

ADAPTIVE INVERSE CONTROL OF WEAKLY NONLINEAR SYSTEMS

Wolfgang J. Klippel

Altenberger Str. 11, 01277 Dresden
74067.1312 @compuserve.com

ABSTRACT

A weak nonlinear plant can be linearized and will track an input signal if the plant is preceded by a nonlinear controller which approximates the inverse of the plant's transfer function. Present techniques for adjusting the controller adaptively to the plant require an additional nonlinear adaptive filter to perform a separate system identification. Straightforward update algorithms can not directly update the filter parameter in the controller because the transfer function of the plant might cause instabilities in the adaptive process. This problem is overcome by performing additional linear filtering to the nonlinear state vector and/or error signal. Novel filtered-A and filtered-E modifications of the stochastic gradient based methods are presented which are capable to update generic as well as special block-oriented nonlinear filter architectures.

1. INTRODUCTION

Most techniques developed for the inverse control of physical systems are based on a linear model for both the plant and the preceding controller. However, some real plants such as electromechanic and electroacoustic transducers (loudspeakers, actuators) are more precisely modeled by a nonlinear system. The nonlinearities are relatively weak and the plant behaves at small amplitudes almost linear but can generate substantial nonlinear distortion of the output signal at higher amplitudes. Nonlinear signal distortion generated by loudspeaker, for example, affects the perceived sound quality and can impair the efficiency of active attenuation systems in professional applications. In an adaptive controller based on a linear filter the nonlinear distortion components increase the residual error and can generate a bias in the linear parameter estimates.

The nonlinearities of the plant can actively be compensated by using a nonlinear controller which approximates the inverse of the plant's transfer function. The inverse preprocessing of the control signal can be realized with nonlinear filters based on a polynomial expansion [1] or on neural networks [2-3]. However, control systems based on such generic architectures

can not be implemented on current digital signal processors (DSP) at low costs. Alternatively, *block-oriented* filter structures (such as the S_M -model [4], Hammerstein-model [5], MMD-model [6]) are composed of both static nonlinear subsystems and dynamic linear subsystems. These architectures have a lower complexity and are very effective if the used filter structure is adequate for the nonlinear mechanism in the plant. If a priori information from physical modeling of the plant is available then it is even possible to derive special block-oriented filter structures with a minimal number of elements and free filter parameters, which are interpretable in a physical sense. Following this approach special nonlinear filter architectures have been derived from the woofer and horn loudspeaker modeling and have been implemented and tested as loudspeaker controllers [7 - 9].

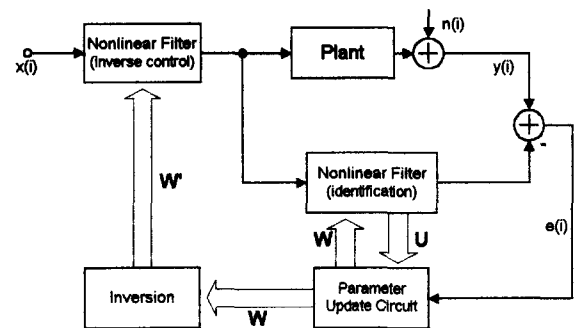


Figure 1: Adaptive inverse control based on forward model identification.

After finding adequate filter structures the interest of research is now focused on the optimal adjustment of the free controller parameters to the particular plant. An adaptive parameter updating is preferable to an off-line adjustment because parameter variations of the plant can be compensated automatically. Straightforward adaptive algorithms available for the generic filter structures (polynomial filter [10] and neural network [3]), require an indirect adjustment of the nonlinear filter by performing an additional nonlinear system identification of the plant. The identification can be performed as a forward or an inverse modeling as shown in Fig. 1 and 2, respectively.

