ADAPTIVE BLIND EQUALIZATION OF TIME-VARYING CHANNELS

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ABSTRACT

In mobile communications, often time-varying multipath is too rapid for a conventional adaptive algorithm to track. This motivates expansion of the time varying channel impulse response over a basis. Rather than estimating the time-invariant coefficients of this basis expansion, it is possible to equalize the channel output directly even when the input is not known. This requires multichannel data as well as a minimal persistence of excitation condition on the input and a coprimeness condition on the multiple channels obtained via fractional sampling and/or multiple antennas. Since the coefficients of the basis expansion will not be constant due to noise and unmodelled dynamics, the equalizer coefficients may also change slowly with time. Adaptive algorithms to track this change are proposed and basis mismatch problems are also investigated in this paper.

1. MOTIVATION AND BACKGROUND

Consider transmitting symbols $s(l)$ every T seconds with a pulse shape (transmit-filter) $f_c^{(tr)}(t)$ using a carrier $\exp(j\omega_c t)$. Denote the analytic form of the transmitted signal as: $s_c(t) = \exp(j\omega_c t)\sum_l s(l) f_c^{(tr)}(t-lT)$. Suppose that due to relative transmitter-receiver motion the propagation (e.g., multipath) channel is changing and has time-varying (TV) impulse response $f_c^{(ch)}(t;\tau) = \sum_{q=1}^{Q} A_q(t)\delta(\tau - d_q(t))$, where Q corresponds to the number of paths and $A_q(t)$, $d_q(t)$ denote each path's TV attenuation and delay, respectively. The need to identify and equalize such TV channels arises in mobile telephony, high-speed modems, and underwater communications [6, 7] . Note that with A_q and d_q constant, time-invariant frequency selective channels are obtained as a special case. Fading channels on the other hand, entail variations. Modeling $\{A_q(t), d_q(t)\}_{q=1}^{Q}$ as stationary random processes has been the traditional approach [6], [11], but the focus herein is on deterministic basis expansion models introduced recently in [9] and [10].

Convolving $s_c(t)$ with $f_c^{(ch)}(t;\tau)$ and removing the carrier we arrive at the received signal-plus-noise model (baseband form): $r_c(t) = \exp(j\omega_c t)\sum_{q=1}^{Q} A_q(t) s_c(t - d_q(t)) + n_c(t)$. To suppress the AWGN $n_c(t)$, we filter $r_c(t)$ through the receive-filter $f_c^{(rec)}(t)$ and obtain $x_c(t) = \sum_l s(l)\ [\sum_{q=1}^{Q} \int_{(l-1)T}^{lT} A_q(\tau)\ f_c^{(tr)}(\tau - lT - d_q(\tau))$ $f_c^{(rec)}(t-\tau)\exp(j\omega_c d_q(\tau))d\tau] + \int_{(l-1)T}^{lT} n_c(\tau) f_c^{(rec)}(t-\tau)d\tau$. Let $f_2(t) := \int_T f_c^{(tr)}(\tau) f_c^{(rec)}(t-\tau)d\tau$ denote the time invariant transmit-receive filters in cascade, and assume:
(a1)constant attenuation and delay over a symbol; i.e., $A_q(\tau) = const. := A_q(l)$, for $\tau \in [(l-1)T, lT]$, and $d_q(\tau) = const. := d_q(l)$, for $\tau \in [(l-1)T, lT]$;
(a2)linearly varying delays across symbols (valid for nominally constant path velocity ν_q); i.e., $d_q(l) = \nu_q l + \epsilon_q$. Under (a1), we have $x_c(t) = \sum_l s(l) h_c(t; t-lT) + v_c(t)$, where

$$h_c(t; t-lT) := \sum_{q=1}^{Q} A_q(l) f_2(t - lT - d_q(l)) e^{j\omega_c d_q(l)} . \quad (1)$$

