A NEW FREQUENCY ESTIMATOR APPLIED TO BURST TRANSMISSION

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Indexing terms: Synchronization, Time Division Multiple Acces frequency estimation.

Abstract: In TDMA communications systems using all feedforward synchronization techniques, the quality of data decoding strictly depends on the estimation accuracy of the synchronization parameters (timing, carrier phase/frequency and preamble detection) extracted from the received signal. The frequency offset estimation is the most critical point. Indeed, an inaccurate frequency estimation can cause cycle slips and then errors during decoding. In this paper, we propose an new frequency estimator, analytically derived from the Maximum Likelihood principle and optimized thanks to variance simulations. Its performance is compared to the Cramer Rao Bound.

INTRODUCTION

In satellite transmissions, operating in TDMA mode, an important constraint is the existence of frequency offsets between reference oscillators of transmitting and receiving stations. A correction of this drift is necessary in order to realize an effective decoding. The use of a frequency estimator (FE) followed by a phase estimator (PE, like the well-known Viterbi-Viterbi one) allows to implement an all feedforward structure, very well suited for burst-type communication systems thanks to its rapid synchronization performance (***Fig. 1***). Here, we analyze the case where an M-PSK modulation is transmitted, over a channel which adds a white gaussian noise. We assume that timing recovery has already and ideally been performed. Samples at the carrier recovery unit input are available at the symbol rate. We exhibit a new frequency estimator, derived from the Maximum Likelihood principle, in non-data aided (NDA) mode. It is applied to a practical case with a bounded normalized frequency offset by $(\Delta f T_s)_{\max}$.

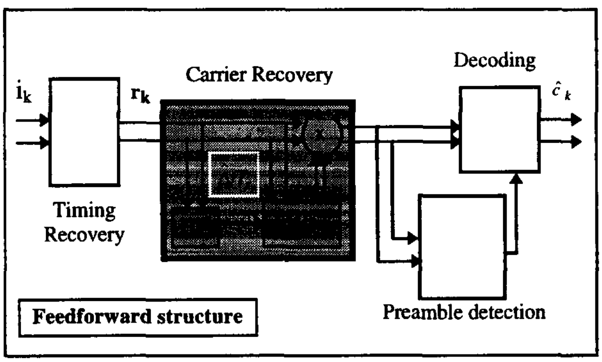

***Fig. 1** Diagram of a feedforward synchronization unit (FE Frequency Estimation, PE: Phase Estimation).*

I ANALYTICAL FORMULATION OF THE ESTIMATOR:

We assume that timing recovery has already and ideally been performed. As the normalized frequency offset is slight ($\Delta f T_s \ll 1$), the intersymbol interference is neglected. Then, we consider the signal, at the output of the timing recovery unit, at the symbol rate:

$$r(kT_s) = c_k e^{j(\theta_0 + 2\pi k \Delta f T_s)} + n(kT_s) \quad (1)$$

where:

- $\{c_k\}$ is a sequence of statistically independent MPSK symbols with $|c_k| = 1$.
- $n(t)$ is a complex additive white gaussian noise, with statistically independent real and imaginary parts, each one having a variance $\sigma^2 = \dfrac{1}{2E_s / No}$.
- T_s is the symbol period.
- θ_0 is the unknown initial phase shift.
- Δf is the frequency offset corresponding to the difference between the received signal and its nominal value.

We have to exhibit an estimator of the random variables couple $(\Delta f, \theta_0)$ from a sequence of N received symbols $\{r_k\}$. The maximization of the likelihood function **[1]**, leads to the expression (2), corresponding to the one given in **[2]** with only one sample per symbol and then no inter symbol interference, with $L = \dfrac{N-1}{2}$:

$$(\hat{\Delta} f, \hat{\theta})_{ML} = Arg\left\{ Max\{\mathrm{Re}(\sum_{k=-L}^{L} e^{-j(\theta + 2\pi M k \Delta f T_s)} r_k^M)\}\right\} \quad (2)$$

Expression (2) can be rewritten as:

$$(\hat{\Delta} f, \hat{\theta})_{ML} = Arg\left\{ Max\{|H(\Delta f T_s)| \,\mathrm{Re}(e^{j(-\theta + Arg[H(\Delta f T_s)])})\}\right\} \quad (3)$$

with $$H(\Delta f T_s) = \sum_{k=-L}^{L} r_k^M e^{-j2\pi M k \Delta f T_s}$$

The joint maximum $(\hat{\Delta} f, \hat{\theta})_{ML}$ is obtained for Δf maximizing $|H(\Delta f)|$ which is independent on θ. As the real part in expression (3) is maximized for $\hat{\theta} = Arg(H(\hat{\Delta} f T_s))$, we only need for frequency estimation to maximize $|H(\Delta f)|$.