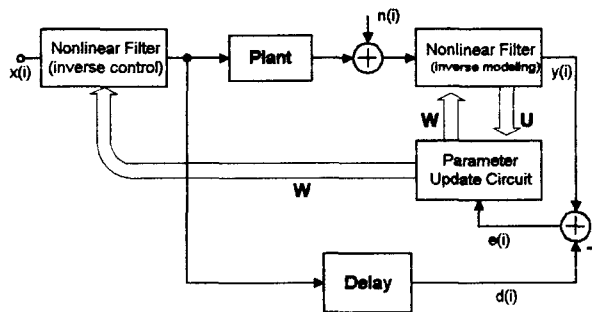


Figure 2: Adaptive inverse control based on inverse model identification.

The additional model filter can be updated by straightforward methods (LMS, LS, RLS, backpropagation method) because this filter is located after or in parallel to the plant. In the forward model identification in Fig. 1 the identified model parameters $\mathbf{W}$ have to be transformed into corresponding control parameters $\mathbf{W}'$ and loaded into the non-adaptive control filter. The transformation can be accomplished by calculating the Volterra kernels from the parameter of the model filter first, inverting the kernel function as described in details in [11] and finally by synthesizing the parameters of the control filter from the inverted system function. Goia [1] simplified the transformation by performing a separate compensation of the first-order system in the plant modeling.

The cumbersome transformation can completely be omitted by using the inverse model identification as shown in Fig. 2. The identified inverse model parameters can directly be copied into the control filter preceding the plant. However, this advantage is paid by a biased parameter estimations under conditions of plant noise additive $n(i)$.

Both approaches require an additional nonlinear filter which has the same complexity as the control filter. This increases the costs of DSP implementation and might be an obstacle for real time processing.

In this paper an alternative approach is presented. It allows direct learning of the nonlinear filter preceding the plant without separate nonlinear system identification. The corresponding block-diagram is shown in Fig. 3. A new update circuit has been developed which has a state vector $\mathbf{A}$ of the nonlinear control filter and the error signal $e(i)$ as inputs and generates an updated parameter vector $\mathbf{W}$ as output. Available update algorithms can not directly be applied here because the transfer function of the nonlinear plant might cause instable behavior of the feedback path.

2. BASIC FILTER AND PLANT MODELING

Following the Volterra series approach the plant is modeled as the sum of m th-order homogeneous power systems with $m = 1, \dots, M$. All the higher-order subsystems ($m > 1$) are summarized in an here not further specified nonlinear subsystem N which is connected in parallel to the first-order system with the

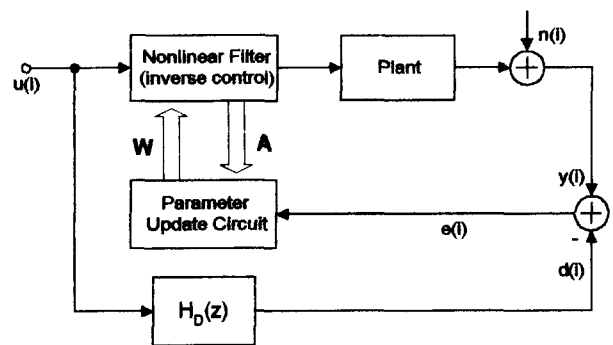


Figure 3: Linearization of loudspeakers based on adaptive inverse control.

linear transfer function $H_0(z)$ as shown in Fig. 4. The output of the nonlinear subsystem $p(i)$ represents the nonlinear "distortion" signal added to the linear output $y(i)$ of the system $H_0(z)$ and the plant noise $n(i)$.

It is assumed that $|p(i)| \ll |y(i)|$, thus the plant behaves as a "weakly" nonlinear system. This assumption is required to ensure that the m th-order distortion products with $m = 2, \dots, M$ generated in the plant can be compensated by a M th-order polynomial filter and that the i th-order distortion components with $M < i \leq M^2$ which are newly generated by cascading two M th-order polynomial systems are sufficiently small.

It is customary to assume that the plant noise $n(i)$ is a zero-mean process and is uncorrelated with the linear signal $y(i)$, the nonlinear distortion components $p(i)$ and the desired signal $d(i)$. Likewise, we assume that the filter input $x(i)$ and the desired response $d(i)$ are single realizations of jointly wide-sense stationary stochastic processes, both with zero mean.