Output $x_c(t)$ is next (over)sampled with period T/M to obtain the discrete time model: $x(n) := x_c(nT/M) = \sum_l s(l) h(n; n-lM) + v(n)$, where $h(n;l) := h_c(nT/M; lT/M)$ and $v(n) := v_c(nT/M)$. Oversampling offers diversity manifested in the M sub-processes defined as $\{x^{(m)}(n) := x(nM + m-1)\}_{m=1}^{M}$ which are expressed in terms of the M subchannels $h^{(m)}(n;l)$ and the corresponding noise $v^{(m)}(n)$ as:

$$x^{(m)}(n) = \sum_{l=0}^{L} h^{(m)}(n;l) s(n-l) + v^{(m)}(n) , \quad (2)$$

where $h^{(m)}(n;l) := h(nM+m-1; lM+m-1) := h_c(T(nM + m-1)/M; T(lM+m-1)/M) = \sum_{q=1}^{Q} A_q(n-l)\ f_2(T(lM + m-1)/M - d_q(n-l))\exp(j\omega_c d_q(n-l))$. Because the variation of A_q and f_2 w.r.t. n is often negligible relative to that of the exponential, it is reasonable to assume:
(a3)$A_q(n-l) \approx A_q(l)$, and $f_2(T(lM+m-1)/M - d_q(n-l)) \approx f_2(T(lM+m-1)/M - d_q(l))$.
Based on (a1)-(a3) we have:

$$h^{(m)}(n;l) := \sum_{q=1}^{Q} h_q^{(m)}(l) b_q(n-l) , \quad b_q(n) := e^{j\omega_c \nu_q n} , \quad (3)$$

where $h_q^{(m)}(l) := A_q(l) f_2(T(lM+m-1)/M + \nu_q l - \epsilon_q) e^{j\omega_c \epsilon_q}$. Using the Fourier basis $\{\exp(j\omega_q n),\ \omega_q := \omega_c \nu_q\}_{q=1}^{Q}$ the TV model in (2) is expressed in (3) as a superposition of Q channels $\{h_q^{(m)}(l)\}_{q=1}^{Q}$. The latter are time-invariant (TI) and allow blind estimation of finitely parameterized TV channels using TI multichannel identification and source separation techniques. The complex exponentials in (3) can be viewed as each path's Doppler arising due to motion – an effect also encountered in radar and sonar where moving targets

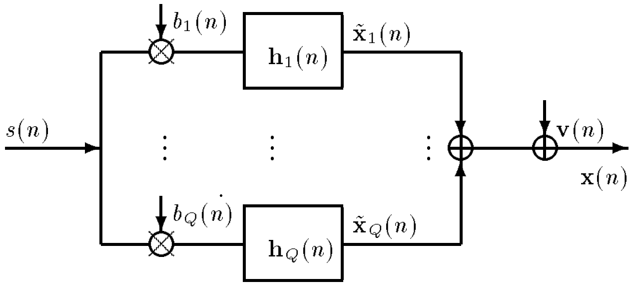

Figure 1. TV - Channel Model

induce TV delays which for narrowband signals manifest themselves as TV phases. Single path Doppler effects were also included in [7], but the challenge here is on blind mitigation of TV intersymbol interference (ISI) and separation of the multiple paths. The unknown frequencies $\omega_c \nu_q$ in (3) can be estimated using tests of cyclostationarity [9].

The basic idea of casting TV multipath models as mixtures of TI ones was first adopted in [10] but the high-variance of high-order statistics motivated second-order subspace methods relying on symbol rate sampling (c.f. (1)-(3) with $m = M = 1$) [8]. However, insufficient diversity in [8] necessitated strong independence assumptions on the bases in order to guarantee identifiability. The independence required by [8] is rarely satisfied in practice for $Q > 2$. Apart from fractional sampling, sufficient diversity is also provided by multiple antennas [4]. Such space and time diversities can be exploited separately or jointly as in the identification methods for TI channels [5, 12].

Collecting $x^{(m)}(n)$'s in an $M \times 1$ vector $\mathbf{x}(n) := [x^{(1)}(n) \ldots x^{(M)}(n)]'$, and defining $\mathbf{h}_q(l)$ and $\mathbf{v}(n)$ similarly, we combine (2) and (3) into the vector model (see also Fig. 1):

$$\mathbf{x}(n) = \sum_{q=1}^{Q} \left[\sum_{l=0}^{L} \mathbf{h}_q(l) b_q(n-l) s(n-l) \right] + \mathbf{v}(n) . \quad (4)$$

Note that $s(n)$ in (4) is not assumed to be white or even random as in [8, 10].

