm{Tr}\left(\sum_{i \neq j} \boldsymbol{T}^H \boldsymbol{R}_i \boldsymbol{T} + \boldsymbol{T}^H \boldsymbol{Q}_n \boldsymbol{T}\right)} = \frac{\mathrm{Tr}(\boldsymbol{T}^H \boldsymbol{R}_j \boldsymbol{T})}{\mathrm{Tr}(\boldsymbol{T}^H \boldsymbol{Q} \boldsymbol{T})} \qquad (8)$$

where Tr(.) is the trace operator and $\boldsymbol{Q}$ is the total interference covariance. $\boldsymbol{T}$ is now chosen to maximize the SINR. It is well known [5] that the $\boldsymbol{T}$ maximizing (8) is the matrix whose columns are the eigenvectors corresponding to the r largest eigenvalues of the matrix pencil $[\boldsymbol{R}_j, \boldsymbol{Q}]$, i.e. the principal eigenvectors of $\boldsymbol{Q}^{-1}\boldsymbol{R}_j$, as $\boldsymbol{Q}$ is full rank. This can easily be shown by rewriting (8) as

$$\mathbf{SINR}_j = \frac{\left\|\boldsymbol{T}^H \boldsymbol{R}_j^{1/2}\right\|_F^2}{\left\|\boldsymbol{T}^H \boldsymbol{Q}^{1/2}\right\|_F^2} = \frac{\left\|\boldsymbol{V}^H \boldsymbol{Q}^{-1/2} \boldsymbol{R}_j^{1/2}\right\|_F^2}{\left\|\boldsymbol{V}\right\|_F^2} \qquad (9)$$

where $\boldsymbol{V}=\boldsymbol{Q}^{1/2}\boldsymbol{T}$ and $\|.\|_F$ denotes the Frobenius matrix norm. With the only restriction being that $\boldsymbol{V}$, and thereby $\boldsymbol{T}$, should be full rank, (9) is maximized by choosing the columns of $\boldsymbol{V}$ to be the eigenvectors corresponding to the r largest eigenvalues of $\boldsymbol{Q}^{-1/2}\boldsymbol{R}\ \boldsymbol{Q}^{-1/2}$. Transforming back from $\boldsymbol{V}$ to $\boldsymbol{T}$ shows that $\boldsymbol{T}$ is given by the principal eigenvalues of $\boldsymbol{Q}^{-1}\boldsymbol{R}_j$ [9,10].

This result is valid for an arbitrary r. The optimal choice of r depends on the structure of the channel and the interference scenario. The relative magnitude of the eigenvalues of $\boldsymbol{Q}^{-1}\boldsymbol{R}_j$ gives an indication of how to choose r. In a practical application, the choice of r is a trade-off between performance (BER) and computational complexity.

The total interference covariance $\boldsymbol{Q}$ is in most cases unknown. It can be estimated as $\hat{\boldsymbol{Q}} = \hat{\boldsymbol{R}}_x - \hat{\boldsymbol{R}}_j$, but experiments indicate that $\boldsymbol{Q}$ can be replaced by $\boldsymbol{R}_x$ without any significant performance loss. This replacement only results in a small change in the subspace spanned by $\boldsymbol{T}$. The user covariance $\boldsymbol{R}_j$ is estimated from filter tap estimates (5).

4. NUMERICAL EXAMPLES

From the above discussion it follows that the optimal r, i.e. the dimension of the reduced data, depends on the eigenstructure of $\boldsymbol{Q}^{-1}\boldsymbol{R}_j$. To illustrate the eigenstructure in a real environment, the eigenvalues of $\boldsymbol{Q}^{-1}\boldsymbol{R}_j$ were estimated from measured data. The measured data consisted of 200 consecutive GSM bursts collected by a base station antenna array in an urban area. The array output is an eight-dimensional vector signal. The user signal of interest was corrupted by one cochannel interferer. 200 GSM bursts correspond to a measurement time of about one second. During this time, the movement of the test mobile is small, and the statistics of the channel parameters can be assumed constant. The resulting normalized eigenvalues are shown in figure 1.

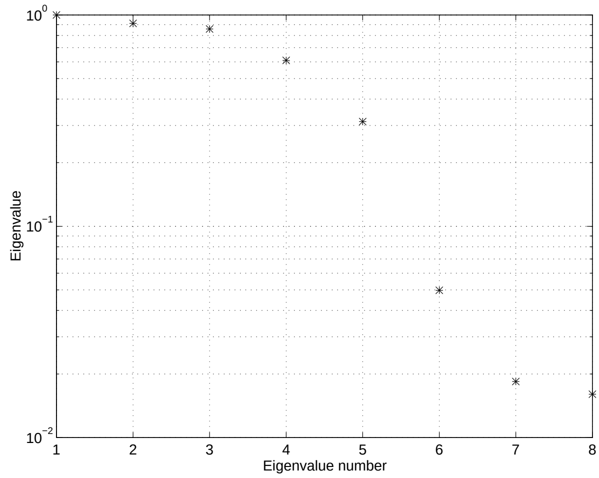

FIGURE 1. Eigenvalues of $\boldsymbol{Q}^{-1}\boldsymbol{R}_j$, estimated from measured GSM data.

