

ON BLIND SIGNAL COPY FOR POLYNOMIAL PHASE SIGNALS

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ABSTRACT

The problem of separating and estimating signals received by an array whose array manifold has an unknown structural form is usually referred to as the blind signal copy problem. In this paper we consider the blind signal copy problem for polynomial phase signals. By deriving the Cramer Rao bound we evaluate the optimal performance achievable by any unbiased estimator. To gain additional insight into this problem we compare the CRB to the bound for the case where the functional form of the array manifold is known. We derive a computationally efficient approximate Maximum-Likelihood (ML) algorithm and compare its performance with the bound.

1. INTRODUCTION

In this paper we consider the problem of separation and estimation of signals received by an array whose array manifold has an unknown structural form. This problem is usually referred to as the “blind signal copy” problem. Blind estimation techniques rely on some temporal or statistical properties of the source signals. For example, fourth-order cumulants methods exploit the non-Gaussian nature of the source signals [1]–[2]. Other methods are based on the discrete-alphabet property of digital signals [3]. In this paper we assume that the source signals are polynomial phase signals.

To formulate the problem consider N signals impinging on an array of M elements. The n -th signal is $s_n(t)e^{j\omega_0 t}$ where ω_0 is the carrier frequency and $s_n(t)$ is a constant amplitude polynomial phase signal of known order P .

$$s_n(t) = \alpha_n e^{j\phi_n(t)} = \alpha_n e^{j \sum_{p=0}^P b_{np} t^p} \quad (1)$$

Assuming that the instantaneous frequency variations over the array are small compared to the center fre-

quency, the demodulated and sampled received signal can be modeled by,

$$\mathbf{x}(t_k) = \sum_{n=1}^N \mathbf{a}_n e^{j\phi_n(t_k)} + \mathbf{n}(t_k) = A\mathbf{s}(t_k) + \mathbf{n}(t_k) \quad (2)$$

where $\mathbf{a}_n$ is the array manifold corresponding to the n -th source, and A is the $M \times N$ matrix consisting of array manifold vectors. We assume that $\{n_m(t), m = 1, \dots, M\}$ is a white zero mean complex circular Gaussian random process with variance η , and noise samples at different sensors are uncorrelated with each other. The identifiability of A is discussed in [1]. A can be at best identified up to permutation and complex scaling of its columns. Therefore, we can assume without loss of generality that $\sum_{k=1}^K |s_n(t_k)|^2 = K$ and $b_{n0} = 0$ for $n = 1, \dots, N$.

The problem can be stated as follows: Given the measurements $\{\mathbf{x}(t_k), k = 1, \dots, K\}$, estimate the signal phase parameters $\{b_{np}, n = 1, \dots, N, p = 1, \dots, P\}$ and the array manifold vectors $\{\mathbf{a}_n, n = 1, \dots, N\}$. In the following, we derive the Cramer Rao bound (CRB) for this problem. Next, we derive a computationally efficient approximate Maximum-Likelihood (ML) algorithm and compare its performance with the bound.

2. THE CRAMER RAO BOUND

Denote the vector of unknown parameters by $\boldsymbol{\psi}$, i.e.,

$$\boldsymbol{\psi} = [\eta, b_{11}, \dots, b_{NP}, \boldsymbol{\theta}^T]^T \quad (3)$$

where b_{np} is the p -th phase parameter of the n -th source and $\boldsymbol{\theta}$ contains the elements of A in a vector form

$$\boldsymbol{\theta} = [\text{Re}\{\mathbf{a}_1^T\}, \text{Im}\{\mathbf{a}_1^T\}, \dots, \text{Re}\{\mathbf{a}_N^T\}, \text{Im}\{\mathbf{a}_N^T\}]^T \quad (4)$$

Note that the complex amplitudes of the sources were absorbed in $\boldsymbol{\theta}$.

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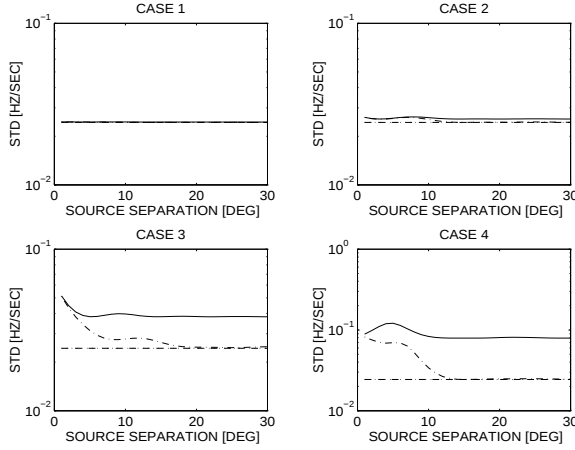


Figure 1: Various bounds on the frequency-rate standard Deviation. $f_s = 60$ Hz. Blind bound (solid line), single source blind bound (dashed line), non-blind bound (dash-dot line), single source non-blind bound (dotted line).

