

A GAUSSIAN SUM FILTERING APPROACH FOR PHASE AMBIGUITY RESOLUTION IN GPS ATTITUDE DETERMINATION

Kenneth J. Turner and Farhan A. Faruqi

Signal Processing Research Centre, QUT
GPO Box 2434, Brisbane Q 4001, Australia
email: k.turner@qut.edu.au

ABSTRACT

The problem of phase ambiguity resolution and filtering for interferometric GPS attitude determination is considered. Traditionally, the resolution of the phase ambiguity and the filtering stages were performed separately, with the filter formulated on the basis that the phase ambiguity is correctly resolved. Should the pre-processing stage not resolve the ambiguity correctly, erroneous results may occur. In response, a unified solution is proposed in which the ambiguity resolution and filtering processes are combined under a Gaussian Sum Filtering (GSF) framework. The GSF naturally accounts for the measurement ambiguity by generating multi-modal probability densities, which leads to a probabilistic interpretation of the attitude estimates. Simulations are performed to illustrate the effectiveness and functionality of the proposed solution.

1. INTRODUCTION

There exists a number of applications and filtering problems in which measurement ambiguities arise. Normally, operating limitations or computationally inefficient pre-processing stages are required to resolve these ambiguities before processing or filtering may be initiated. In general, these pre-processing methods are formulated in a deterministic sense. Subsequently, no statistical measures are available to gauge the confidence and significance of the estimates.

A typical example of a practical problem which contains inherent measurement ambiguity is the Global Positioning System (GPS) attitude determination system [4, 7, 9, 13, 14]. This system utilises satellite carrier phase interferometric techniques in order to estimate the orientation of an antenna array. The attitude (Φ^L) is defined as the departure angles from a fixed reference or locally oriented axes system (L) to a vehicle fixed or body aligned axes system (B) such that,

$$\Phi^L = \begin{bmatrix} \varphi & \theta & \psi \end{bmatrix} \quad (1)$$

where φ , θ and ψ are the respective Roll, Pitch and Yaw angles.

Using three or more satellite signals with at least three antennae, as shown in Figure 1, it is possible to determine the platform attitude so long as the carrier cycle number ambiguities can be resolved. Ambiguities arise once the antenna separation becomes larger than the wavelength (λ) of the carrier signal. This is generally the case, since other

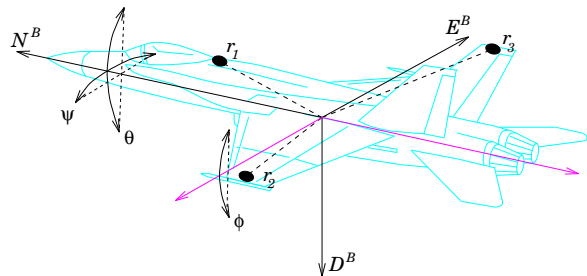


Figure 1. GPS attitude determination

stages in the navigation system require that the antennae be placed at the furthest points from one another [12].

Applications of GPS attitude determination systems include maritime attitude determination to aid multi-beam echo sounding, precise aircraft attitude determination to aid Synthetic Aperture Radar (SAR) systems and attitude stabilisation and control of low-orbit satellites [8].

2. PHASE AMBIGUITY RESOLUTION

Historically, a deterministic formulation involving carrier phase differences between the antennae is employed to resolve the phase cycle number (k) ambiguity. Details of these methods are given in [13].

In order to attain more robust estimates in noisy environments, redundant phase measurements and differences are taken. Using this redundancy it is possible to formulate an expression for the residual mean square error for every combination of the phase ambiguities (k). The phase ambiguity set with the lowest mean square error, under an exhaustive search, is chosen as the estimate for the phase ambiguity. It is possible, that the result of this method may result in some spurious solutions becoming equally, or more likely than the true ambiguity. Since the proceeding filter assumes that the ambiguity is fully resolved, unexpected results may occur.