One can rewrite the expression to be maximized:

$$|H(\Delta f)|^2 = V_0 + \mathrm{Re}\left\{ \sum_{n=1}^{2L} V_n e^{-j2\pi M n \Delta f T_s} \right\} \quad (4)$$

with $$V_n = \sum_{k=-L+n}^{L} (r_k r_{k-n}^*)^M$$

The idea is to simplify expression (4) to keep only the most significant terms. As shown in **[3]**, the contribution of each correlation terms in the estimator performance is not equal. Indeed, when D takes low values, noise is coarsely filtered, while that large values of D make the number of terms in the computation of the correlation decreases.

Keeping only one term leads to the estimator given in **[2]** and **[1, 3]**.

Let's keep only two terms in the simplified expression of (4):

$$\left|H_{m,n}(\Delta f)\right|^2 = \mathrm{Re}\left\{V_m e^{-j2\pi m \Delta f T_s} + V_n e^{-j2\pi n \Delta f T_s}\right\} \quad (5)$$

We have to determine the couple (V_m, V_n) which gives the most accurate estimation. Resorting to simulations one can compute the estimator variance for each possible couple and then obtain the optimal couple $(V_m, V_n)_{opt}$. ***Fig. 1*** shows the estimator variance versus the index of each element V_m and V_n with $|\Delta f T_s| < 4.10^{-3}$ when Eb/No=5 dB.

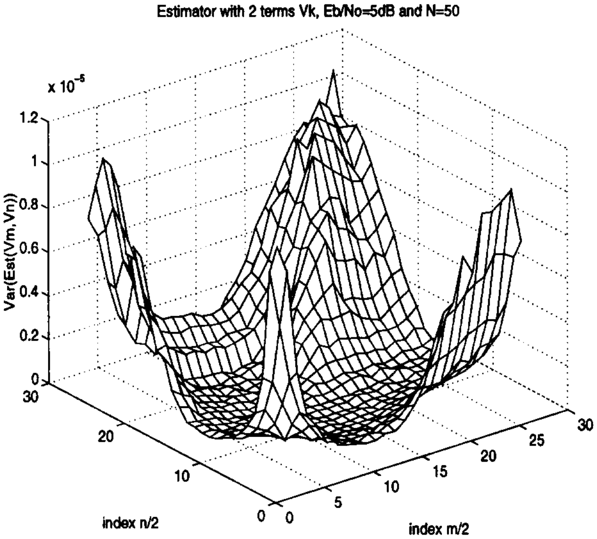

Fig. 2 *Variance of the 2 terms estimator with N =50 symbols, at Eb/No=5 dB.*

In order to increase the improvement, one can take into account more than two terms. ***Fig. 3*** shows the estimator variance results taking 1, 3, 5, 10, 40 and all the V_k terms to compute the estimation. These results are compared to the Cramer Rao bound.

For Eb/No>10dB, the maximization of the simplified expression of $|H(\Delta f)|$ with one or more correlation terms is close to the Cramer Rao Bound. The performance improvement due to the use of several V_k terms is quite unsignificant. But, for lower signal to noise ratio, the variance improvement is more obvious.

The maximization of expression (5) is complex because to minimize the variance, optimal V_n terms, close one from eachother, have to be chosen. A factorization can be done to derive a simple analytical expression of the estimator. Then, it is possible to take into account (2p+1) V_n terms such that the deviation angle due to the first p last terms compensates the one due to the last p terms, as shown in ***Fig. 4*** for p=1.

The optimal V_{Dopt} term and both its closest right and left neighbors are used to obtain the so called IRCFE3C estimator (expression (6)):

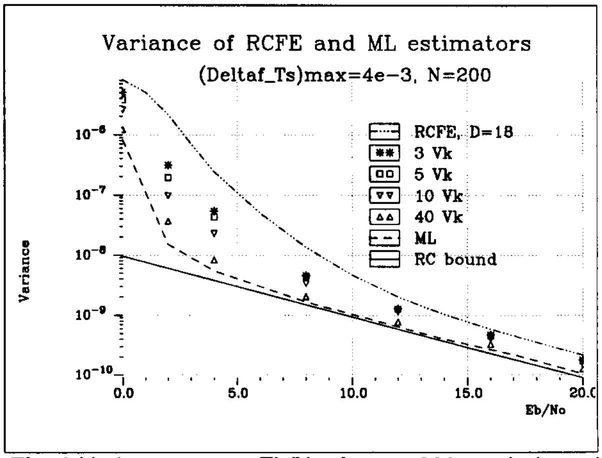

Fig. 3 *Variance versus Eb/No, for N =200 symbols and several numbers of V_k terms.*