Knowing the bases $\{b_q(n)\}_{q=1}^{Q}$ and measuring only the vector output $\{\mathbf{x}(n)\}_{n=0}^{N-1}$, this paper deals with: (i)FIR adaptive zero-forcing equalization to track slow variations in $h_q(l)$'s (note that rapid variations are accounted for by the bases); and (ii)model mismatch issues to assess possible perturbations in the basis sequences (Section 3).

2. DIRECT BLIND EQUALIZERS

In [1] it is shown that the input can be blindly recovered within a delay d (non-identifiable in blind setups) with K^{th}-order FIR equalizers $\{\mathbf{g}_q^{(d)}(k)\}_{k=0}^{K}$:

$$\sum_{k=0}^{K} \mathbf{x}'(n-k)\, \mathbf{g}_q^{(d)}(k) = s_q(n-d)\ ,\ q = 1, \ldots, Q\ , \quad (5)$$

where $s_q(n-d) := b_q(n-d)s(n-d)$ denotes the recovered input modulated by the qth basis. If we define $\mathbf{s}_q'(n) := [b_q(n)s(n) \ldots b_q(n-L-K)s(n-L-K)]$, the input and the channel matrices are given as follows:

$$\mathbf{S}_b := \begin{bmatrix} \mathbf{s}_1'(N-1) & \ldots & \mathbf{s}_Q'(N-1) \\ \vdots & \vdots & \vdots \\ \mathbf{s}_1'(K) & \ldots & \mathbf{s}_Q'(K) \end{bmatrix} ,\ \mathbf{H} := \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_Q \end{bmatrix}$$

where

$$\mathbf{H}_q := \begin{bmatrix} \mathbf{h}_q'(0) & \ldots & \mathbf{0}' \\ \vdots & \ddots & \vdots \\ \mathbf{h}_q'(L) & \ldots & \mathbf{h}_q'(L-K) \\ \vdots & \ddots & \vdots \\ \mathbf{0}' & \ldots & \mathbf{h}_q'(L) \end{bmatrix} .$$

Then, noise-free matrix version of (4) is:

$$\mathbf{X} := \begin{bmatrix} \mathbf{x}'(N-1) & \ldots & \mathbf{x}'(N-1-K) \\ \vdots & \vdots & \vdots \\ \mathbf{x}'(K) & \ldots & \mathbf{x}'(0) \end{bmatrix} = \mathbf{S}_b \mathbf{H}\ , \quad (6)$$

where N, K, M, L denote the data length, equalizer order, oversampling rate, and maximum channel order respectively. Matrix $\mathbf{S}_b$ is, $\mathbf{S}_b := [\mathbf{S}_1 \ldots \mathbf{S}_Q]$ where each $\mathbf{S}_q$ is an $(N-K) \times (L+K+1)$ Hankel matrix formed from the modulated input sequence $s_q(n) = s(n)b_q(n)$ and each $\mathbf{H}_q$ is a block Toeplitz matrix of dimension $(L+K+1) \times M(K+1)$. We make the following assumptions:

A1. $N - K \geq M(K+1)$ satisfied by collecting sufficient data.
A2. Matrix $\mathbf{H}$ is at least fat; $M(K+1) \geq Q(L+K+1)$
A3. Matrix $\mathbf{H}$ is full rank; $\text{rank}(\mathbf{H}) = Q(L+K+1)$.
A4. The bases $b_q(n)$ are sufficiently varying and $s(n)$ is persistently exciting of sufficient order to assure $\text{rank}(\mathbf{S}_b) = Q(L+K+1)$; again, $s(n)$ can be random or deterministic.

Using **A2** and **A4** we can infer $\text{rank}(\mathbf{X}) = Q(L+K+1)$. To determine the channel order L and number of bases Q given two known upper bounds $\tilde{K}_1 > \tilde{K}_2$ on K, we use the ranks of the corresponding matrices $\tilde{\mathbf{X}}_1$, $\tilde{\mathbf{X}}_2$ to write

$$Q = \frac{\text{rank}(\tilde{\mathbf{X}}_1) - \text{rank}(\tilde{\mathbf{X}}_2)}{\tilde{K}_1 - \tilde{K}_2}\ ,\quad L = \frac{\text{rank}(\tilde{\mathbf{X}}_1)}{Q} - (\tilde{K}_1 + 1)\ .$$

Having Q and L, the oversampling rate M will determine K to satisfy $M(K+1) \geq Q(L+K+1)$.