3. GAUSSIAN SUM FILTERING

To address the carrier phase ambiguity problem a Gaussian Sum Filter (GSF) is investigated [1, 3, 5, 10, 11]. Intuitively, one may view the measurement ambiguity as manifesting itself as a multi-modal measurement noise probability density function. This is readily handled by the fundamental structures and procedures which form the basis of the GSF.

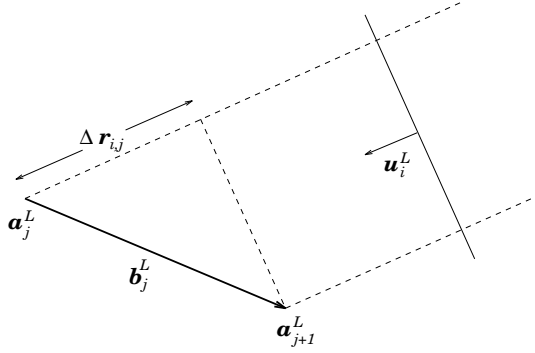


Figure 2. Carrier Phase Difference Measurements

The density functions manipulated by the GSF take the form of convex sums of Gaussian probability density functions [11] as shown below,

$$p(\mathbf{x}) = \sum_{i=1}^n \alpha_i \frac{1}{(2\pi)^{\frac{m}{2}} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} [\mathbf{x} - \boldsymbol{\mu}_i]^T \Sigma_i^{-1} [\mathbf{x} - \boldsymbol{\mu}_i] \right\}$$

where,

$$\sum_{i=1}^n \alpha_i = 1, \quad |\alpha_i| > 0 \quad (2)$$

given that n is the number of approximating terms in the sum, m is the length of the state vector and α_i is the weight for the i th term.

The filter structure updates the individual $\boldsymbol{\mu}_i$ and Σ_i using an Extended Kalman Filter (EKF), while updating the weighting terms α_i using an interpretation of Bayes' Theorem. Subsequently, the GSF may be viewed as a convex sum of Extended Kalman Filters operating in parallel.

As each phase difference measurement is processed, additional terms are generated for each of the possibilities arising from the measurement ambiguity. The weighting terms are then calculated, with terms containing a negligible α_i being removed. Subsequently, there is no need to conduct an exhaustive search of all of the ambiguity combinations, as a large proportion of the search space may be neglecting using this probabilistically justified basis. The process is repeated for a number of measurements until only one significant term remains.

4. THE MEASUREMENT MODEL

Figure 2 depicts the measurement construction for a single phase difference measurement. A baseline vector ($\mathbf{b}_j^L$) is constructed as an inter-antenna displacement vector. Since the baseline length is negligible when compared to distance from the satellite to the receiver, it is acceptable to assume that the satellite signals propagate, in the direction of the line of sight, as plane waves. Projection of the baseline vector onto the line of sight vector to the satellite ($\mathbf{u}_i^L$) results in the ranging difference ($\Delta \mathbf{r}_{ij}$) defined as,

$$\Delta \mathbf{r}_{ij} = \mathbf{u}_i^L \cdot \mathbf{b}_j^L = \lambda \left(\frac{\Delta \phi_{ij}}{2\pi} + k \right) \quad (3)$$

for the i th satellite at the j th baseline, where $\Delta \phi_{ij}$ is the wrapped phase difference and k an integer number of carrier

cycles. A line of sight vector may be constructed by considering the approximate coordinates of the satellites and the interferometer, which are attained through a standard GPS navigation solution. It is seen that attitude solution is relatively insensitive to small deviations from the true line of sight vector [6].

