

A MODEL-BASED APPROACH TO ACTIVE NOISE CANCELLATION USING LOUDSPEAKER ARRAY

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ABSTRACT

This paper presents a new model-based adaptive noise cancellation system using loudspeaker array and error sensor array which can be used to reduce the noise in a specific three-dimensional region. First, open loop system transfer functions are designed using a theoretical propagation model. The transfer functions thus found are regarded as the nominal values for the complete system. Second, to compensate for deviations from the theoretical model, the transfer functions are adapted using error measures from error sensor array by *LMS* algorithm. Computer simulation results shows that our approach is effective for noise reduction in 3-D space. Experiments using real-time active noise control hardware also confirms the performance of the system.

1. INTRODUCTION

With increased concerns for environmental protection and industrial safety, noise control has generated increasing interest in recent years. The traditional method of noise reduction using absorbing materials and erected barriers works effectively only at middle and high frequencies, typically above 1000Hz. Thus, active method which is based on the principle of destructive interference between the primary noise source and the actively generated acoustic waves is introduced [1]. This approach works best at low frequencies when the wavelength of noise is long compared to the dimensions of its surroundings.

In many existing active control methods, the noise is reduced in very limited locations or along a single direction. For example, in active noise control inside passenger cars [2], the system is designed to reduce road noise level at passengers' seat locations; in noise reduction inside a duct [3][4], noise is reduced along the path of the duct. However, in most applications, it is desirable to reduce noise in a specific 3-D region. In view of this, a model-based adaptive noise control system using loudspeaker and error sensor array is developed in this paper. We also assembled a real-time demonstration unit to test the performance of the designed system. In this study, we concentrated on reduction of broadband noise with a maximum frequency of 1000Hz. Based on theoretical and simulation results, the performance of our system can be further improved if this maximum frequency is reduced.

The block diagram of the proposed feedforward noise control system is shown in Figure 1. It consists of a noise source

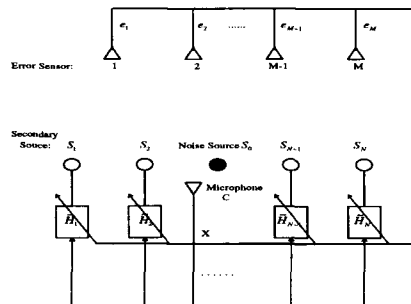


Figure 1. The adaptive noise control system

S_0 , an input microphone C to measure the noise, N transfer functions or filters $\hat{H}_1 - \hat{H}_N$ to drive the N secondary sources $S_1 - S_N$, and M error sensors which measure the system performance and provide error signal inputs to adaptive system.

In most previous works, the following method to design the control FIR filters has been adopted: First, the user specifies the order of the control filters. Then the error signals in target region are minimized, thus yielding the weights of the filters. This method works satisfactorily when we are only concerned with noise reduction at a few measure points or along a single direction. But for noise reduction in a specific three dimensional region, the above method becomes computationally intensive and the adaptive algorithm may not converge to achieve uniform noise reduction in the region. In view of the above problems, in this paper, we propose an alternative approach for noise control system design. First, open loop system transfer functions $H_1 - H_N$ are designed using a theoretical propagation model. The transfer functions thus found can in principle achieve global noise reduction in a specific 3-D region. They are regarded as the nominal values for the complete system. Second, to compensate for deviations from the theoretical model, the transfer functions are adapted using error measures from error sensor array by *LMS* algorithm.

