PERFORMANCE OF DECOUPLED DIRECTION FINDING BASED ON BLIND SIGNAL SEPARATION

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ABSTRACT

In recent years a number of methods for blindly separating superimposed digitally modulated signals arriving at an antenna array have been proposed. These techniques are efficient at the up-link (mobile to base) in a mobile communication system. However, for solving the base-to-mobile beamforming problem it may be necessary to also estimate the directions-of-arrival (DOAs) of the various signal paths. We present an optimal decoupled DOA estimation procedure based on information from the blind separation algorithm. Its performance is evaluated in the presence of spatially correlated noise and array modeling errors. The proposed technique has computational advantages as compared to traditional DOA estimation, because the different signal waveforms are treated in a separated fashion. Yet, the decoupled approach is shown to be substantially less sensitive to modeling errors and interference.

1. INTRODUCTION

During the past decade, a number of signal structure based approaches for blind signal estimation have appeared in the literature. Besides fractionally spaced equalization, an important application of these algorithms is signal separation in base station antenna arrays. The idea is to use knowledge of the signal structure to derive signal waveform estimates without requiring expensive direction-of-arrival (DOA) estimation. The signal properties that have been exploited include constant modulus [1, 2], non-Gaussianity [3, 4], cyclic correlation properties [5] and finite alphabet structure [6, 7, 8].

However, these methods apply only on the up-link (mobile to base), since they require an estimate of the communication channel. In a TDD (Time-Division Duplex) system, the up-link channel can be expected to be at least close to the down-link channel, implying

that the derived blind beamformer can be used also for down-link beamforming. Unfortunately, most surface-covering mobile communication system use FDD (Frequency-Division Duplex), which means that different carriers are used for up-link and down-link communication. The up-link beamformer may then be useless at the down-link, because the fading is likely to be completely different at the new carrier. The following three approaches have been proposed to solve the down-link beamforming problem: 1) Estimate the DOAs of all signal paths. Compute the steering vectors at the new carrier frequency (if necessary) and form a transmit beamformer that nulls all undesired signal paths (e.g. [9, 10]); 2) Incorporate mobile-to-base feedback of the signal quality [11]; 3) Assume that the up-link and down-link fading *statistics* are the same. Track the up-link fading statistics and use these to construct an "on-the-average optimal" down-link beamformer [12].

All of these have drawbacks: 1) requires a calibrated array, 2) requires modifications in existing communication systems and cost bandwidth, whereas 3) may perform poorly in severe multipath scenarios. The approach suggested here is to use DOA estimation as in 1), but exploiting preliminary signal estimates from the separation algorithm. This approach is demonstrated to alleviate the sensitivity to calibration errors and interference.

2. DOA ESTIMATION

Suppose an m-element antenna array receives the waveforms of d narrowband emitters. At baseband, the array output is modeled by the following familiar equation

$$\mathbf{x}(t) = \sum_{k=1}^{d} \mathbf{a}_k s_k(t) + \mathbf{n}(t) \ , \ t = 1, \dots, N. \qquad (1)$$

Here, $s_k(t)$ is the k:th waveform, $\mathbf{n}(t)$ is the noise vector, whereas $\mathbf{a}_k$ is the *spatial signature* of user k. The

noise is assumed to be circularly symmetric with unknown and arbitrary covariance matrix, and the signal waveforms are uncorrelated. Assuming frequency-flat fading, the spatial signatures are modeled by

$$\mathbf{a}_k = \sum_{l=1}^{d_k} \rho_{kl}\mathbf{a}(\theta_{kl}) , \qquad (2)$$

where ρ_{kl} is the reflection coefficient and θ_{kl} is the DOA of the l:th signal path emanating from user k. The number of such paths is denoted d_k. The function $\mathbf{a}(\theta)$ is termed the array manifold, and it represents the response of the array to a unit signal from the DOA θ. Clearly, any DOA estimator requires information of the functional form of $\mathbf{a}(\theta)$. In most practical cases, the manifold must be measured experimentally (calibration), which is a costly and time-consuming procedure. In addition, the resulting manifold is inevitably subject to errors.