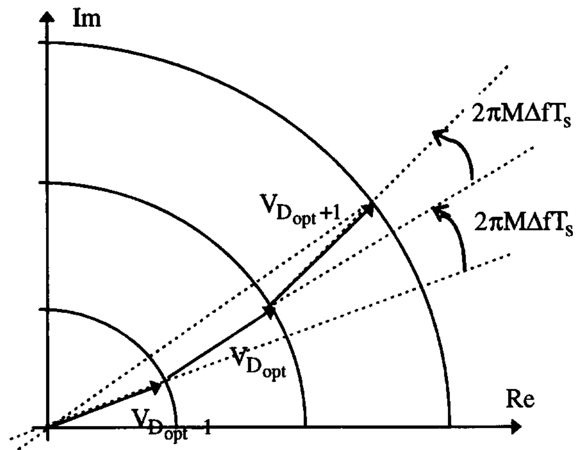

Fig. 4 *Representation in the complex plane of the contributions of the V_k terms.*

$$\hat{\Delta f}T_s = \frac{1}{2\pi M D_{opt}} \mathrm{Arg}(V_{D_{opt}-1} + V_{D_{opt}} + V_{D_{opt}+1}) \quad (6)$$

A generalization of V_k terms can be derived using the same kind of nonlinear functions F as for the RCFE estimator **[3]**.

$$V_k = \sum_{k=-L+k}^{L} F\left(r_n r_{n-k}^*\right)$$

with $F(r) = \frac{r^M}{|r|^q}$ and $0 \le q \le M$.

It is shown in **[1]** that the best nonlinear function to minimize the variance is obtained with q=M-1. Moreover, that realizes a good trade-off between performance and complexity.

The bloc window accumulator, used for the noise filtering operation, can be replaced with an integrating IIR filter. The filter output has to be taken into account only when the noise filtering is sufficiently effective (after L symbols filtered). Thus, the first estimation can be used to correct the first L symbols. This solution offers flexibility in case of demodulation of bursts with very different length. Indeed, with short bursts (L_b=400 symbols for instance and L= L_b), the filtering operation is equivalent to the bloc window accumulator (BWA).

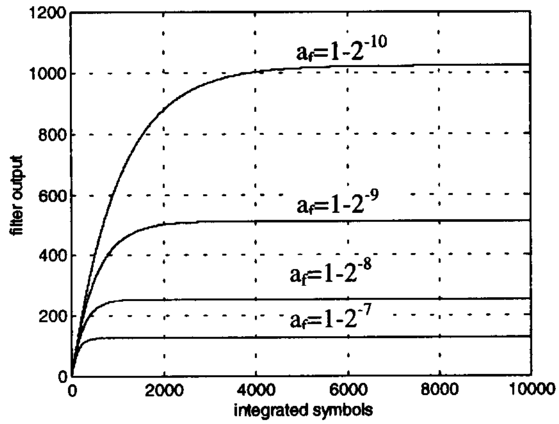

Fig. 5 Filter response for an Heaviside input signal.

But for longer bursts, the integrating filter takes into account more samples than allowed by the last BWA and improve the first estimation.
For an Infinite Impulse Response integrating filter (whose Z-transform is given in ***fig. 6***) with an integrating tap a_f=1-2^{-x}, one can show that an equivalent accumulation window $L_{éq}$ exists. Its size varies with x. So, ***fig. 5*** presents the output of the filter for an Heaviside signal input. The output filter is bounded by 2^x and the equivalent accumulation window is $L_{éq}$#2^{x+2}. Such a filter allows to integer information of 2^{x+2} samples, and to code it on x bits (except a sign bit).

II BRIEF COMPLEXITY ANALYSIS

Fig. 6 shows a way to implement this algorithm, using judiciously polar/cartesian and cartesian/polar conversions. Compared to the RCFE estimator , whose complexity is detailed in **[1]** , the new IRCFE(2p+1)C estimation algorithm only implies a slight increase of complexity (3 real multipliers instead of 1 in the RCFE and a modification of the processing rate).