Considering $\mathbf{X} = \mathbf{S}_b\mathbf{H}$, under **A2**, **A3** we can assert the existence and uniqueness of a $\mathbf{G}$ which contains the equalizer coefficients. Specifically, with $\mathbf{G} = \mathbf{H}^{\dagger}$, the pseudoinverse of $\mathbf{H}$, we find $\mathbf{X}\mathbf{G} = \mathbf{S}_b$; hence, given $\mathbf{H}$ FIR zero-forcing equalizers $\mathbf{G}$ exist and are unique.

2.1. Batch Algorithm

To find the equalizers, set $n = N-1, \ldots K$ in (5) and form the equation $\mathbf{X}\, \boldsymbol{g}_q^{(d)} = \mathbf{s}_q^{(d)} := \tilde{\mathbf{B}}_q^{(d)}\, \mathbf{s}^{(d)}$, where $\boldsymbol{g}_q^{(d)} := [\mathbf{g}_q^{(d)'}(0) \ldots \mathbf{g}_q^{(d)'}(K)]'$, $\mathbf{s}_q^{(d)} := [b_q(N-1-d)s(N-1-d) \ldots b_q(K-d)s(K-d)]'$, $\tilde{\mathbf{B}}_q^{(d)} := \text{diag}[b_q(N-1-d) \ldots b_q(K-d)]$, and $\mathbf{s}^{(d)} := [s(N-1-d) \ldots s(K-d)]'$. We now use Matlab's notation $\mathbf{X}(i_1 : i_2, :)$ to denote a submatrix of $\mathbf{X}$ formed by the i_1 through i_2 rows and all columns of $\mathbf{X}$ to define $\mathbf{X}_{0,d} := \mathbf{X}(d+1 : N-K, :)$, $\mathbf{X}_d := \mathbf{X}(1 : N-K-d, :)$, $\mathbf{B}_q^{(0,d)} := \tilde{\mathbf{B}}_q^{(d)}(d+1 : N-K, d+1 : N-K) = \text{diag}[b_q(N-$

$1-2d)\ldots b_q(K-d)]$, and $\mathbf{B}_q^{(d)}:=\tilde{\mathbf{B}}_q^{(d)}(1:N-K-d,1:N-K-d) = \mathrm{diag}[b_q(N-1-d)\ldots b_q(K)]$.

Due to the structure of $\mathbf{S}_b$, it holds that $\mathbf{X}_{0,d}\,\boldsymbol{g}_{q1}^{(0)} = \mathbf{B}_{q1}^{(0,d)}\,\mathrm{s}^{(0)}(d+1:N-K)$ and $\mathbf{X}_d\,\boldsymbol{g}_{q2}^{(d)} = \mathbf{B}_{q2}^{(d)}\mathrm{s}^{(d)}(1:N-K-d)$. Since $\mathrm{s}^{(0)}(d+1:N-K) = \mathrm{s}^{(d)}(1:N-K-d)$ we can cross multiply with portions of $\tilde{\mathbf{B}}_q^{(d)}$ to obtain:

$$\mathbf{B}_{q2}^{(d)}\,\mathbf{X}_{0,d}\,\boldsymbol{g}_{q1}^{(0)} \;=\; \mathbf{B}_{q1}^{(0,d)}\,\mathbf{X}_d\,\boldsymbol{g}_{q2}^{(d)}\;. \tag{7}$$

The pair of equalizers $(\boldsymbol{g}_{q1}^{(0)},\boldsymbol{g}_{q2}^{(d)})$ will be identifiable (up to a scale) as the eigenvector corresponding to the minimum eigenvalue of $\mathcal{X}_{q1,q2}^{(0,d)}$ in

$$\mathcal{X}_{q1,q2}^{(0,d)}\boldsymbol{g}_{q1,q2}^{(0,d)} := \begin{bmatrix}\mathbf{B}_{q2}^{(d)}\mathbf{X}_{0,d} & -\mathbf{B}_{q1}^{(0,d)}\mathbf{X}_d\end{bmatrix}\begin{bmatrix}\boldsymbol{g}_{q1}^{(0)}\\ \boldsymbol{g}_{q2}^{(d)}\end{bmatrix} = \mathbf{0}\;, \tag{8}$$