2. THE ADAPTIVE CONTROL SYSTEM

2.1. Design of open loop system

Suppose that the noise signal S_0 is an omnidirectional point source given by

$$p_{S_0}(t) = Ae^{j\omega t}. \quad (1)$$

For such a source, we propose to use a linear loudspeaker array to reduce the noise in a specific 3-D quiet zone. In the following example, the quiet zone refers to the shaded

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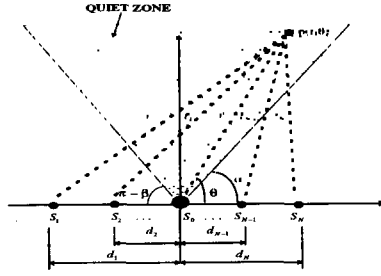


Figure 2. Geometry of the active noise control simulation model

region shown in Figure 2. The signals from each loudspeaker should be of the same frequency as the noise source. Let the signal generated by the loudspeaker S_i be

$$p_{S_i}(t) = H_i p_{S_0}(t), \quad (2)$$

then the signal at a location (r, θ) (see Figure 2) is given by

$$p(r, \theta, t) = \frac{Ae^{j\omega t}}{K} \left(\frac{1}{r} e^{-jkr} + \sum_{i=1}^N \frac{H_i}{r_i} e^{-jkr_i} \right) \quad (3)$$

where K is a constant which represents the sound attenuation factor after propagation. In the above equation, we have assumed that all the loudspeakers can be modeled as omnidirectional sources as well. Also, $k = 2\pi f/c_0$ is the wave number of the signal, c_0 is the speed of sound which is around 343 m/s in air at 20°C. Assuming $r \gg d_i$, so that $r - r_i \approx d_i \cos \theta$ and $r \approx r_i$, then

$$p(r, \theta, t) = \frac{Ae^{j(\omega t - kr)}}{K \cdot r} \left(1 + \sum_{i=1}^N H_i e^{jk d_i \cos \theta} \right). \quad (4)$$

The first part of the equation represents the signal received by the measure point (r, θ) without control. Thus, to reduce noise in the quiet zone, we choose H_i 's such that

$$J = \int_{\alpha}^{\beta} \left| 1 + \sum_{i=1}^N H_i e^{jk d_i \cos \theta} \right|^2 d\theta \quad (5)$$

is minimized. It should be noted that the values of H_i thus designed depend on the signal frequency f . Hence by repeatedly solving the above minimization problem for different frequencies within the specified frequency range, the frequency response of the transfer function from primary source to loudspeaker S_i can be found as $H_i(\omega)$.

In practice, the noise source is not available as input to $H_i(\omega)$. As shown in Figure 1, the microphone C is used to measure the noise. Unfortunately, since the loudspeakers and microphones are assumed to be omnidirectional, signals from secondary sources will be measured by C as acoustical feedbacks. Let $x(t)$ be the signal measured by C , then

$$x(t) = p_{S_0}(t) + \sum_{i=1}^N K_i p_{S_i}(t - \tau_i) \quad (6)$$

where $\tau_i = d_i/c_0$ is the propagation delay from loudspeaker S_i to C , and K_i is the corresponding signal gain. Thus,

$$X(\omega) = \left[1 + \sum_{i=1}^N K_i e^{-j\omega \tau_i} H_i(\omega) \right] P_{S_0}(\omega) \quad (7)$$

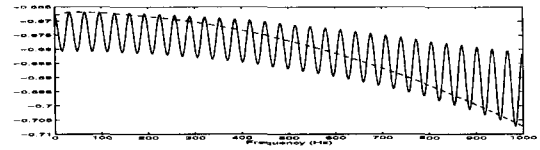


Figure 3. Frequency response of $H_2(z)$.

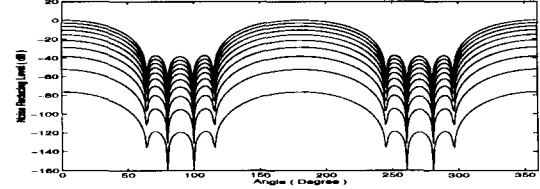


Figure 4. Noise reducing level using desired frequency response.

and

$$P_{S_0}(\omega) = \frac{1}{1 + \sum_{i=1}^N K_i e^{-j\omega \tau_i} H_i(\omega)} X(\omega). \quad (8)$$

The frequency response of the nominal transfer functions with $x(t)$ from microphone C as input to the secondary sources should therefore be

$$\tilde{H}_l(\omega) = \frac{H_l(\omega)}{1 + \sum_{i=1}^N K_i e^{-j\omega \tau_i} H_i(\omega)}, l = 1 \dots N. \quad (9)$$

