

STRUCTURAL SUBBAND DECOMPOSITION: A NEW CONCEPT IN DIGITAL SIGNAL PROCESSING

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ABSTRACT

This paper introduces the concept of structural subband decomposition of sequences, a generalization of the polyphase decomposition of sequences, and outlines a number of applications of this concept, such as efficient FIR filter design and implementation, adaptive filtering, and fast computation of discrete transforms.

1. STRUCTURAL SUBBAND DECOMPOSITION

Any finite- or infinite-length sequence $\{x[n]\}$ with a z -transform $X(z)$ can be written as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{k=0}^{M-1} X_k(z^M)z^{-k}$$

$$= [1 \ z^{-1} \ \dots \ z^{-(M-1)}] \begin{bmatrix} X_0(z^M) \\ X_1(z^M) \\ \vdots \\ X_{M-1}(z^M) \end{bmatrix} \quad (1)$$

where

$$X_k(z) = \sum_{n=-\infty}^{\infty} x[Mn+k]z^{-n}, \quad 0 \leq k \leq M-1. \quad (2)$$

The right-hand side expression of Eq. (1) is called the *polyphase decomposition* of the transform $X(z)$ with the functions $X_k(z)$ being the *poly-phase components* of $X(z)$ [1]. If $x_k[n]$ denotes the inverse z -transform of $X_k(z)$, then

$$x_k[n] = x[Mn+k], \quad 0 \leq k \leq M-1,$$

i.e., the sequence $x_k[n]$ is simply obtained by down-sampling $x[n]$ by a factor of M with k indicating the phase of the sub-sampling process.

A generalization of the polyphase decomposition of Eq. (1), called the *structural subband decomposition* of $X(z)$, is given by [2]

$$X(z) = [1 \ z^{-1} \ \dots \ z^{-(M-1)}] T \begin{bmatrix} V_0(z^M) \\ V_1(z^M) \\ \vdots \\ V_{M-1}(z^M) \end{bmatrix} \quad (3)$$

where $T = [t_{ij}]$ is an $M \times M$ nonsingular matrix. Relation between the polyphase components $X_k(z)$ and the generalized polyphase components $V_k(z)$ are given by:

$$\begin{bmatrix} V_0(z) \\ V_1(z) \\ \vdots \\ V_{M-1}(z) \end{bmatrix} = T^{-1} \begin{bmatrix} X_0(z) \\ X_1(z) \\ \vdots \\ X_{M-1}(z) \end{bmatrix} \quad (4)$$

Depending on the application, the matrix T can have various forms. Some novel applications of structural decomposition are outlined next.

2. EFFICIENT FIR FILTER DESIGN AND IMPLEMENTATION

Consider an FIR filter $H(z)$ with an impulse response $\{h[n]\}$ of length N with $N = P \times M$, where P and M are positive integers. By applying the structural subband decomposition of Eq. (2) to $H(z)$ we can express it in the form:

$$H(z) = [1 \ z^{-1} \ \dots \ z^{-(M-1)}] T \begin{bmatrix} G_0(z^M) \\ G_1(z^M) \\ \vdots \\ G_{M-1}(z^M) \end{bmatrix} \quad (5a)$$

or, equivalently as

$$H(z) = \sum_{k=0}^{M-1} I_k(z) G_k(z^M), \quad (5b)$$

where $I_k(z)$ is given by

$$I_k(z) = \sum_{j=0}^{M-1} t_{k+1,j+1} z^{-j}, \quad k = 0, 1, \dots, M-1. \quad (6)$$

Realizations of $H(z)$ based on the structural subband decomposition are shown in Figure 1. It should be noted that the delays in the implementation of the sub-filters $G_k(z^M)$ in Figure 1 can be shared leading to a canonic realization of the overall structure.

Computational complexity of the overall structure can be reduced by choosing "simple" invertible transform matrices T . One such matrix is the $M \times M$ Hadamard matrix R_M which is given by

$$R_M = \underbrace{R_2 \otimes R_2 \otimes \dots \otimes R_2}_{L\text{-terms}} \quad (7)$$

where $R_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and \otimes denotes the Kronecker product. From Eq. (7) it can be seen that M must be a power-of-two, i.e., $M = 2^L$.