The CRB for $\boldsymbol{\psi}$ is given by $\text{CRB}(\boldsymbol{\psi}) = [F(\boldsymbol{\psi})]^{-1}$ where $F(\boldsymbol{\psi})$ is the Fisher information matrix for $\boldsymbol{\psi}$. For Gaussian complex observations $\{\mathbf{x}(t_k)\}$ the entries of the Fisher Information Matrix (FIM) can be written as,

$$[F(\boldsymbol{\psi})]_{il} = 2\text{Re} \left\{ \left[\frac{\partial \mathbf{m}_x(\boldsymbol{\psi})}{\partial \psi_i} \right]^H R_x^{-1} \left[\frac{\partial \mathbf{m}_x(\boldsymbol{\psi})}{\partial \psi_l} \right] \right\} \quad (5)$$

$$+ \text{tr} \left\{ R_x^{-1}(\boldsymbol{\psi}) \frac{\partial R_x(\boldsymbol{\psi})}{\partial \psi_i} R_x^{-1}(\boldsymbol{\psi}) \frac{\partial R_x(\boldsymbol{\psi})}{\partial \psi_l} \right\}$$

where $R_x(\boldsymbol{\psi})$ is the covariance matrix of the observation vector $[(\mathbf{x}(t_1))^T, \dots, (\mathbf{x}(t_K))^T]^T$ and $\mathbf{m}_x(\boldsymbol{\psi})$ is its mean.

In our case $R_x(\boldsymbol{\psi}) = \eta I$ where I is the identity matrix of dimension MK and

$$\mathbf{m}_x(\boldsymbol{\psi}) = [(As(t_1))^T, \dots, (As(t_K))^T]^T \quad (6)$$

It follows that $\partial \mathbf{m}_x(\boldsymbol{\psi}) / \partial \eta = 0$ and

$$\frac{\partial \mathbf{m}_x(\boldsymbol{\psi})}{\partial \mathbf{b}} = [(D_{\mathbf{b}}(t_1))^T, \dots, (D_{\mathbf{b}}(t_K))^T]^T$$

$$\frac{\partial \mathbf{m}_x(\boldsymbol{\psi})}{\partial \boldsymbol{\theta}} = [(D_{\boldsymbol{\theta}}(t_1))^T, \dots, (D_{\boldsymbol{\theta}}(t_K))^T]^T \quad (7)$$

where

$$D_{\mathbf{b}}(t_k) = [AS_1(t_k), \dots, AS_P(t_k)]$$

$$D_{\boldsymbol{\theta}}(t_k) = S(t_k) \otimes I_M$$

$$S_p(t_k) = \text{diag} \left[\frac{\partial s_1(t_k)}{\partial b_{1p}}, \dots, \frac{\partial s_N(t_k)}{\partial b_{Np}} \right]$$

$$S(t_k) = [s_1(t_k), \dots, s_N(t_k)] \otimes [1, j] \quad (8)$$

and $\otimes$ denotes the Kronecker product

The derivatives of $R_x(\boldsymbol{\psi})$ with respect to $\mathbf{b}$ and $\boldsymbol{\theta}$ are all equal to zero, and the derivative with respect to the noise variance η is simply

$$\frac{\partial R_x(\boldsymbol{\psi})}{\partial \eta} = I_{MK} \quad (9)$$

Substituting (7) and (9) in (5) we get the following expressions for the elements of the FIM,

$$F_{\eta\eta} = \frac{MK}{\eta^2}$$

$$F_{\eta b} = 0$$

$$F_{\eta \boldsymbol{\theta}} = 0$$

$$F_{\mathbf{b}\mathbf{b}} = \frac{2}{\eta} \sum_{k=1}^K \text{Re}[D_{\mathbf{b}}^H(t_k) D_{\mathbf{b}}(t_k)]$$

$$F_{\mathbf{b}\boldsymbol{\theta}} = \frac{2}{\eta} \sum_{k=1}^K \text{Re}[D_{\mathbf{b}}^H(t_k) D_{\boldsymbol{\theta}}(t_k)]$$

$$F_{\boldsymbol{\theta}\boldsymbol{\theta}} = \frac{2}{\eta} \sum_{k=1}^K \text{Re}[D_{\boldsymbol{\theta}}^H(t_k) D_{\boldsymbol{\theta}}(t_k)] \quad (10)$$

Let us define the following N -by- N matrices that summarize the temporal characteristics of the sources.