After $\tilde{H}_l(\omega)$, $l = 1, \dots, N$ have been determined, we design stable transfer functions $\tilde{H}_l(s)$ whose frequency response complies with that of $\tilde{H}_l(\omega)$. With the help of MATLAB or other control system design tools, this can be done easily. For practical implementation, the control system should be discrete-time in nature. Therefore, as a first step, we convert the system transfer functions $\tilde{H}_l(s)$ into discrete-time systems $\tilde{H}_l(z)$. Further, in order to simplify the implementation of the adaptive algorithm in next section, we convert $\tilde{H}_l(s)$ into FIR filters.

To illustrate the above design methodology, consider the following example. Refer to Figure 2. Let $\alpha = \pi/3$, $\beta = 2\pi/3$ and $N = 4$. Four loudspeakers are put along the 0° direction and their distances from the microphone are ± 0.1 m, ± 0.2 m respectively. As a first step, the desired frequency response of the transfer functions $H_l(\omega)$, $l = 1, 2, 3, 4$, are computed by minimization of J in equation (5). Since the four loudspeakers are arranged symmetrically relative to the primary source, $H_1(\omega) = H_4(\omega)$ and $H_2(\omega) = H_3(\omega)$. Thus two FIR filters are designed. The order of the filters was chosen to be 63. Figure 3 shows the frequency response of $H_2(z)$, where the dashed curve is the desired frequency response, and the solid curve is the actual frequency response of the designed transfer functions.

Figure 4 shows the noise reduction level using the desired frequency response. Figure 5 shows the noise reduction level using the designed transfer function. In Figure 4, the ten curves from bottom to top represent noise reductions in decibel (dB) for noise frequencies from 100 Hz to 1000 Hz, in uniform step. In Figure 5, the ten curves from bottom to

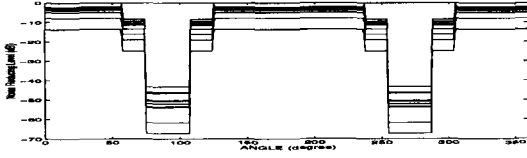


Figure 5. Noise reducing level using designed FIR filters.

top indicate noise reductions for noise frequencies 300Hz, 600Hz, 200Hz, 100Hz, 1000Hz, 400Hz, 500Hz, 700Hz, 900Hz and 800Hz. The result shows that in the angular range of $[60^\circ, 120^\circ]$ and $[240^\circ, 300^\circ]$, especially within $\pm 15^\circ$ range from 90° and 270° , significant reduction in noise level has been achieved using the designed transfer functions. In particular, when the noise frequency is 300Hz, the noise reduction is up to 70dB. In addition, at these frequencies the noise level is reduced in other directions as well. Inspection of Figure 4 also shows that, in general, reduction in noise level is higher when the noise frequency is lower if desired frequency response can be achieved. However, due to deviations between the designed FIR filters' frequency response and the desired one, the actual results will be somewhat different. In view of above results, if we replace the FIR filters by IIR filters, the performance of the system will be improved. However, IIR filters will sometimes lead to the unstable system. Finally, it is worth noting that the procedure to design $\tilde{H}_l(z)$'s is the same as that for $H_i(z)$'s.

From above discussion, it can be seen that if accurate propagation model can be established and the associated parameters are measured, the proposed noise cancellation scheme can reduce noise effectively.