As another example, a 10-element uniform linear antenna array was employed at the receiver, in a simulated scenario with three cochannel users. The signal emitters were placed at -30°, 0° and 45° relative the antenna broadside. BPSK data was transmitted in bursts of 150 bits, of which 19 bits were used as a training sequence. The (fading) channel associated with each user was modelled according to (2) with 5 taps, each corresponding to a local scatter distribution [8], modelling the angular spread of the multipath. The eigenvalues of the resulting $\boldsymbol{Q}^{-1}\boldsymbol{R}_j$, for an element signal to noise ratio of 6dB, and with j being the user at 0°, are shown in figure 2.

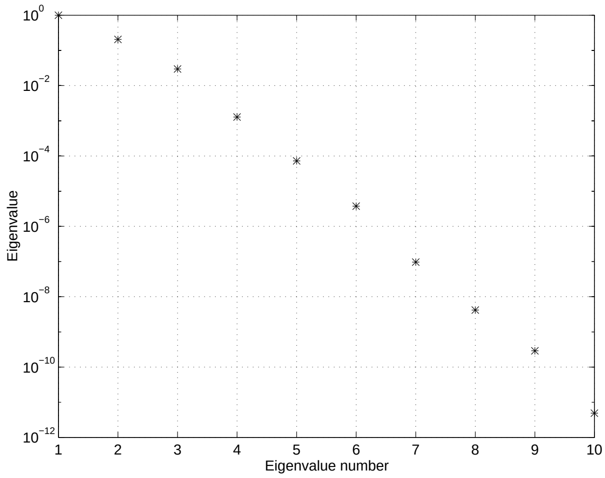

FIGURE 2. Eigenvalues of $Q^{-1}R_j$ (simulated data).

Comparing to figure 1, the eigenvalues are more rapidly decreasing. This is due to the larger angular spread in the real data compared to the simulated. The transform matrix $\boldsymbol{T}$ was evaluated for $r \in [2, 3, 4, 5]$. Figure 3 shows the BER performance of the DWILSP RAKE-combiner in the reduced BSP, as well as in the ESP. Somewhat surprisingly, it shows that choosing a proper dimension (i.e. r not too large, nor too small) of $\boldsymbol{T}$ yields an improvement in BER relative to ESP. A heuristic explanation for this is that the prefiltering supplied by the BSP transformation increases the SINR. This improves the convergence properties of the DWILSP algorithm. Also, the prefiltering prevents the DWILSP algorithm to converge to a signal in the wrong subspace, i.e. another user signal.

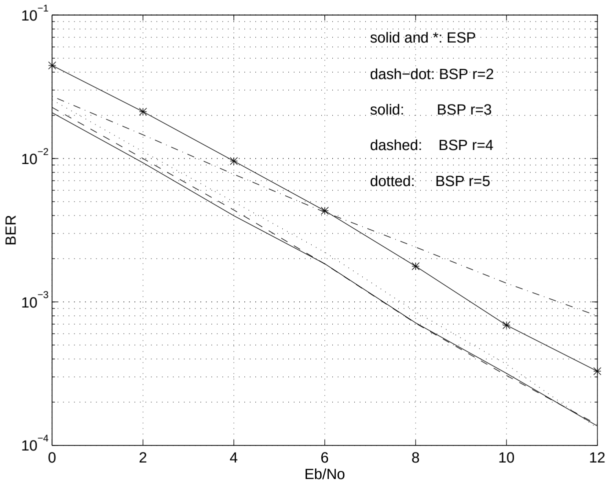

FIGURE 3. The BER of the DWILSP RAKE-combiner with and without beamspace transformation applied.

Most significant is the reduction of total computational cost. Table 1 shows the computational cost using different r, relative to ESP operation.

TABLE 1. Complexity using BSP-transformation.

r :	2	3	4	5
Complexity relative to ESP (%)	14	23	33	44

In table 1, the cost for the transformation operation itself (6) is included, but not the calculation of the transform matrix $\boldsymbol{T}$. This because $\boldsymbol{T}$ can be updated slowly, making the average cost insignificant in the comparison.