Blind estimation procedures are often employed for circumventing the need for array calibration. However, the estimated signal waveforms can also be used for DOA estimation. The problem of interest herein is to estimate the DOAs $\{\theta_{kl}\}_{l=1}^{d_k}$ associated with a particular user k, given observations $\{\mathbf{x}(t)\}_{t=1}^{N}$ and signal waveform estimates $\{s_k(t)\}_{t=1}^{N}$. Similar[1] to [13], one may postulate a model for user k:

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta}_k)\mathbf{b}_k\, s_k(t) + \mathbf{j}(t) , \qquad (3)$$

where

$$\mathbf{A}(\boldsymbol{\theta}_k) = [\mathbf{a}(\theta_{k1}),\ldots,\mathbf{a}(\theta_{kd_k})] \qquad (4)$$
$$\mathbf{b}_k = [\rho_{k1},\ldots,\rho_{kd_k}]^T \qquad (5)$$

and where $\mathbf{j}(t)$ represents the contribution of the "interfering" signals plus noise. Given an estimate $s_k(t)$ provided by the blind algorithm[2], we can model $\mathbf{j}(t)$ as a white Gaussian noise and derive the maximum likelihood (ML) estimate of $\boldsymbol{\theta}_k = [\theta_{k1},\ldots,\theta_{kd_k}]^T$. Assuming $\mathbf{j}(t)$ to be circularly symmetric $\mathbf{j}(t) \in \mathcal{N}(0,\mathbf{Q})$, the model (3) is identical to the parameterized signals model considered in [14].

The following notation is introduced:

$$\hat{\boldsymbol{\Pi}}^{\perp} = \mathbf{I} - \hat{\mathbf{Q}}^{-1/2}\mathbf{A}(\mathbf{A}^*\hat{\mathbf{Q}}^{-1}\mathbf{A})^{-1}\mathbf{A}^*\hat{\mathbf{Q}}^{-1/2} \quad (6)$$
$$\hat{\mathbf{Q}} = \hat{\mathbf{R}}_{xx} - \hat{\mathbf{r}}_{xs}\hat{r}_{ss}^{-1}\hat{\mathbf{r}}_{xs}^* \qquad (7)$$
$$\hat{\mathbf{r}}_{xs} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}(t)s_k^*(t) , \qquad (8)$$

where the argument of $\mathbf{A} = \mathbf{A}(\boldsymbol{\theta}_k)$ has been suppressed for notational simplicity. Further, $(\cdot)^*$ is Hermitian

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[1] In [13], only the case where $d_k = 1$ is treated.
[2] With some abuse of notation, we use the symbol $s_k(t)$ both for the true signal and its estimate.

transpose, $\mathbf{Y}^{-1/2}$ denotes a Hermitian square-root of a positive definite matrix $\mathbf{Y}$ and the sample covariances $\hat{\mathbf{R}}_{xx}$ and $\hat{r}_{ss}$ are defined similarly to (8). From [14], the exact ML estimate of $\boldsymbol{\theta}_k$ is obtained by solving the following d_k-dimensional optimization problem

$$\hat{\boldsymbol{\theta}}_k = \arg\min_{\boldsymbol{\theta}_k} f(\boldsymbol{\theta}_k) \qquad (9)$$
$$f(\boldsymbol{\theta}_k) = \|\hat{\boldsymbol{\Pi}}^{\perp}\hat{\mathbf{Q}}^{-1/2}\hat{\mathbf{r}}_{xs}\|^2 . \qquad (10)$$

Once $\hat{\boldsymbol{\theta}}_k$ has been computed, the complex signal amplitudes derive from the expression

$$\hat{\mathbf{b}}_k = (\mathbf{A}^*\hat{\mathbf{Q}}^{-1}\mathbf{A})^{-1}\mathbf{A}^*\hat{\mathbf{Q}}^{-1}\hat{\mathbf{r}}_{xs}\hat{r}_{ss}^{-1} . \qquad (11)$$

Note that the decoupled ML method requires a d_k-dimensional search for each signal waveform, as opposed to a full $\sum d_k$-dimensional optimization that is necessary when the blind signal estimates are not exploited. It may also be noted that the above exact ML approach differs from that of [13] in that the data are weighted by $\hat{\mathbf{Q}}^{-1/2}$ rather than $\hat{\mathbf{R}}^{-1/2}$. It may be argued that the weightings are asymptotically equivalent for large N.

3. PERFORMANCE ANALYSIS

Note that the estimated signal waveform $s_k(t)$ is treated as being exact, which means that the above method can also be viewed as an extension of the decoupled ML algorithm of [15]. Indeed, the optimality of the decoupled approach in the absense of modeling errors follows from [15]. However, in practice the signals are estimated using a blind signal separation algorithm. Thus, the effects of such modeling errors is of interest. Using a first order analysis, it is straightforward to show that the ratio of the DOA MSE error due to $\tilde{s}_k(t)$ and that due to $\mathbf{j}(t)$ is asymptotically (for large N) proportional to $\mathrm{E}[|\tilde{s}_k(t)|2]/r_{ss}$, which in turns is essentially the symbol-error-rate (SER) of signal k. In a practical system, the operating region is better than SER=1%, which indeed suggests that the estimated signal waveforms can be regarded as exact.