2.2. Adaptive control algorithm

In practice, errors in propagation model are inevitable. Therefore the transfer functions $\tilde{H}_l(z)$ needs to be adjusted accordingly. In this paper, we use the well-known multiple error filtered-x LMS algorithm [5, 6] to adapt the secondary sources' system transfer functions. Figure 6 shows the block diagram of this adaptive system. In the figure, $x(n)$ is the sample reference signal fed to the adaptive digital filters \tilde{H}_l , whose output signals are $y_l(n)$; $c_{lm}(j)$, $j = 0, \dots, J-1$, is the impulse response of digital filter which models the acoustic propagation function from l -th secondary loudspeaker to the m -th error microphone, and $d_m(n)$ is the output of the m -th error sensor due to the primary source. The finite impulse responses of $\tilde{H}_l(z)$ are denoted by $\tilde{h}_l(n)$, $n = 0, \dots, L-1$. Based on the system configuration in Figure 6, we have

$$y_l(n) = \sum_{k=0}^{L-1} \tilde{h}_l(k)x(n-k) \quad (10)$$

and

$$e_m(n) = d_m(n) + \sum_{l=1}^N \sum_{j=0}^{J-1} c_{lm}(j)y_l(n-j). \quad (11)$$

Let $r_{lm}(n)$ be the reference signal filtered by the response of the path from the l -th secondary source to the m -th sensor, given by

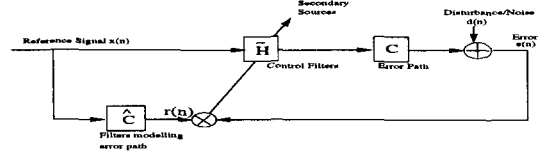


Figure 6. Block diagram of filtered-x LMS adaptive system

$$r_{lm}(n) = \sum_{j=0}^{J-1} c_{lm}(j)x(n-j) \quad (12)$$

and define the cost function as

$$J(n) = \sum_{m=1}^M e_m^2(n), \quad (13)$$

then the gradient of the error surface is

$$\frac{\partial J(n)}{\partial \tilde{h}_l(k)} = 2 \sum_{m=1}^M e_m(n)r_{lm}(n-k). \quad (14)$$

Each filter coefficient can be updated using the LMS algorithm:

$$\tilde{h}_l^{(n+1)}(k) = \tilde{h}_l^{(n)}(k) - \alpha \sum_{m=1}^M e_m(n)r_{lm}(n-k) \quad (15)$$

where α is the step size of adaptive algorithm. The stability of this filtered-x LMS algorithm is affected by α and the accuracy of the filters \hat{C} which model the response of the true secondary paths C .

3. SIMULATION RESULTS

Computer simulations using MATLAB and Simulink toolbox was conducted to evaluate the noise control performance and system stability of the proposed system. Refer to Figure 2. The simulation model uses the arrangement as the example in Section 2. Three error sensors are arranged in position $\theta = [\pi/2.9, \pi/2, 2\pi/3]$ and $r = 10$ m inside a quiet zone with $\alpha = \pi/3$ and $\beta = 2\pi/3$. Forty thousands (40000) samples of broadband noise were generated assuming a sampling rate of 4kHz. For simplicity, we have used $\hat{C} = C$. The step size α was chosen to be less than $1/\sigma L$, where L is the order of FIR filters \tilde{H}_l and σ^2 is the variance of the noise. Simulation shows that with this choice of α , the adaptive system remains stable.

Figure 7 gives the power spectral density of the noise measured at a direction of 90° . In this figure, the top curve is the power spectral density of the measured noise with the active noise control system turned off; the bottom curve is the power spectral density with the open-loop system turned on. From the figure, it can be seen that the active noise control system is capable of reducing noise significantly especially in low frequency range.

Table 1. Theoretical and actual hardware implementation results for noise reducing level (in dB).