It is interesting to compare the performance of the DWILSP RAKE-combiner to the performance of the optimal maximum likelihood sequence estimator (MLSE). Using the same 5-tap channel as in the previous example, figure 3 shows the BER of the DWILSP RAKE-combiner, with and without BSP transformation (r=3), as well as the BER of the MLSE. The MLSE was run using the true channel parameters and interference covariance matrix, thus serving as a benchmark.

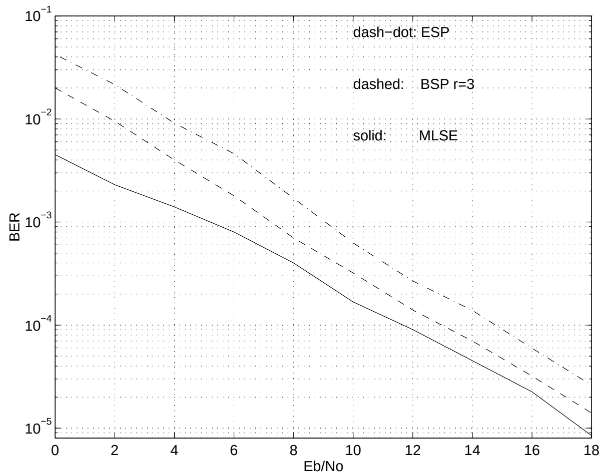

FIGURE 4. The BER of the DWILSP RAKE-combiner with and without beamspace transformation, compared to the optimal MLSE.

With the BSP transformation employed, the DWILSP RAKE-combiner comes close to the performance of the optimal MLSE at high signal to noise ratios. It should be noted, that if the MLSE is run with parameters estimated from the training sequence, the performance is far from the ideal shown in figure 4 [2].

5. CONCLUSIONS

Based on the underlying statistics of the mobile communication signal model, rather than any parametrizations in terms of the array manifold or directions of arrival etc., a beamspace transformation was derived. The data model used in this derivation is the same as the one used to derive the DWILSP RAKE-combiner receiver algorithm.

The main reason for applying the beamspace transformation was to reduce the computational complexity, and thereby enabling the use of a large array. However, when processing the reduced beamspace data with the DWILSP RAKE-combiner, a further advantage was revealed. The increase in SINR that follows from a proper reduction of the data dimension improves the convergence properties of the algorithm. This potentially reduces the number of necessary training bits.

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Appendix A: The Decoupled Weighted Iterative Least Squares with Projections (DWILSP) Algorithm

Consider the model (1) with each filter $\boldsymbol{H}_i(t,q)$ replaced with a vector $\boldsymbol{h}_i$. Collecting N snapshots in matrix form, and regarding user 1 as the signal of interest yields

$$\boldsymbol{X} = \boldsymbol{h}_1\boldsymbol{s}_1 + \sum_{i=2}^{d}\boldsymbol{h}_i\boldsymbol{s}_i + \boldsymbol{N} = \boldsymbol{h}_1\boldsymbol{s}_1 + \boldsymbol{J} \tag{10}$$

DWILSP solves the problem

$$\min_{\boldsymbol{h}_1, \boldsymbol{s}_1}\left\|\boldsymbol{W}(\boldsymbol{X} - \boldsymbol{h}_1\boldsymbol{s}_1)\right\|^2 \tag{11}$$

by iteratively minimizing with respect to $\boldsymbol{h}_1$ and $\boldsymbol{s}_1$. In each iteration, the signal estimates, $\hat{\boldsymbol{s}}_1$, are projected to the closest discrete points in the signal constellation, $\bar{\boldsymbol{s}}_1 = \mathrm{Proj}(\hat{\boldsymbol{s}}_1)$. The scheme is repeated until convergence of $\bar{\boldsymbol{s}}_1$. DWILSP thus provides estimates of both the signal, as well as the channel vector. The weighting matrix W should ideally be chosen as $\boldsymbol{R}_j^{-1/2}$, but using $\boldsymbol{R}_x^{-1/2}$ gives asymptotically equivalent estimates.

Appendix B: The DWILSP RAKE-Combiner

The DWILSP RAKE-Combiner is an extension of DWILSP for time-dispersive channels. Based on the model (3), DWILSP is employed to obtain estimates of the L+1 delayed versions of user signal i. These estimates are combined to yield a final signal estimate as

$$\tilde{s}_i(t) = \mathrm{Proj}\left(\sum_{k=0}^{L}\frac{1}{\sigma_{ik}^4}\hat{s}_{ik}(t)\right) \tag{12}$$

where σ_{ik}^4 is the square of the variance $E\left[\left|\hat{\boldsymbol{s}}_{ik} - \bar{\boldsymbol{s}}_{ik}\right|^2\right]$. The channel estimates provided by the algorithm can be used to track the user covariance $\boldsymbol{R}_j$, (5).

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