The baseline vector ($\mathbf{b}_j^L$) may be expressed in terms of a Body or vehicle fixed vector ($\mathbf{b}_j^B$) and the transformation from Body to Local coordinates ($[\mathbf{C}(\boldsymbol{\Phi}^L)_B^L]$), which is a function of the attitude vector ($\boldsymbol{\Phi}^L$) [2],

$$[\mathbf{C}(\boldsymbol{\Phi}^L)_B^L] = \begin{bmatrix} c\theta c\psi & s\theta s\varphi c\psi - c\varphi s\psi & s\theta c\varphi c\psi + s\varphi s\psi \\ c\theta s\psi & s\theta s\varphi s\psi + c\varphi c\psi & s\theta c\varphi s\psi - s\varphi c\psi \\ -s\theta & c\theta s\varphi & c\theta c\varphi \end{bmatrix}$$

with $s = \sin$ and $c = \cos$. Subsequently, the phase difference measurement may be written as,

$$\Delta \phi_{m_{ij}} = \frac{2\pi}{\lambda} \mathbf{u}_i^L \cdot ([\mathbf{C}(\boldsymbol{\Phi}^L)_B^L] \mathbf{b}_j^B) - 2\pi k + \eta_{ij} \quad (4)$$

where η_{ij} is the measurement noise. Now, one may define k as,

$$k = k_n + \Delta k; \quad \Delta k = 0, \pm 1, \pm 2, \dots, \pm k_{\max} \quad (5)$$

given that Δk is the integer phase ambiguity, $k_{\max}$ is the maximum phase deviation considered and k_n is the nominal estimate for the integer phase difference. The nominal integer phase difference is defined in terms of the nominal attitude estimate ($\hat{\boldsymbol{\Phi}}^L$) as,

$$k_n = \lfloor \frac{1}{\lambda} \mathbf{u}_i^L \cdot ([\mathbf{C}(\hat{\boldsymbol{\Phi}}^L)_B^L] \mathbf{b}_j^B) \rfloor \quad (6)$$

Subsequently, Equation (4) may be written in the form,

$$\Delta \phi_{m_{ij}} = \frac{2\pi}{\lambda} \mathbf{u}_i^L \cdot ([\mathbf{C}(\boldsymbol{\Phi}^L)_B^L] \mathbf{b}_j^B) - 2\pi k_n + \{2\pi \Delta k + \eta_{ij}\} \quad (7)$$

The term in braces represents the combined measurement noise whose probability density function is multi-modal in nature. The phase disturbance (η_{ij}) is modelled as a zero mean, white Gaussian process. This assumption is imposed to ensure the brevity of this discussion and is indeed not a limitation of the GSF. For example, one may model the multi-path effects by a non-Gaussian process [7], which in turn may be represented as a weighted sum of Gaussian processes.

Finally, to conform to the GSF structure, the first partial derivative of Equation (7) is taken with respect to the attitude in order to linearise the measurement function. Thus, the linearised error measurement may be expressed as,

$$\delta(\Delta \phi_{m_{ij}}) = [\mathbf{H}_{ij}] \delta \boldsymbol{\Phi} + \{2\pi \Delta k + \eta_{ij}\} \quad (8)$$

where $[\mathbf{H}_{ij}]$ is defined as,

$$[\mathbf{H}_{ij}] = \left. \frac{\partial \frac{2\pi}{\lambda} \mathbf{u}_i^L \cdot ([\mathbf{C}(\boldsymbol{\Phi}^L)_B^L] \mathbf{b}_j^B)}{\partial \boldsymbol{\Phi}^L} \right|_{\boldsymbol{\Phi} = \hat{\boldsymbol{\Phi}}} \quad (9)$$