provided that the nullity $\nu(\mathcal{X}_{q1,q2}^{(0,d)}) = 1$. For equalizers corresponding to $(0,d) = (0,L+K)$, this is true provided that: (i) $\mathrm{rank}(\mathbf{S}_b) = Q(L+K+1)$, (ii) **A2** holds as an equality, and (iii) **A3** is satisfied. Having a zero delay equalizer, we can compute any d delay equalizer using the relation (7). Equalizers corresponding to different delays and bases perform differently depending on the channel. The equalizer that has minimum norm (induced by the quadratic $\boldsymbol{g}'\mathbf{R}_v\boldsymbol{g}$) among all $\boldsymbol{g}$ that is a column of $\mathbf{G}$ will amplify the noise the least, therefore perform better (see also [2]). An average of the outputs of different equalizers weighted by the inverse of their norm often yields more reliable estimates of the input.

An alternative approach that also exploits the structure in the input matrix utilizes that $range(\mathbf{X}'\mathbf{X}) = range(\mathbf{H})$ (see [5]). From the data matrix $\mathbf{X}$, $\tilde{\mathbf{H}} = \mathbf{F}\mathbf{H}$ can be calculated where $\mathbf{F}$ is a $Q(L+K+1)\times Q(L+K+1)$ full rank ambiguity matrix. $\mathbf{F}$ can be calculated identical to how the equalizer coefficients were calculated above. Notice that $\mathbf{F}$ is $Q(L+K+1)\times Q(L+K+1)$ and knowledge of all the columns of $\mathbf{F}$ enables calculation of $\mathbf{H}$. Also, the block Toeplitz structure of $\mathbf{H}$ can be exploited to bring an additional criterion on the selection of $\mathbf{F}$ which may yield more accurate estimation of its columns.

2.2. Adaptive Algorithms

Equation (8) can be recast in a least squares framework by setting the first coefficient of $\boldsymbol{g}_{q1,q2}^{(0,d)}$ to 1 and can be rewritten as $\overline{\mathcal{X}}\;\overline{\boldsymbol{g}} = \overline{\mathrm{x}} = -\mathbf{x}_1$, where $\overline{\mathcal{X}}$ is $\mathcal{X}_{q1,q2}^{(0,d)}$ without its first column, $\mathbf{x}_1$ is the vector containing the elements of that column, and $\overline{\boldsymbol{g}}$ is $\boldsymbol{g}_{q1,q2}^{(0,d)}$ without its first element. It is well known that RLS is a recursive way of computing $\overline{\boldsymbol{g}}_{LS} = (\overline{\mathcal{X}}'\;\overline{\mathcal{X}})^{-1}\;\overline{\mathcal{X}}'\;\overline{\mathrm{x}}$ which also solves the least squares problem. We use this algorithm to update the vector of equalizer coefficients.

One could also be interested in using the computationally less intensive LMS algorithm at the expense of less accuracy and slower convergence. In the absence of a training sequence (desired input), we consider the elements of $\mathbf{x}_1$ as our desired sequence that we would like $\hat{\mathrm{g}}_t'\hat{\eta}_t$ to estimate. Here $\hat{\eta}_t$ are the rows of $\overline{\mathcal{X}}$, and $\hat{\mathrm{g}}_t$ is the estimate of the vector of equalizer coefficients at time t. At each iteration the vector of equalizer coefficients is updated by the relations

$$\hat{\mathrm{g}}_{t+1} = \hat{\mathrm{g}}_t + \mu\epsilon_{t+1}^*\hat{\eta}_t\;,\quad \epsilon_t = \hat{\mathrm{g}}_t'\hat{\eta}_t - x_{1t} \tag{9}$$

where μ is the step size parameter and x_{1t} denotes t^{th} scalar entry of $\mathbf{x}_1$.

2.3. Basis Mismatch

So far we have assumed perfect knowledge of the basis. It is of interest to analyze the errors $\Delta\mathbf{h}$, $\Delta\mathbf{g}$, $\Delta\mathrm{s}$ in the vector channel, equalizer, and input estimates, when the bases are only known (or estimated) within a mismatch error $\Delta b_q(n)$. Specifically, we will assume $|\Delta b_q(n)| \ll 1$ for all $q \in [1,Q]$ and utilize first-order perturbation analysis in order to obtain channel and input perturbations, $\Delta\bar{\mathbf{h}} := [\Delta\bar{\mathbf{h}}_1'(0)\ldots\Delta\bar{\mathbf{h}}_Q'(0)\ldots\;\Delta\bar{\mathbf{h}}_1'(L)\ldots\Delta\bar{\mathbf{h}}_Q'(L)]'$ and $\Delta\bar{\mathrm{s}} := [\Delta s(N-1)\ldots\Delta s(0)]'$, that match the received data in the least-squares (LS) sense.