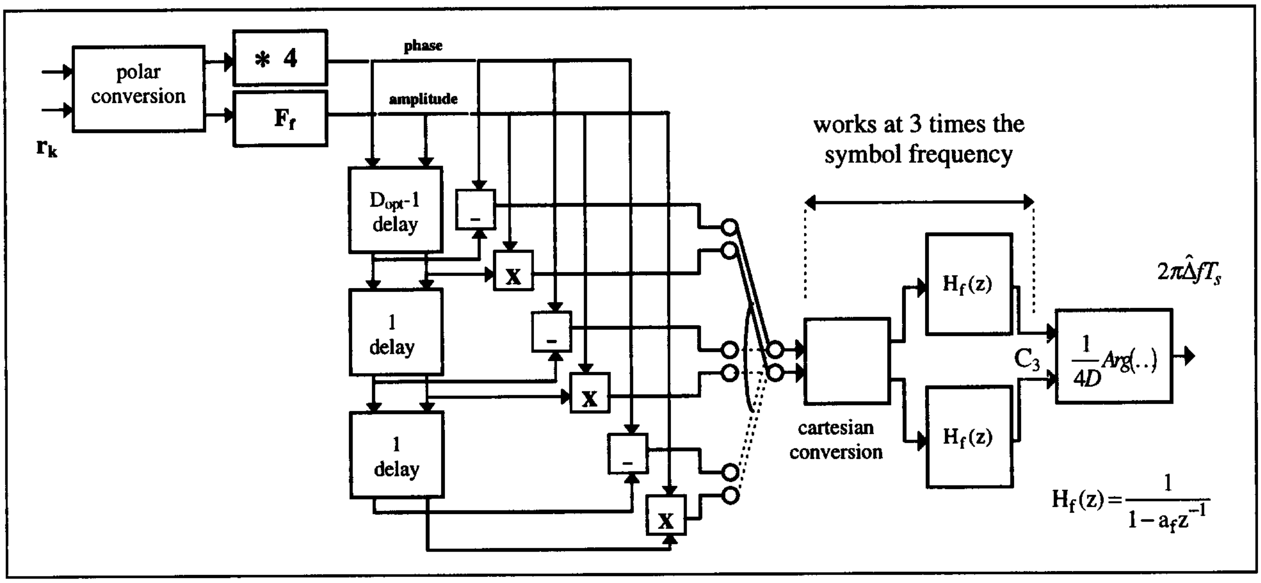

Fig. 6 Implementation example of the IRCFE3C estimator with a filtering integration.

III GLOBAL CARRIER FREQUENCY AND PHASE RECOVERY UNIT:

After frequency estimation, a phase estimation can be used to compute a phase estimate of the received symbol.
The well-known Viterbi-Viterbi algorithm [7] delivers a phase estimation at the middle of a window used to filter thermal noise. Its expression is:

$$\hat{\theta}_k = \frac{1}{M} Arg\left\{ \sum_{n=-(L-1)/2}^{(L-1)/2} F_p(r_{k+n}) e^{jMArg(r_{k+k})} \right\} \quad (7)$$

where $F_p(r)$ is a non linear function similar to the one used in the frequency estimator described above.

According to **[4]**, an optimal window length, function of the residual frequency offset, can be found by minimizing the phase error variance when the non linear function is $F_p(r) = |r^M|$. It is given by:

$$L_{opt} = Int\left[\frac{1.165}{\pi M |\delta f T_s|} \right] \quad (8)$$

where $Int(x)$ means the closest odd integer to x.

As shown in **[1]**, this whole carrier frequency/phase recovery unit is well suited to burst transmissions. Indeed, in such transmissions, to realize the carrier recovery for each new burst, even very short (sometimes a few hundreds symbols) and at low signal to noise ratio (about Eb/No=2 dB) is a severe constraint. Without coding, bit error rate for a classic QPSK modulation does not allow to satisfy a sufficient quality of service.
In order to remedy such a disadvantage, classical convolutionnal coding can be used. For a given data rate and a given transmitting power, energy per transmitted bit decreases and transmitted data rate increases because of the insertion of redundancy. We note:

$$(Eb/No)_t = (Eb/No)_u + 10\log(R) \quad (9)$$

Performance of the whole carrier recovery unit is given for several configurations of parameters $(\Delta f T_s)_{max}$, $(L_b)_{min}$, Eb/No, and rate coding R.

With a R=¾ convolutionnal coding and a normalized frequency offset $(\Delta f T_S)_{max}=8.10^{-3}$, BER performance is close to the theoretical optimum (***fig. 7***). Even with short bursts (L_b=400 symbols), the carrier recovery unit does not degrade significantly BER performance. Thus, with this burst length, BER=10^{-3} is obtained for $(Eb/No)_u$=4.4 dB. With long bursts, frequency estimation tracking allows to improve BER performance. BER=10^{-3} is reached for $(Eb/No)_u$=4.25 dB. Decreasing the signal to noise ratio, BER performance is still close to the theoretical limit.