The remaining partial derivatives are evaluated as follows,

$$\begin{aligned}\frac{\partial[C(\Phi^L)_B^L]}{\partial\varphi} &= \begin{bmatrix} 0 & s\theta c\varphi c\psi + s\varphi s\psi & -s\theta s\varphi c\psi + c\varphi s\psi \\ 0 & s\theta c\varphi s\psi - s\varphi c\psi & -s\theta s\varphi s\psi - c\varphi c\psi \\ 0 & c\theta c\varphi & -c\theta s\varphi \end{bmatrix} \\ \frac{\partial[C(\Phi^L)_B^L]}{\partial\theta} &= \begin{bmatrix} -s\theta c\psi & c\theta s\varphi c\psi & c\theta c\varphi c\psi \\ -s\theta s\psi & c\theta s\varphi s\psi & c\theta c\varphi s\psi \\ -c\theta & -s\theta s\varphi & -s\theta c\varphi \end{bmatrix} \\ \frac{\partial[C(\Phi^L)_B^L]}{\partial\psi} &= \begin{bmatrix} -c\theta s\psi & -s\theta s\varphi s\psi - c\varphi c\psi & -s\theta c\varphi s\psi + s\varphi c\psi \\ c\theta c\varphi & s\theta s\varphi c\psi - c\varphi s\psi & s\theta c\varphi c\psi + s\varphi s\psi \\ 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

Though not addressed in the following simulated analysis, it is worth noting the mechanisms by which the ambiguity may be resolved in rotating systems. By considering a situation in which the rotation of the body (ω^B) is available, one may formulate a predictor mechanism for the diffusion of the attitude estimates between GPS epochs. This is performed in a similar fashion as that for the measurement function formulation, in that the Euler angle evolution relations [2],

$$\frac{d}{dt} \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\varphi & \tan\theta \cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi \sec\theta & \cos\varphi \sec\theta \end{bmatrix} \omega^B \quad (10)$$

are linearised about the attitude estimate.

The body rates (ω^B) may be made available through inertial gyroscope measurements or through interpretation of the interferometric phase rates. It is seen that the inertially aided system, once resolved, no longer requires online ambiguity resolution and the results are consistent with those for the non-rotating case. The case in which the body rates are inferred from the differential phase measurements is decidedly more complicated and embodies a sizeable discussion in its own right.

5. SIMULATION RESULTS

To illustrate the effectiveness of this method, consider a configuration which utilises three satellites and three antennas. The antennae are fixed at known points to a platform which is at a constant yet unknown orientation. Each of the antennae are placed at a distance of 1 m from the platform centre, at equal angles of $2\pi/3$. The carrier wavelength (λ) for GPS is approximately 19 cm and the standard deviation for GPS carrier phase noise is set to 0.25 radians. The true platform attitude is set to zero, with an initial estimate of -0.15 radians for each of the attitude parameters. The initial estimates are assumed uncorrelated with variances of $(0.2)^2 \text{ rad}^2$. For these initial conditions it is sufficient to consider only $k_{\max} = 2$ integer phase ambiguities.

Figure 3 shows the marginal probability densities for (a) the initial conditions, (b) after processing the first differential phase measurement and (c) after processing all of the differential phase measurements for the first epoch. Figure 3(b) is particularly informative in that the multimodality of the conditional density is evident. Combining densities of this form, with differing orientations, for each differential phase measurement in the current GPS