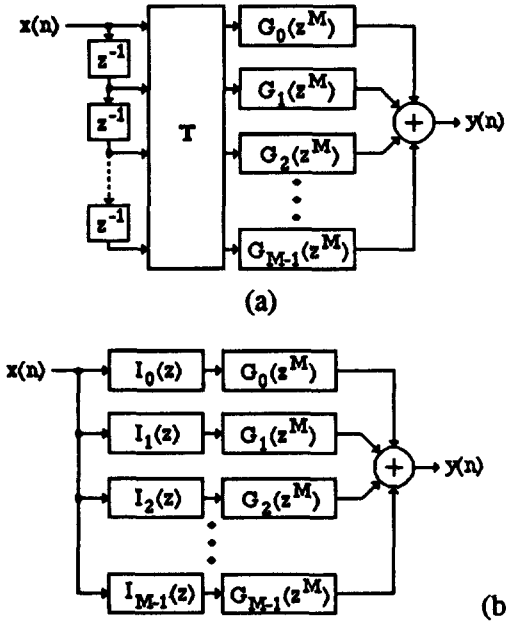


Figure 1

For an M -branch decomposition, note also that the subfilter $I_0(z)$ has a lowpass magnitude response given by $\sin(M\omega/2)/\sin(\omega/2)$, whereas, the subfilter $I_1(z)$ has a highpass magnitude response given by $\sin[M(\pi - \omega)/2]/\sin[(\pi - \omega)/2]$. Each of the remaining subfilters $I_k(z)$, for $k \neq 0, 1$ have a bandpass magnitude response. Each of the

branches in the realization of Figure 3(b) thus contributes to the overall response essentially within a "subband" associated with the corresponding interpolator. For some narrow-band FIR filters, it is possible to drop from the subband decomposition structure branches that do not contribute significantly to the overall frequency response, thus resulting in a computationally efficient realization.

The structural subband decomposition of an FIR transfer function $H(z)$ also simplifies considerably the filter design process. To this end, two different design approaches have been advanced recently. In one approach, each branch is designed one-at-a-time using either a least-squares minimization method or a minimax optimization method [2]. In the second approach, each subfilter is designed using a frequency-sampling method [3].

The structural subband decomposition-based structure can be computationally more efficient than the conventional polyphase decomposition-based structure in realizing decimators and interpolators that employ linear-phase Nyquist filters. To this end, it is necessary to use transform matrices that transfer the filter-coefficient symmetry to the sub-filters.

3. SUBBAND ADAPTIVE FILTERING

The structural subband implementation of FIR filters can be applied to adaptive filtering, resulting in a structure which unifies the direct form and the transform-domain implementations of the *least mean squares* (LMS) algorithm [4, 5]. The adaptive subband structure is illustrated in Figure 2 below. Note that the structure used in the transform-domain LMS algorithm is obtained from Figure 2 for $M = N$, i.e. $P = 1$, in which case each of the subfilters consists of a single coefficient [6].

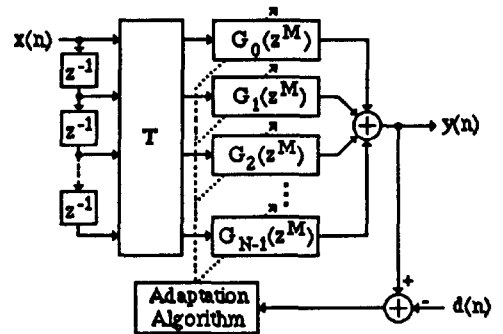


Figure 2

In this structure, the input signal is first processed by a fixed orthogonal transform T of length M , and the resulting transformed signals are then

filtered by sparse adaptive subfilters $G_i(z^M)$. The choice of a transform T with good frequency selection decreases the correlation among the transformed signals, which can be used to obtain a significant improvement in the convergence speed of the LMS algorithm for colored input signals. To this end, the discrete Fourier transform (DFT) or the discrete cosine transform (DCT) has been found to be useful [4]. Also, the subband structure has the flexibility of allowing that subbands not contributing greatly to the overall frequency response be removed, reducing the number of operations needed for the filter implementation, such as in adaptive line enhancer (ALE) applications. Other adaptive methods, such as the *recursive least squares* (RLS) algorithm, can also be used to update the coefficients of the subfilters. In the case of a subband decomposition using a "good" transform, the subband signals are approximately orthogonal; so, each of the sparse filters $G_i(z^M)$ can be independently adapted in parallel by the RLS algorithm, resulting in a reduction in the computational complexity, and speeding up of the convergence.

The adaptive FIR filter structure based on the structural subband decomposition has also been shown to be considerably more fault-tolerant than the conventional direct form FIR adaptive filter structure [7].

5. EFFICIENT DISCRETE TRANSFORM COMPUTATION

Fast algorithms for calculating the discrete transform coefficients which makes use of frequency separation property of the structural subband decomposition have been developed [8]-[10]. For example, the N -point DFT $X(k)$ of a sequence $x(n)$ of length N is simply given by

$$X[k] = X(z) \Big|_{z=e^{j2\pi k/N}}, \quad 0 \leq k \leq N-1. \quad (9)$$

Consider a 2-branch structural subband decomposition of $X(z)$ based on the Hadamard transform. Then we can write

$$\begin{aligned} X(z) &= \begin{bmatrix} 1 & z^{-1} \end{bmatrix} R_2 \begin{bmatrix} V_0(z^2) \\ V_1(z^2) \end{bmatrix} \\ &= (1+z^{-1})V_0(z^2) + (1-z^{-1})V_1(z^2). \end{aligned} \quad (10)$$

