A NOVEL FREQUENCY DOMAIN FILTERED-X LMS ALGORITHM FOR ACTIVE NOISE REDUCTION

Toshifumi Kosaka*, Stephen J. Elliott** and Christopher C. Boucher***

* Tokyo National College of Technology, Kunugidamachi, Hachioji Tokyo, 193 Japan
** *** ISVR University of Southampton, Highfield Southampton SO17 1BJ UK
*kosaka@tokyo-ct.ac.jp, *** ccb@soton.ac.uk

ABSTRACT

A Frequency Domain implementation of the LMS Algorithm has significant advantages. In broadband applications it is important to use the correct window function before Fourier transformation to obtain an unbiased estimation of the required cross correlation function and to eliminate wrap-around effects.

In the Frequency Domain Filtered-X LMS Algorithm described in this paper, the control filter is updated in the frequency domain as a background task, while control filtering is performed in time domain, to minimize processing delays.

The frequency domain algorithm showed better performance than the conventional time domain algorithm in simulations of single channel active control systems. The algorithm is also able to improve the convergence of multiple channel systems by compensating for the coupling between the control channels.

1. INTRODUCTION

The Filtered-X LMS algorithm is widely used in active noise reduction [1]. A Frequency Domain implementation of the LMS algorithm using the FFT has two advantages. One is reducing computing complexity for long control filter lengths. The other is its ability to converge rapidly when the adaptation parameter is correctly adjusted in each frequency bin.

Frequency Domain LMS Algorithms suggested previously for active control have implemented the control filtering and the adaptation in the frequency domain [2, 3], which causes a one block processing delay in the controller.

In the Frequency Domain Filtered-X LMS Algorithm used in this paper, the updating of the control filter is performed in the frequency domain as a background task, while control filtering is performed in the time domain, thus minimizing the delay in the controller.

Time and Frequency Domain Filtered-X LMS algorithm in active noise reduction of single channel system are simulated which illustrate the importance of using the correct window function and of zeropadding the data vector. A multi-channel system, which uses one reference two secondary sources and two error sensors, is also described.

2. COMPARISON OF WINDOW FUNCTIONS

The Frequency Domain LMS algorithm involves the estimation of a cross spectral density. There is a problem in choosing the correct window function, that is applied before the FFT calculation, to obtain an unbiased estimation of this cross spectral density and to eliminate wrap-around effects.

To illustrate their effects we have used a frequency domain block averaging method to identify the first 256 points of a decaying exponential impulse response. We compared the effects using Hanning window and zero padding + the use of a rectangular window on the estimation of such an impulse response using the block averaging method as shown in Fig. 1 with a block length of 512 points. The estimation of the primary path impulse response is obtained by

$$w(i) = IFFT(\overline{S_{xy}(k)} / \overline{S_{xx}(k)}) \quad , \tag{1}$$

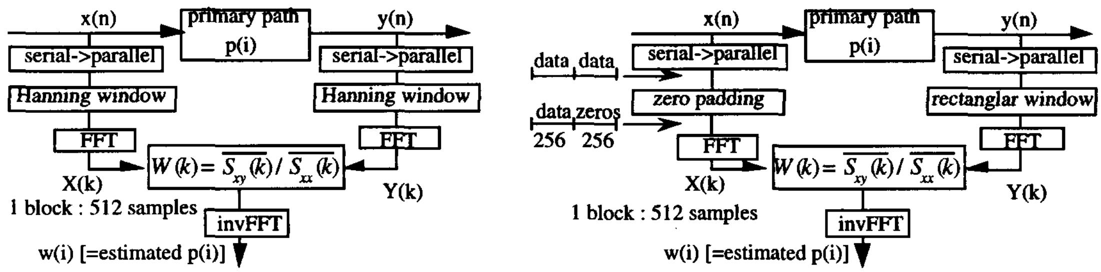
where $\overline{S_{xy}(k)}$ and $\overline{S_{xx}(k)}$ are the block-averaged cross spectral density and power spectral density respectively. The correct estimation of the time impulse response is expected for the first 256 points, that is the causal part, of the block length of w(i).