To isolate the basis mismatch effect from the receiver noise we consider $\mathbf{v}(n) \equiv \mathbf{0}$ in (4) and define $\mathcal{H}_q := [\mathbf{h}_q(0)\ldots\mathbf{h}_q(L)]$, $\mathrm{s}(n) := [s(n)\ldots s(n-L)]'$. Suppose we use $b_q(n)$ in our algorithms although the true basis is $b_q(n)+\Delta b_q(n)$, and hence the noise free data model is: $\mathbf{x}(n) = \sum_{q=1}^{Q}(b_q(n)+\Delta b_q(n))\mathcal{H}_q\mathrm{s}(n)$. Using $b_q(n)$ instead of $b_q(n)+\Delta b_q(n)$ will result in perturbed estimates $\mathcal{H}_q+\Delta\mathcal{H}_q$ and $\mathrm{s}(n)+\Delta\mathrm{s}(n)$. The latter correspond to a model $\hat{\mathbf{x}}(n) = \sum_{q=1}^{Q} b_q(n)(\mathcal{H}_q+\Delta\mathcal{H}_q)(\mathrm{s}(n)+\Delta\mathrm{s}(n))$. Using first order approximations and assuming $||\Delta\mathcal{H}_q|| \ll ||\mathcal{H}_q||$, $||\Delta\mathrm{s}(n)|| \ll ||\mathrm{s}(n)||$, the error $\mathbf{e}(n) := \hat{\mathbf{x}}(n) - \mathbf{x}(n)$ turns out to be:

$$\mathbf{e}(n) = \sum_{q=1}^{Q} b_q(n)\mathcal{H}_q\Delta\mathrm{s}(n) + \sum_{q=1}^{Q}[b_q(n)\Delta\mathcal{H}_q - \Delta b_q(n)\mathcal{H}_q]\mathrm{s}(n).$$

Vectorizing $\{\Delta\mathcal{H}_q\}_{q=1}^{Q}$ to $\Delta\mathbf{h}$ and $\{\Delta\mathrm{s}(n)\}_{n=0}^{N-1}$ to $\Delta\mathrm{s}(n)$, we seek the minimizers $(\Delta\bar{\mathbf{h}},\Delta\bar{\mathrm{s}})$ of $||\mathbf{e}||^2 := N^{-1}\sum_{n=0}^{N-1}||\mathbf{e}(n)||^2$. They turn out to be the LS solution of:

$$[\,\mathbf{A}\;\mathbf{C}\,]\begin{bmatrix}\Delta\bar{\mathbf{h}}\\ \Delta\bar{\mathrm{s}}\end{bmatrix} = \Delta\mathbf{C}\,\mathrm{s}\;, \tag{10}$$

where $\mathrm{s} := [s(N-1)\ldots s(-L)]'$, $\mathbf{C} := \sum_{q=1}^{Q} diag[b_q(N-1)\;\mathcal{H}_q\ldots b_q(0)\mathcal{H}_q]$, $\Delta\mathbf{C} := \sum_{q=1}^{Q} diag[\Delta b_q(N-1)\mathcal{H}_q\ldots\Delta b_q(0)\mathcal{H}_q)$, $\mathbf{A} := [\mathbf{A}'(N-1)\ldots\mathbf{A}'(0)]'$, and $\mathbf{A}(n) := [b_1(n)\mathrm{s}'(n)\otimes\mathbf{I}\ldots b_Q(n)\mathrm{s}'(n)\otimes\mathbf{I}]$ ($\otimes$ denotes Kronecker product).

Note that $\Delta\bar{\mathbf{h}}$, $\Delta\bar{\mathrm{s}}$ obtained from (10) can serve only as a reference for the channel and input errors that result from small perturbations of the basis. It is possible for an algorithm to have smaller $\Delta\mathbf{h}$ and/or $\Delta\mathrm{s}$, but a larger matching error $||\mathbf{e}||^2$. Similar analysis is also possible for $\Delta\mathbf{g}$.