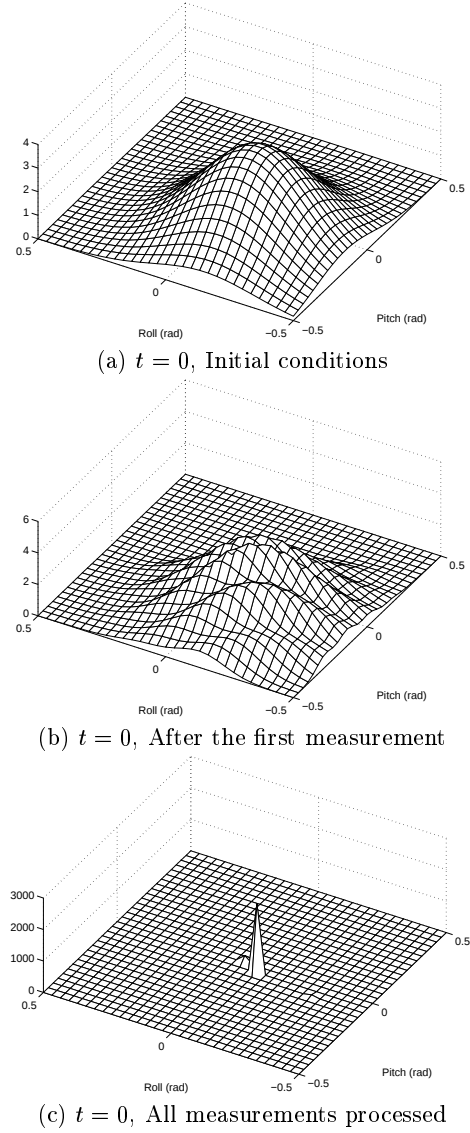


Figure 3. Simulated Example

epoch gives the conditional density shown in Figure 3(c). From this figure one can see that a spurious mode is dominant, and hence the phase ambiguity remains unresolved. It should also be noted that the desired solution remains with a significant probability.

Figure 4 gives the marginal probability densities for the attitude's Roll and Pitch parameters for the initial conditions, and the first two iterations. Each iteration processes every baseline's differential phase measurements for each of the satellite signals. It is seen that, similar to Figure 3(c), an ambiguity exists after the first iteration ($t = 1$) for which a spurious mode is dominant. After processing the measurements for the second epoch ($t = 2$), this ambiguity is resolved correctly.

The resulting filter estimates for the epochs $t \geq 2$ are consistent with estimates obtained through an Extended Kalman Filter as though the phase ambiguity is known correctly for all time. The classical method in which the

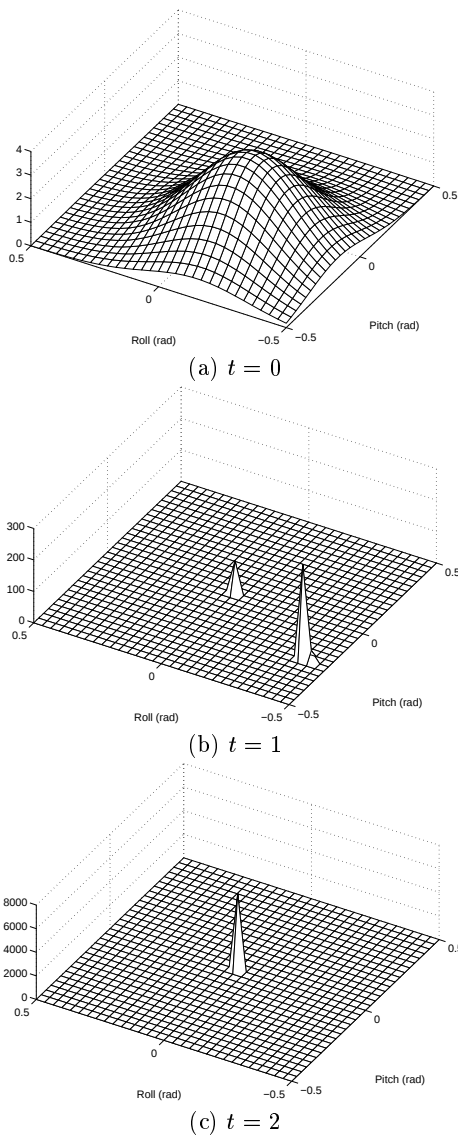


Figure 4. Simulated Example

ambiguity resolution and attitude filtering are performed sequentially would produce biased estimates as an artifact of the incorrectly resolved ambiguity at the first epoch. It should be noted that through processing additional measurements from either redundant baselines or satellites, the ambiguity may be resolved more accurately.

6. CONCLUSION

An alternative methodology for phase ambiguity resolution in GPS attitude determination was discussed. This procedure utilised a Gaussian Sum Filter in order to characterise the multi-modality of the conditional densities for the attitude estimates. This multi-modality is inherently induced through the ambiguous nature of the differential phase measurements. It is seen that the GSF formulation provides the desired ambiguity resolution in simulated scenarios.

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