The nonlinear control filter preconnected to the plant is represented by a nonlinear state expander fed by the input signal $x(i)$ and generating the state vector

$$\mathbf{A}(i) = [a_0(i) \ a_1(i) \ \dots \ a_L(i)]^T.$$

This vector is weighted by the parameter vector

$$\mathbf{W}(i) = [w_0(i) \ w_1(i) \ \dots \ w_L(i)]^T$$

and summarized by a following linear combiner to the filter output signal

$$z(i) = \mathbf{A}(i)^T \mathbf{W}(i).$$

This model is straightforward for polynomial filters where the nonlinear Volterra state expander generates the products of the delayed input samples $x(i)$ in all combinations as described by Mathews [10].

To apply this approach to the block-oriented filter structures it is necessary to develop the static nonlinear systems into a series expansion (e.g. power series) and to separate the linear parameters and the coefficients of the series expansions from the linear and nonlinear operations. That leads to a nonlinear state expander which comprises not only a tapped delay line and multipliers but can also contain linear filters and static nonlinear systems with constant parameters.

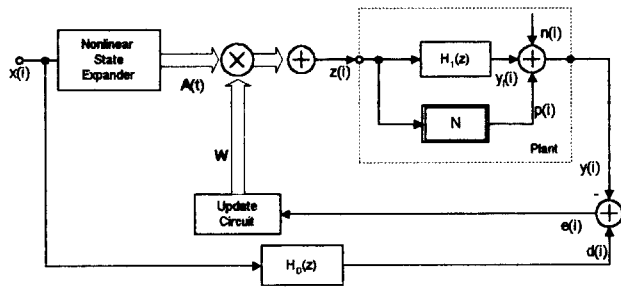


Figure 4: Basic modeling of the direct inverse control system.

3. OPTIMAL PARAMETER ADJUSTMENT

According to Fig. 4 the error signal at the discrete time i is

$$e(i) = d(i) - y(i) = d(i) - [\mathbf{A}(i)^T \mathbf{W}]^* h_1 - n(i) - p(i) \quad (1)$$

where $*$ denotes the convolution operator and it is understood that the Z -transform of h_1 is $H_1(z)$.

Contrary to the usual approach in adaptive filtering the cost function is defined here as the mean squared *filtered* error

$$MSFE \equiv J = E \left[(e(i) * h_e)^2 \right] \quad (2)$$

where the Z-transform of h_e is a causal filter function $H_e(z)$. Inserting Eq. (1) in Eq. (2) and differentiating the cost function with respect to each component of the weight vector yield

$$\nabla(J) \equiv \frac{\partial J}{\partial \mathbf{W}} = 2\mathbf{R}\mathbf{W} + 2\mathbf{Z} - 2\mathbf{P}$$

where

$$\begin{aligned}\mathbf{R} &= E\left[(\mathbf{A}(i) * h_i * h_e)(\mathbf{A}(i) * h_i * h_e)^T\right], \\ \mathbf{P} &= E\left[(d(i) * h_i * h_e)(\mathbf{A}(i) * h_i * h_e)^T\right], \\ \mathbf{Z} &= E\left[(p(i) * h_e)(\mathbf{A}(i) * h_i * h_e)^T\right].\end{aligned}$$

To obtain the minimum mean-square error the parameter vector $\mathbf{W}$ is set at its optimal value $\mathbf{W}^*$, where the gradient is zero. Assuming that the correlation matrix $\mathbf{R}$ is nonsingular, the optimal weight vector is

$$\mathbf{W}^* = \mathbf{R}^{-1}[\mathbf{P} - \mathbf{Z}] \quad (3)$$

This result is the Wiener-Hopf equation for the special case of nonlinear inverse optimal filtering. The additional vector $\mathbf{Z}$ describes the crosscorrelation between the filtered nonlinear distortion signal $p(i)$ and the filtered nonlinear state vector. This vector is almost independent on the parameter $\mathbf{W}$ as long as the control filter and the plant behave as weakly nonlinear systems.