$$(R)_{nl} = \sum_{k=1}^K s_n(t_k) s_l^*(t_k)$$

$$(R_p)_{nl} = \sum_{k=1}^K s_n(t_k) \frac{\partial s_l^*(t_k)}{\partial b_{lp}}$$

$$(R_{ps})_{nl} = \sum_{k=1}^K \frac{\partial s_n(t_k)}{\partial b_{np}} \frac{\partial s_l^*(t_k)}{\partial b_{ls}} \quad (11)$$

Note that

$$D_{\boldsymbol{\theta}}^H(t_k) D_{\boldsymbol{\theta}}(t_k) = [S^H(t_k) S(t_k)] \otimes I_M \quad (12)$$

It follows that

$$F_{\boldsymbol{\theta}\boldsymbol{\theta}} = \frac{2}{\eta} \text{Re} \left\{ \left(R^* \otimes \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix} \right) \otimes I_M \right\} \quad (13)$$

where the source correlation matrix R is defined in (11). In a similar fashion we can show that

$$F_{\mathbf{b}\mathbf{b}} = \frac{2}{\eta} \text{Re} \begin{bmatrix} A^H A \times R_{11}^* & \cdots & A^H A \times R_{1P}^* \\ \vdots & \ddots & \vdots \\ A^H A \times R_{P1}^* & \cdots & A^H A \times R_{PP}^* \end{bmatrix} \quad (14)$$

where $\times$ denotes the Hadamard product and R_{ps} is defined in (11). We can also show that

$$F_{\boldsymbol{\theta}\mathbf{b}} = \frac{2}{\eta} \text{Re}[G_1^H, \dots, G_P^H] \quad (15)$$

where

$$G_p = (R_p^T \otimes [\mathbf{r}_M, j\mathbf{r}_M]) \times (\mathbf{r}_{2N} \otimes A^H) \quad p = 1, \dots, P \quad (16)$$

and $\mathbf{r}_M$ is a length- M row vector of ones.

Combining the above results we get the following expression for the FIM

$$F(\boldsymbol{\psi}) = \frac{2}{\eta} \text{Re} \begin{bmatrix} \frac{MK}{2\eta} & 0 & 0 & 0 & 0 \\ 0 & (A^H A)R_{11}^* & \cdots & (A^H A)R_{1P}^* & G_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & (A^H A)R_{P1}^* & \cdots & (A^H A)R_{PP}^* & G_P \\ 0 & G_1^H & \cdots & G_P^H & Q \end{bmatrix} \quad (17)$$

where

$$Q = \left(R^* \otimes \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix} \right) \otimes I_M \quad (18)$$

We refer to the bound derived in this paper as the “blind CRB.” To gain some insight into the problem under consideration we compare the blind CRB to the bound for array manifold with known functional form which was derived in [4] and will be referred to as the “non-blind” CRB.

In the example of Figure 1 we consider a pair of linear FM chirps (second order polynomial phase signals). We use an 8 element uniform linear array with half wavelength spacing. We assume that the number of samples K is 120. The direction of the first source is fixed at 0 degrees, while the direction of the second sources is varied according to the source separation. We fix the SNR at 0 dB and plot the standard deviation of the frequency rate estimate as a function of the source separation. The source signals are given by,

$$s_n(t) = e^{j2\pi 0.25\xi_n f_s (t - t^2/T)} \quad n = 1, 2 \quad (19)$$

where f_s is the sampling frequency and $T = K/f_s$ is the duration of the observation interval. $\xi_1 = 1$, and ξ_2 assumes the values $\{-1, 0.8, 0.9, 0.95\}$ for cases 1, 2, 3, and 4, respectively. In Case 1, each of the source signals and its derivatives with respect to the phase parameters are approximately orthogonal to the other source signal and its derivatives. In this case we expect a single source performance. The other cases represent transition from orthogonal sources to nearly coherent sources. For each case we plot the following bounds: The bound derived in this paper (blind CRB), its single source version (single-source blind CRB), the non-blind CRB and its single source version. In all cases the single-source blind CRB coincides with the single source non-blind CRB, indicating that in the single source case the assumption of unknown structure of the array manifold does not cause any degradation in performance. In the case of approximately orthogonal signals (case 1), all 4 bounds coincide, indicating that both blind and non-blind bounds predict single source performance. A similar situation occurs in case 2, where the source signals are neither approximately orthogonal nor close to coherent. In cases 3 and 4, however, the sources are nearly coherent, and the two-source bounds depart from each other and from the single source bound. In these cases, the assumption of unknown structure of the array manifold causes significant performance degradation relative to the case where the functional form of the array manifold is perfectly known. Furthermore, it is not possible to achieve single source performance.