The asymptotic (for large N) variance of the DOA estimation errors due to noise plus interference follows immediately from [14]. To include the effect of modeling errors, we assume the data were actually generated according to

$$\mathbf{x}(t) = \Big(\mathbf{A}(\boldsymbol{\theta}_k) + \tilde{\mathbf{A}}(\boldsymbol{\theta}_k)\Big)\mathbf{b}_k\, s_k(t) + \mathbf{j}(t) , \qquad (12)$$

where $\tilde{\mathbf{A}}(\boldsymbol{\theta}_k)$ denotes a zero-mean unstructured error term. The following model is adopted from [16]

$$\mathrm{E}[\tilde{\mathbf{a}}(\theta_{kl})\tilde{\mathbf{a}}^*(\theta_{km})] = \nu_{lm}\boldsymbol{\Psi} \qquad (13)$$
$$\mathrm{E}[\tilde{\mathbf{a}}(\theta_{kl})\tilde{\mathbf{a}}^T(\theta_{km})] = 0 . \qquad (14)$$

The direction-dependent covariance terms are collected into the matrix

$$\mathbf{\Upsilon} = \{\nu_{lm}\}_{l,m=1}^{d_k} \tag{15}$$

The following result gives an approximate expression for the covariance matrix of the estimation error. The proof follows essentially [17] and [14]. The details will be presented elsewhere.

Theorem 1 *Assume that the SER is low, so that the effects of signal estimation errors can be neglected. Further, assume that* $\mathbf{j}(t)$ *is circularly symmetric (not necessarily Gaussian) with covariance matrix* $\mathbf{Q}$. *Then, up to first order in* $1/N$ *and* $\{\nu\}_{ii}$, *the mean square error (MSE) matrix of the DOA estimate (9)–(10) is given by*

$$E\left[(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k)(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k)^T\right] \approx \mathbf{H}^{-1}(\mathbf{H}\, r_{ss}/N + \tilde{\mathbf{H}})\mathbf{H}^{-1}, \tag{16}$$

where

$$\mathbf{H} = 2r_{ss}^2\, \mathrm{Re}(\mathbf{BD}^*\tilde{\mathbf{\Pi}}\mathbf{DB}^*) \tag{17}$$

$$\tilde{\mathbf{H}} = 2r_{ss}^4\, \mathbf{b}_k^*\mathbf{\Upsilon}^T\mathbf{b}_k\, \mathrm{Re}(\mathbf{BD}^*\tilde{\mathbf{\Pi}}\mathbf{\Psi}\tilde{\mathbf{\Pi}}\mathbf{DB}^*) \tag{18}$$

$$\mathbf{B} = \mathrm{diag}\{\rho_{k1}, \ldots, \rho_{kd_k}\} \tag{19}$$

$$\tilde{\mathbf{\Pi}} = \mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{A}(\mathbf{A}^*\mathbf{Q}^{-1}\mathbf{A})^{-1}\mathbf{A}^*\mathbf{Q}^{-1} \tag{20}$$

$$\mathbf{D} = [\partial\mathbf{a}(\theta_{k1})/\partial\theta_{k1}, \ldots, \partial\mathbf{a}(\theta_{kd_k})/\partial\theta_{kd_k}]. \tag{21}$$

It should be emphasized that the above result does not rely on any assumption regarding the PDFs of $\mathbf{j}(t)$ and $\tilde{\mathbf{A}}$. Further, the assumption that $\mathbf{j}(t)$ is circularly symmetric is made for simplicity only. In the case of binary phase shift keying (BPSK) signals this assumption is violated. The algorithm should then be applied to a real data vector, formed by stacking the real and imaginary parts of $\mathbf{x}(t)$. The array response vectors $\mathbf{a}(\theta)$ must be similarly modified. The formula for the MSE matrix given above still holds, but the $2\mathrm{Re}(\cdot)$ in the definitions of $\mathbf{H}$ and $\tilde{\mathbf{H}}$ should be dropped.

As a final remark we note that the above result is given for a fixed "scattering vector" $\mathbf{b}_k$. In a fading scenario, $\mathbf{b}_k$ is usually modeled as random. The different matrices involved in the DOA MSE expression should then be averaged also over the fading parameters.