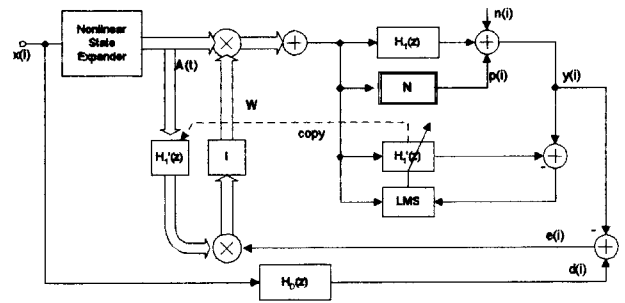


Figure 5: Nonlinear inverse control with A-filtered LMS Algorithm.

4. GRADIENT-BASED ADAPTATION

Although it is possible to solve the Wiener-Hopf equation (3) directly it is more practical in real time implementation to use an iterative method. Beginning with an initial value $\mathbf{W}(0)$ the next guess of the parameter vector is determined by the simple recursive relation

$$\mathbf{W}(i+1) = \mathbf{W}(i) + \frac{1}{2} \mu [-\nabla(J)] = \mathbf{W}(i) + \mu [\mathbf{p} - \mathbf{Z} - \mathbf{R}\mathbf{W}(i)]$$

OR

$$\mathbf{W}(i+1) = \mathbf{W}(i) + \mu E[(e(i) * h_e)(\mathbf{A}(i) * h_1 * h_e)] \quad (4)$$

leading to the steepest-descent algorithm for this particular problem.

Omitting the expectation operator in Eq. (4) results to a stochastic gradient-based method

$$\mathbf{W}(i+1) = \mathbf{W}(i) + \mu \left[(e(i) * h_e)(\mathbf{A}(i) * h_1 * h_e) \right] \quad (5)$$

belonging to the family of the LMS-algorithms. In contrast to the straightforward gradient based algorithms the update Eqs. (4) and (5) require additional filtering of the signals prior to their multiplication. Whereas $H_1(z)$ corresponds with the first-order system of the plant, the filter function $H_2(z)$ can be chosen arbitrarily. However, there are two configurations which are of special interest:

4.1. Filtered-A LMS Algorithm

For $H_e(z)=1$ the additional filter in the error path can be omitted and the update equation reduces to

$$\mathbf{W}(i+1) = \mathbf{W}(i) + \mu [e(i)(\mathbf{A}(i) * h_1')]$$

related to the block diagram presented in Fig. 5. Each element of the nonlinear state vector requires a separate linear filter $H_1'(z)$ approximating the first-order system function $H_1(z)$. The adjustment of these filters is relatively uncritical but if the responses $H_1'(z)$ and $H_1(z)$ deviate more than $\pm 90^\circ$ in phase the update circuit might become unstable. Fig. 5 shows a straightforward way to identify the first-order system function of the weak nonlinear plant.

If the state expander is completely linear (such as a tapped delay line in an FIR-filter) the filtering of the state vector $\mathbf{A}$ can be replaced by single filtering of the input signal $x(t)$ prior to the linear expansion which leads to the straightforward filtered-X LMS algorithm.

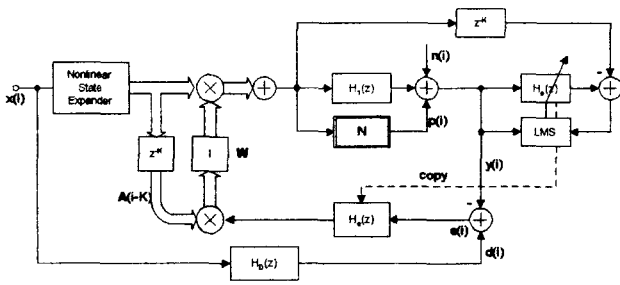


Figure 6: Nonlinear inverse control with E-filtered LMS Algorithm.