Using a R=½ convolutionnal coding and a normalized frequency offset $(\Delta f T_S)_{max}=4.10^{-3}$, ***fig. 7*** show BER performance of the carrier recovery unit. The great BER improvement (due to a large redundancy) allows to realize « a kind of zoom » on the influence of cycle slips **[5, 6]** on the burst decoding processing. Indeed, a cycle slip is a slip of the estimated phase in discordance with the actual evolution of the available samples phase. This phenomenon generates error packets which provoke a large degradation of the BER. Thus, with L_b=200 symbols, the frequency offset estimation is not accurate enough to deliver a small residual frequency offset at the Viterbi-Viterbi phase estimator input. Then, performance of this step is considerably degraded. With L_b=400 symbols, the frequency offset estimation is more accurately performed and the behavior of the whole carrier recovery unit is improved (the cycle slips rate is reduced as shown in **[1]**).

IV CONCLUSION

In this paper, we proposed an all feedforward carrier recovery technique, avoiding the insertion of preambles. It is based on a new frequency estimation technique, derived from the Maximum Likelihood principle and close to the algorithm. Moreover, this IRCFE(2p+1)C frequency estimator performs better than the RCFE one **[3]**, with only a small increase of complexity (as long as p is close to 1).

The great performance improvement is due to the optimization in the choice of the correlation terms. Once the frequency drift is reduced, phase estimation in performed by the help of the well-known Viterbi-Viterbi phase estimation algorithm.

We showed that good BER performance is obtained even at very low signal to noise ratio (about $(Eb/No)_t$=2 dB) and for short bursts.

The improvement due to the use of the IRCFE3C estimator can be seen in 3 ways:

- Either it allows to increase the maximum admitted normalized frequency offset $(\Delta f T_S)_{max}$, keeping the BER performance, the operating point and the minimum burst length.
- Or it allows to decrease the minimum burst length, keeping the BER performance, the admitted $(\Delta f T_S)_{max}$ and the operating point.
- Or it allows to improve the BER, conserving the minimum burst length, the admitted $(\Delta f T_S)_{max}$ and the operating point.

As the carrier phase recovery is the most critical point for such a TDMA system, variations of these main parameters generate an hypersurface, characterizing global performance for the whole modem. That wil be the subject of a future paper.

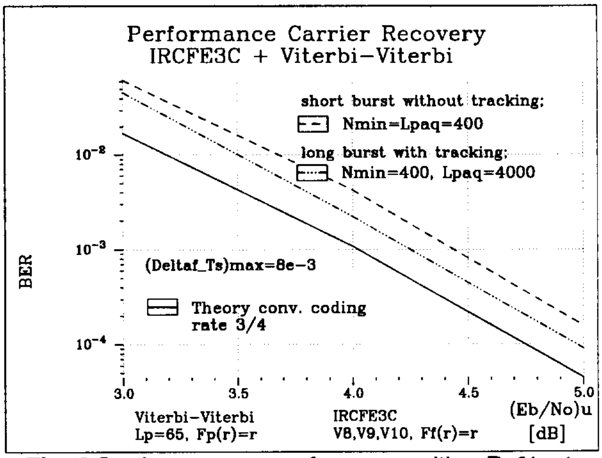

Fig. 7 *Carrier recovery performance with a R=¾ rate coding.*

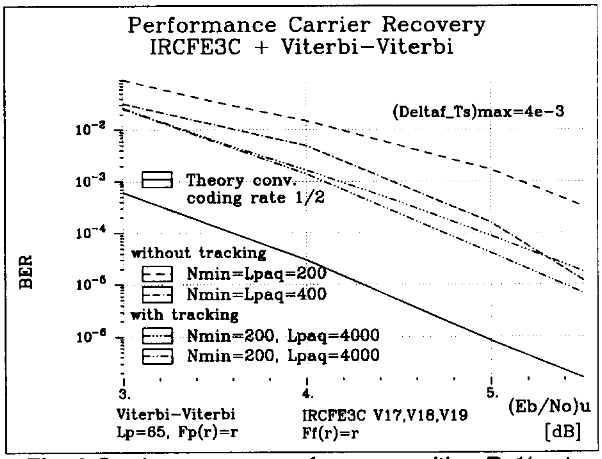

Fig. 8 *Carrier recovery performance with a R=½ rate coding.*

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