4. EXAMPLE

As a measure of the sensitivity of the MSE expression to the assumption of perfectly estimated signals, we choose to consider an extremely difficult scenario involving two mobiles, each subject to a coherent reflection. The first signal is the SOI (signal of interest), and it arrives at the $m = 8$ element standard ULA (uniform linear array) from the DOAs $\boldsymbol{\theta}_1 = [80°, 88°]^T$ relative the endfire of the array. The amplitudes are
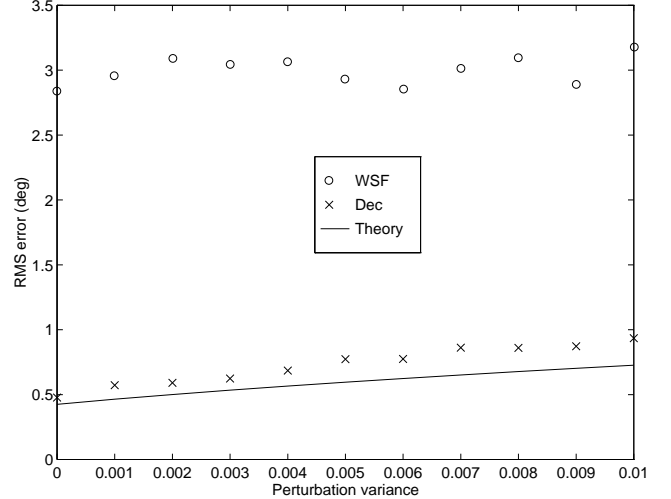

Figure 1: *Theoretical and empirical RMS error for the source at* 80° *versus the variance of the array response errors.*

$\mathbf{b}_1 = [1, 0.5]^T$, corresponding to signal-to-noise ratios of 0 dB and -3 dB respectively. The corresponding parameters for the interfering signal are $\boldsymbol{\theta}_2 = [100°, 92°]^T$ and $\mathbf{b}_2 = [1, 0.5]^T$. Hence, the separation between the direct and reflected path of the SOI is approximately one Rayleigh beamwidth, whereas the two reflected paths are within 0.5 beamwidths. The signal constellation is quadrature phase shift keying (QPSK) with $r_{ss} = 1$, and the background noise is temporally white $\mathcal{N}(0, \mathbf{I})$. The task is to estimate the DOAs $\boldsymbol{\theta}_1$ based on a batch of $N = 200$ symbols. The array response is subject to a Gaussian perturbation with $\mathbf{\Psi} = \mathbf{I}$ and $\mathbf{\Upsilon} = \nu\,\mathbf{I}$, where ν is varied from 0 to 0.01 (i.e., up to 10% errors on the individual components of $\mathbf{a}(\theta)$).

The SOI is first estimated using the DWILSP (decoupled weighted iterative least squares with projection) algorithm [8], and using a 10 symbol periods long training sequence. In this severe scenario, an SER of approximately 0.5% is obtained. The proposed algorithm is then applied, and the RMS DOA error is calculated from 1024 independent trials. In $\leq 0.2\%$ of the trials, the DWILSP algorithm failed to capture the SOI, and these outliers were removed when computing the RMS error. As a reference, the weighted subspace fitting (WSF) algorithm [18] was applied, assuming $\sum d_k = 4$ signals. The WSF algorithm yielded DOA errors larger than 10° in $13\% - 24\%$ of the trials (depending on ν), and these were also removed when computing the RMS error. The results for θ_{11} are displayed in Figure 1, whereas Figure 2 shows the results for θ_{12}. The results for the proposed algorithm are labeled Dec (Decoupled DOA estimator) in the plots.
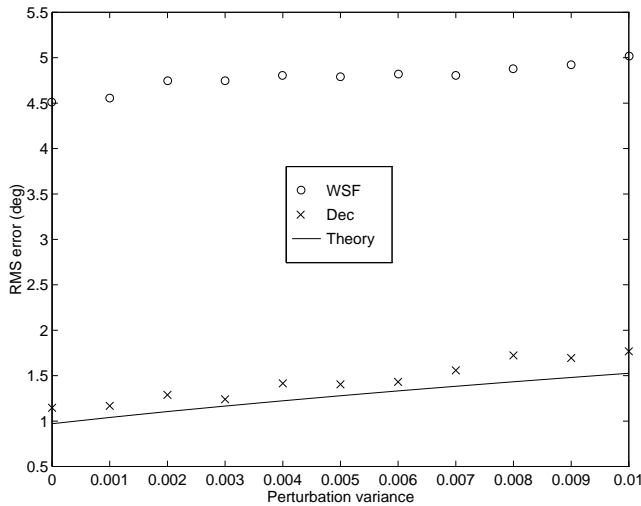
Despite the relatively large SER, the agreement be-

Figure 2: *Theoretical and empirical RMS error for the source at 88° versus the variance of the array response errors.*

tween the theoretical and empirical RMS errors is satisfactory. A standard DOA algorithm which does not make use of estimated signals is clearly not useful in this difficult scenario.