4.2 Filtered-E LMS Algorithm

The presented filtered-A LMS algorithm is impractical if the dimension of the state vector $\mathbf{A}$ is high and/or the filter function $H_1'(z)$ is very complex. In such cases it is advisable to omit additional filtering of the state vector and to use an additional filter $H_e(z)$ in the error path to hold the LMS algorithm stable. If the filter function

$$H_e(z) = \frac{z^{-K}}{H_1(z)}$$

is just the inverse of the first-order system function $H_1(z)$, including an additional time delay to make $H_e(z)$ causal, the update algorithm reduces to

$$\mathbf{W}(i+1) = \mathbf{W}(i) + \mu \left[(e(i) * h_e) \mathbf{A}(i-K) \right]$$

The corresponding block diagram is shown in Fig. 6. Besides the time delay of the nonlinear state vector there is only one additional filter in the error path required which performs an additional shaping of the error spectrum. The time delayed inverse of the first-order system function is identified with an additional linear adaptive filter in a straightforward configuration and instantly copied to the error filter. Plant noise $n(i)$ might generate a bias in the estimate of $H_e(z)$ but that is acceptable for the error filter.

5. CONCLUSION

The filtered-A and filtered-E LMS algorithms presented here allow direct updating of the nonlinear filter controlling the plant. Instead of performing a complete identification of the nonlinear plant in the accuracy required for the controller the new approach only needs a rough estimate of the plant's transfer function to perform additional prefiltering of the nonlinear state vector and/or the error signal prior to their correlation. This technique ensures stability of the adaptive process as long as the plant behaves as a weak nonlinear system and the input-output relationship can be approximated by a linear system function. That is not a hindrance for practical applications because almost all generic filter architectures used in the control filter are bound to the same

requirement to approximate the plant's inverse and to provide sufficient compensation of the nonlinear distortion.

This technique is not limited to the gradient-based algorithms presented in this paper but can also be applied to the recursive least-square algorithm.

6. REFERENCES

- [1] F. X.Y. Gao, "Adaptive Linear and Nonlinear Filters," PhD Thesis, University of Toronto, 1991.
- [2] S. Low and O.J. Hawksford, "A Neural Network Approach to the Adaptive Correction of Loudspeaker Nonlinearities," presented at the 95th Convention 1993 October 7-10, New York, preprint 3751.
- [3] P. Chang, C. Lin and B. Yeh, "Inverse Filtering of a Loudspeaker and Room Acoustics using Time-delay Neural Networks," J. Acoust. Soc. Am. 95 (6), June 1994, pp. 3400-3408.
- [4] W. Klippel, "Identification of Nonlinear S_M -Systems," IEEE Workshop on Nonlinear Signal and Image Processing, June 20-22, 1995 Neos Marmara, Greece.
- [5] I.W. Hunter, M.J. Korenberg, "The Identification of Nonlinear Biological Systems: Wiener and Hammerstein Cascade Models," Biol. Cybern., vol. 55, pp. 135-433, 1987.
- [6] W. Frank, "Low complexity 3rd order Nonlinear Filtering, IEEE Workshop on Nonlinear Signal and Image Processing," June 20-22, 1995 Neos Marmara, Greece.
- [7] A.J. Kaizer, "Modeling of the Nonlinear Response of an Electrodynamical Loudspeaker by a Volterra Series Expansion," J. Audio Eng. Soc. 35 (1987) 6, p. 421.
- [8] W. Klippel, "The Mirror filter - a New Basis for Reducing Nonlinear Distortion Reduction and Equalizing Response in Woofer Systems," J. Audio Eng. Soc. 32 (9), pp. 675-691, (1992).
- [9] W. Klippel, "Filter Structures to Compensate for Nonlinear Distortions of Horn Loudspeakers," presented at the 99th Convention of Audio Engineering Society, 1995, October 6 - 9, New York, preprint 4102.
- [10] V.J. Mathews, "Adaptive Polynomial Filters," IEEE Signal Processing Magazine, pp. 10 - 26, July (1991).
- [11] M. Schetzen, "The Volterra and Wiener Theories of Nonlinear Systems," Wiley and Sons, New York, 1980.
- [12] B. Widrow, D. Shur, and S. Shaffer, "On Adaptive Inverse Control," in Proceedings of the 15th Asilomar Conference on Circuits, Systems and Computers, 9-11 Nov. 1981, Pacific Grove, CA (IEEE, New York, 1982), pp. 185-189.