3. SIMULATIONS

We tried the LMS and the RLS with bases $b_1(n) = 1$, $b_2(n) = \exp(j2\pi n/50)$. For both algorithms we had L=3, K=0, M=8. N=300 (RLS) N=500 (LMS) QPSK samples were generated. All results were averaged over 100 Monte Carlo runs, except for the eye diagrams that show one realization of the data and the equalized output. The RLS estimate is initialized with **0** whereas the LMS curves were initialized with a batch estimate based on the minimum number of data required by **A1**. In Figure 5 the eye diagrams illustrate how utilizing all the delays improves the input estimate for a channel of $M = 8$, $L = 2$ and K chosen to be 0 (see [3] for channel coefficients). The plot on the left in Figure 5 is the output of the zero-delay equalizer, and the plot on the right is an average of the ouputs of all the equalizers inversely weighted by their norms at SNR=20 dB. This weighting was also observed to perform better than straight averaging in the simulations.

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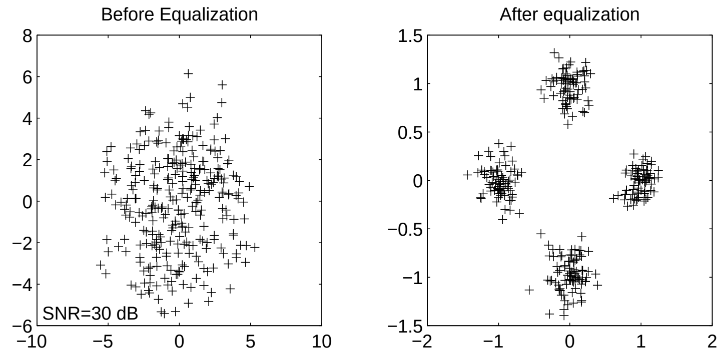

Figure 2. RLS algorithm, approximate initialization

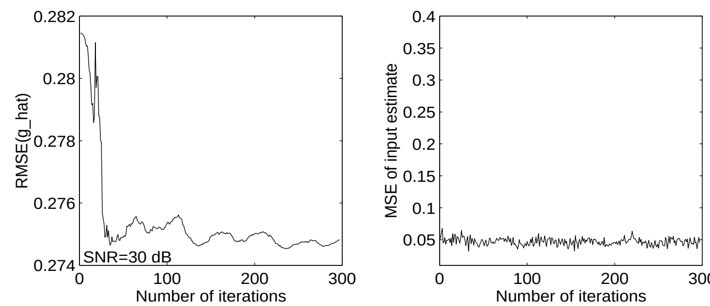

Figure 3. RLS with the number of iterations

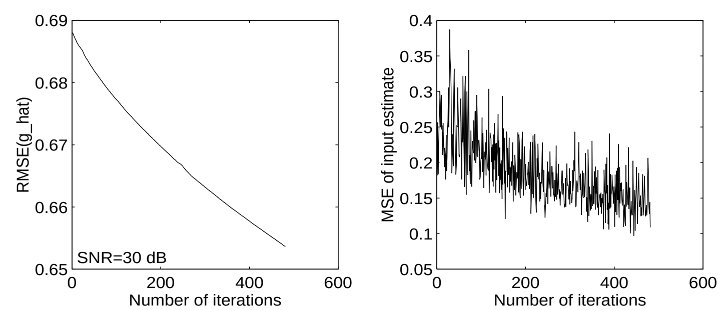

Figure 4. Performance of the LMS algorithm

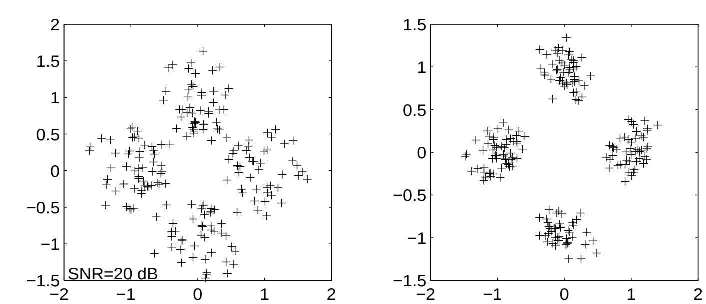

Figure 5. Zero-delay vs average equalizers