Frequency (Hz)	90°			80°			Angle			70°			60°		
	The.	Act.	Adp.	The.	Act.	Adp.	The.	Act.	Adp.	The.	Act.	Adp.	The.	Act.	Adp.
200	54.28	25.73	28.02	54.25	28.08	29.98	54.21	20.45	17.21	18.86	10.97	17.79	10.05	5.09	8.94
250	60.16	26.23	30.00	60.13	30.96	30.19	60.10	21.64	19.05	17.28	11.19	18.10	8.19	3.00	6.10
300	67.33	29.05	31.64	67.29	33.98	32.31	67.25	24.40	21.26	15.99	14.23	22.41	6.72	7.04	6.94
350	55.05	24.05	28.72	55.01	26.75	24.23	54.97	17.95	20.01	14.68	10.56	17.21	5.52	5.25	3.09
400	50.96	23.44	23.22	50.93	25.75	23.13	50.89	17.05	18.32	13.32	9.54	20.21	4.50	5.08	4.23
450	49.71	18.14	20.12	49.67	19.43	17.89	49.63	12.50	11.42	12.13	6.21	13.93	3.63	3.18	0.30
500	50.72	11.09	13.32	50.68	15.75	14.30	50.64	10.82	12.19	11.30	5.90	8.02	2.88	0.48	-1.36
550	55.01	9.95	11.00	54.97	11.77	9.21	54.91	11.89	10.32	10.85	5.77	10.00	2.24	-0.82	-2.99

* Act.: actual result without adaptation, The.: theoretical result, Adp.: actual result using adaptive algorithm

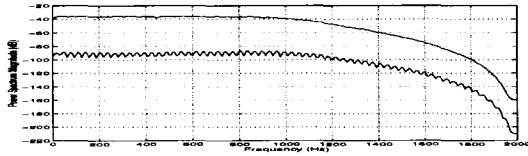


Figure 7. Power spectral density in direction of 90°.

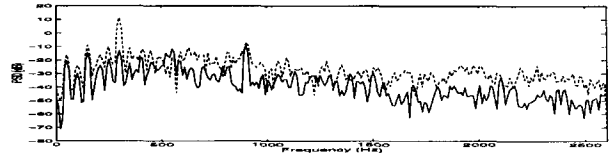


Figure 9. Power spectral density.

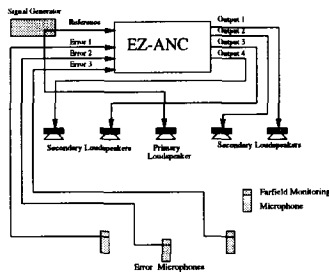


Figure 8. Experimental Hardware Arrangement.

4. HARDWARE DEMONSTRATION

Real-time implementation of the proposed active noise control system is achieved using the EZ-ANC active control development system made by the Causal Systems. This system includes software package and a stand-alone digital signal processing board with one reference signal input port, five error microphone input ports, six control output ports and a serial communication port for personal computer access. Because of the limitation of the hardware function, we only tested the system with monotone noise. The hardware arrangement of the whole system is shown in Figure 8. The distance between the loudspeakers are the same as the previous example, distance between the primary source and the three error microphones are 1.5m, and the corresponding $\theta = [7\pi/18, \pi/2, 11\pi/18]$.

The real-time demonstration system is placed in a $3m \times 4m$ room, and the loudspeakers we used in the experiment are the popular low-cost computer speakers. Figure 9 shows the noise reducing level at 300Hz, at position $(r, \theta) = (2, 75^\circ)$. The dotted curve is the power spectrum density of the received signal without active noise control and the solid curve is with the control. Table 1 gives the noise reducing level at different positions for single frequency noise radiated from the primary loudspeaker with and without adaptive control. From the table we can see that, along the direction of the error microphone position, noise is reduced significantly with adaptive control. Therefore, if the number of the error microphones is increased, better results can be expected. Due to restrictions on hardware capability and a

very small room, our experimental results are not as good as the theoretical prediction. However, the experiment results clearly demonstrated that the proposed method is effective in reducing noise in a specified 3-D region, especially in low frequency range.

5. CONCLUSION

In this paper, we have proposed a effective model-based adaptive control system to reduce low frequency noise in a specific three-dimensional region. The system uses a loudspeaker array and a error sensor array. Two steps are used to design the system: (1) Open loop system transfer functions are designed using a theoretical propagation model. The transfer functions thus found are used as the nominal values for the complete system. (2) To compensate for deviations from the theoretical model, the transfer functions are adapted using error measures from error sensor array by filter-x *LMS* algorithm. Computer simulation result and experimental result using real-time system have both demonstrated that the proposed system is effective for broadband noise reduction in three dimensional region.

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