5. CONCLUSIONS

A decoupled algorithm for DOA estimation based on preliminary signal waveform estimates has been proposed. The MSE matrix of the DOA estimates was derived, assuming small errors due to finite sample effects and array response errors. The effects of erroneous signal estimates were shown to be negligible. A computer simulation of a difficult scenario with closely spaced DOAs and multipath indicated that the theoretical MSE expression accurately predicts the empirical performance with symbol error rates as large as 0.5% and array perturbations up to 10%.

6. REFERENCES

[1] R.P. Gooch and J. Lundel, "The CM Array: An Adaptive Beamformer for Constant Modulus Signals", In *Proc. ICASSP 86*, pages 2523–2526, Tokyo, Japan, 1986.

[2] B. Agee, "Blind Separation and Capture of Communication Signals Using a Multitarget Constant Modulus Beamformer", In *Proc. IEEE MILCOM*, pages 340–346, 1989.

[3] J.-F. Cardoso, "Source Separation Using Higher Order Moments", In *Proc. ICASSP 89*, pages 2109–2112, Glasgow, Scotland, May 1989.

[4] L. Tong, Y. Inouye, and R. Liu, "Waveform-Preserving Blind Estimation of Multiple Independent Sources", *IEEE Trans. on SP*, **41**:2461–2470, Jul. 1993.

[5] B.G. Agee, A.V. Schell, and W.A. Gardner, "Spectral Self-Coherence Restoral: A New Approach to Blind Adaptive Signal Extraction Using Antenna Arrays", *Proc. IEEE*, **78**:753–767, Apr. 1990.

[6] H. Liu and G. Xu, "A Deterministic Approach to Blind Symbol Estimation", *IEEE SP Letters*, **1**:205–207, Dec. 1994.

[7] S. Talwar, M. Viberg, and A. Paulraj, "Blind Estimation of Multiple Co-Channel Digital Signals Arriving at an Antenna Array: Part I, Algorithms", *IEEE Trans. on SP*, pages 1184–1197, May 1996.

[8] A. Ranheim and P. Pelin, "Decoupled Blind Symbol Estimation Using an Antenna Array", In *Proc. 8:th Signal Processing Workshop on SSAP*, pages 136–139, Corfu, Greece, June 1996.

[9] S. Andersson, M. Millnert, M. Viberg, and B. Wahlberg, "An Adaptive Array for Mobile Communication Systems", *IEEE Trans. on Veh. Tec.*, **40**(1):230–236, 1991.

[10] P. Zetterberg and B. Ottersten, "The Spectrum Efficiency of a Base-Station Antenna Array System for Spatially Selective Transmission", *IEEE Trans. on VT*, **44**(3):651–660, 1995.

[11] D. Gerlach and A. Paulraj, "Adaptive Transmitting Antenna Arrays with Feedback", *IEEE SP Letters*, **1**:150–152, Oct. 1994.

[12] G. Raleigh, S.N. Diggavi, V. Jones, and A. Paulraj, "A Blind Adaptive Transmit Antenna Algorithm for Wireless Communication", In *Proc. ICC*, 1995.

[13] B. Agee, "The Copy/DF Approach to Signal-Specific Emitter Location", In *Proc. 25:th Asilomar Conf. Sig., Syst., Comput.*, pages 994–999, Pacific Grove, CA, Nov. 1991.

[14] P. Stoica, M. Viberg, B. Ottersten, and T. Kailath, "Optimal Localization of Partially Known Signals in Unknown Noise Fields", In *Proc. ICASSP 94*, Adelaide, Australia, April 1994.

[15] J. Li, B. Halder, P. Stoica, M. Viberg, and T. Kailath, "Decoupled Maximum Likelihood Angle Estimation for Signals with Known Waveforms", *IEEE Trans. on SP*, **43**:2154–2163, Sept. 1995.

[16] A. Swindlehurst and T. Kailath, "A Performance Analysis of Subspace-Based Methods in the Presence of Model Errors - Part I: The MUSIC Algorithm", *IEEE Trans. SP*, **SP-40**(7):1758–1774, July 1992.

[17] M. Viberg and A.L. Swindlehurst, "Analysis of the Combined Effects of Finite Samples and Model Errors on Array Processing Performance", *IEEE Trans. SP*, **42**:3073–3083, Nov. 1994.

[18] M. Viberg and B. Ottersten, "Sensor Array Processing Based on Subspace Fitting", *IEEE Trans. SP*, **SP-39**(5):1110–1121, May 1991.