3. AN APPROXIMATE ML ALGORITHM

Without loss of generality we can rewrite the signal model in Eq. (2) in the following way,

$$\mathbf{x}(t_k) = A\Lambda\mathbf{s}(t_k) + \mathbf{n}(t_k) \quad k = 1, \dots, K \quad (20)$$

where Λ is a diagonal matrix with real positive entries and $(A^H A)_{nn} = M$. It can be shown that the ML estimator minimizes the following cost function with respect to the phase and the array parameters.

$$J = \text{tr}\{\Lambda A^H A \Lambda \sum_{k=1}^K \mathbf{s}(t_k) \mathbf{s}^H(t_k)\} - 2\text{Re} \left[\text{tr}\{\Lambda A^H \sum_{k=1}^K \mathbf{x}(t_k) \mathbf{s}^H(t_k)\} \right] \quad (21)$$

As discussed in [4] polynomial phase signals tend to be orthogonal to each other, unless they are identical

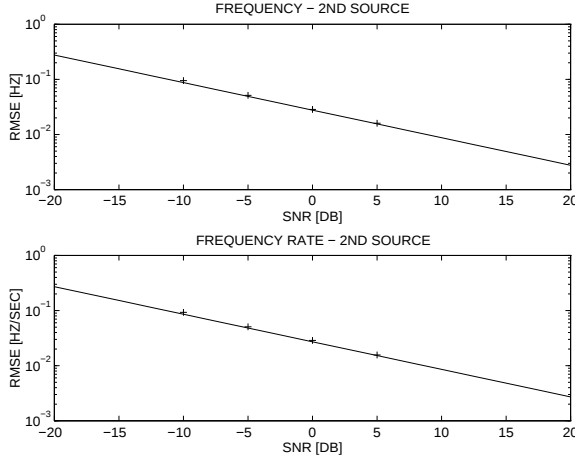


Figure 2: RMSE of frequency and frequency rate vs. SNR. $f_s = 60$ Hz. CRB (solid line), Monte-Carlo results ('+')

(or nearly identical). In this case $\sum_{k=1}^K \mathbf{s}(t_k) \mathbf{s}^H(t_k) \approx KI$, where I is the identity matrix of dimension N . Using the above approximation we get the following ML algorithm:

1. Evaluate the following cost function

$$J_1(b_1, \dots, b_P) = \sum_{m=1}^M \left| \sum_{k=1}^K x_m(t_k) e^{-j(b_1 t_k + \dots + b_P t_k^P)} \right|^2 \quad (22)$$

2. Identify N local peaks in J_1 . $\{\hat{b}_{n1}, \dots, \hat{b}_{nP}\}$ are the arguments associated with the n -th peak.

3. Obtain the estimates of $\mathbf{a}_n$, $n = 1, \dots, N$ as follows:

$$\hat{\mathbf{a}}_n = \sum_{k=1}^K \mathbf{x}(t_k) e^{-j \sum_{p=1}^P \hat{b}_{np} t_k^p} \quad (23)$$

Although the above approximate ML algorithm is significantly simpler than the exact ML algorithm, it still requires the identification of N local peaks in the cost function. To further reduce the computational complexity we suggest the following algorithm.

1. $\mathbf{y}(t_k) = \mathbf{x}(t_k)$ for $k = 1, \dots, K$.
2. For $n = 1, \dots, N$ do the following:
 - a. Let

$$\{\hat{b}_{n1}, \dots, \hat{b}_{nP}\} = \arg \max_{b_1, \dots, b_P} \sum_{m=1}^M \left| \sum_{k=1}^K y_m(t_k) e^{-j(b_1 t_k + \dots + b_P t_k^P)} \right|^2 \quad (24)$$

- b. Remove temporarily the n -th source from $\{y(t_k)\}$ by

$$\begin{aligned} \mathbf{y}(t_k) &\leftarrow \mathbf{y}(t_k) e^{-j \sum_{p=1}^P \hat{b}_{np} t_k^p} \\ \mathbf{y}(t_k) &\leftarrow \mathbf{y}(t_k) - \frac{1}{K} \sum_{k=1}^K \mathbf{y}(t_k) \\ \mathbf{y}(t_k) &\leftarrow \mathbf{y}(t_k) e^{j \sum_{p=1}^P \hat{b}_{np} t_k^p} \end{aligned} \quad (25)$$

for $k = 1, \dots, K$

3. Remove permanently the N -th source from $\{x(t_k)\}$ in a similar fashion to (25).
4. $N \leftarrow N - 1$. If $N \geq 1$ go to Step 1.

To demonstrate the performance of the above algorithm we present the results of a simulated experiment and compare them with the CRB. In this experiment we used the scenario of Section 2 with $\xi_2 = 0.7$. We fixed the source separation at 5 degrees, and considered SNR values of -10, -5, 0, and 5 dB. At each SNR value we performed 100 Monte-Carlo runs. The results are shown in Figure 2, where we plot the RMS errors of the phase parameter estimates as a function of the SNR. The phase parameters are represented by the initial frequency of the chirp, and its frequency rate. At the SNR values under consideration there is a good agreement with the CRB.

4. REFERENCES

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