Therefore $X[k]$ can be expressed as:

$$\begin{aligned} X[k] &= (1+W_N^{-k})X_0(< k >_{N/2}) \\ &\quad + (1-W_N^{-k})X_1(< k >_{N/2}) \end{aligned} \quad (11)$$

where we have used the notations $W_N = e^{-j2\pi/N}$, $< k >_N = k \text{ modulo } N$. In above, $X_0[< k >_{N/2}]$ is

the $(N/2)$ -point DFTs of the length- $(N/2)$ sequence $x_0[n]$, and $X_1[< k >_{N/2}]$ is the $(N/2)$ -point DFTs of the length- $(N/2)$ sequence $x_1[n]$, generated by the 2-branch structural subband decomposition of $x[n]$:

$$\begin{aligned} x_0[n] &= \frac{1}{2} \{x[2n] + x[2n+1]\}, \\ x_1[n] &= \frac{1}{2} \{x[2n] - x[2n+1]\}, \end{aligned} \quad 0 \leq k \leq N-1. \quad (12)$$

The computation of the N -point DFT using Eq. (15) requiring the computation of two $(N/2)$ -point DFTs has been referred to as the *subband DFT* [8]. The above process can be continued to decompose the sub-sequences $x_0[n]$ and $x_1[n]$ provided $N/2$ is an even integer. The process terminates when the final sub-sequences are of length-2.

By exploiting the spectral contents of the sub-sequences, an efficient DFT algorithm can be developed. For example, if $x[n]$ is known to have most of its energy contents in the low frequencies, a reasonable approximation of the overall transform can be obtained by discarding the term $X_1[< k >_{N/2}]$ in Eq. (11) arriving at an approximation to the N -point DFT $X[k]$ given by

$$\hat{X}[k] \equiv (1+W_N^{-k})X_0(< k >_{N/2}). \quad (13)$$

The structural subband decomposition concept has also been applied to the efficient computation of the discrete cosine transform (DCT) [11, 12]. The N -point DCT of a length- N sequence $x[n]$ is given by

$$C[k] = \sum_{n=0}^{N-1} x[n] \cos\left(\frac{(2n+1)\pi k}{2N}\right), \quad 0 \leq k \leq N-1. \quad (14)$$

Using Eq. (12), Eq. (14) can be reexpressed as

$$\begin{aligned} C[k] &= 2 \cos\left(\frac{\pi k}{2N}\right) \bar{C}_0[k] \\ &\quad + 2 \sin\left(\frac{\pi k}{2N}\right) \bar{S}_1[k], \quad 0 \leq k \leq N-1, \end{aligned} \quad (15)$$

where

$$\bar{C}_0[k] = \begin{cases} C_0[k], & 0 \leq k \leq \frac{N}{2}-1, \\ 0, & k = \frac{N}{2}, \\ -C_0[N-k], & \frac{N}{2}+1 \leq k \leq N-1, \end{cases} \quad (16)$$

and

$$\bar{S}_1[k] = \begin{cases} S_1[k], & 0 \leq k \leq \frac{N}{2} - 1, \\ 2 \sum_{n=0}^{(N-2)/2} (-1)^n x_1[n], & k = \frac{N}{2}, \\ S_1[N-k], & \frac{N}{2} + 1 \leq k \leq N-1, \end{cases} \quad (17)$$

with $C_0[k]$ denoting the $(N/2)$ -point DCT of $x_0[n]$, and $S_1[k]$ denoting the $(N/2)$ -point DST (discrete sine transform) of $x_1[n]$. The computation of the N -point DCT using Eq. (15) requiring the computation of an $(N/2)$ -point DCT and an $(N/2)$ -point DST has been referred to as the *subband DCT* [12]. The above process can be continued to decompose the sub-sequences $x_0[n]$ and $x_1[n]$ provided $N/2$ is an even integer. The process terminates when the final sub-sequences are of length-2.

As in the case of subband DFT, by exploiting the spectral contents of the sub-sequences, an efficient DFT algorithm can be developed. For example, if $x[n]$ is known to have most of its energy contents in the low frequencies, a reasonable approximation of the overall transform can be obtained by discarding the terms in Eq. (15) associated with high frequencies resulting in approximation to the N -point DCT given by

$$C[k] \equiv \begin{cases} 2 \cos\left(\frac{\pi k}{2N}\right) \bar{C}_0[k], & k = 0, 1, \dots, \frac{N}{2} - 1, \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

The subband DCT computation has been shown to result in less visible border artifacts when applied to very low bit rate image compression than that resulting from the direct DCT computation used in the JPEG standard [11,12].

Another interesting application of the structural subband decomposition is in the development of a fast dual-tone multi-frequency (DTMF) tone detection scheme using the subband nonuniform DFT computation [13].

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