The results of using a Hanning window and a zeropadding + rectangular window are illustrated in Fig. 2, in which 1024 block averages have been used. These results can be understood by assuming that the estimated impulse response is equal to the true response multiplied by a scaling function, which is defined to be

$$m(j) = \frac{q(j)}{q_1} \quad , \tag{2}$$

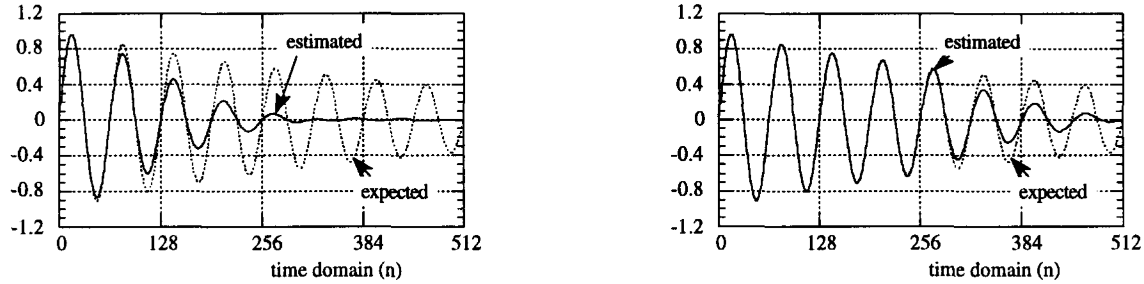
$$q(j) = \frac{1}{N} \sum_{i=0}^{N-j-1} f_x(i)\, f_y(i+j) \quad , \quad q_1 = \frac{1}{N} \sum_{i=0}^{N-1} f_x^2(i) \quad , \tag{3}$$

where $f_x(i)$ and $f_y(i)$ are window functions located at the input and output signals respectivelyand, is illustrated for the various windows in Fig. 3. Similarly a leakage function can be defined, which quantifies the influence of the



(a) Hanning window (b) Zero padding +rectangular window

Fig. 1 Comparison of window functions in an identification example



(a) Hanning window (b) zero padding +rectangular window

Fig. 2 The estimation of primary path using the block averaging method with different window functions

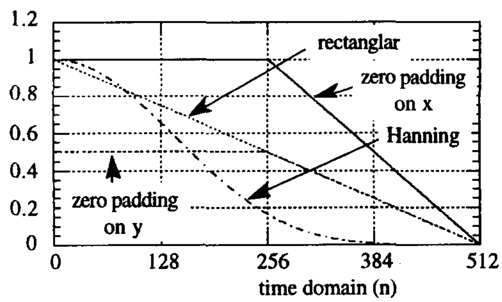


Fig. 3 The scale function of the estimated impulse response depending on the window functions

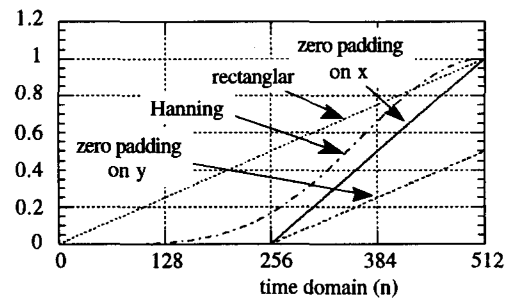


Fig. 4 The leak function of the estimated impulse response depending on the window functions

noncausal part of the true impulse response on the estimated response, as plotted in Fig. 4. These results illustrate the danger of using the Hanning window or the rectangular one used in previous work [4].

3. SINGLE CHANNEL FREQUENCY DOMAIN FILTERED-X ALGORITHM

The performance of the Time Domain Filtered-X algorithm and the Frequency Domain Filtered-X algorithm have been compared in simulation. A Frequency Domain Filtered-X algorithm with zero padding + rectangular window is used as shown in Fig. 5 in which the control filtering is performed in the time domain and its adaptation performed in the frequency domain. The Frequency Domain Filtered-X LMS algorithm can be written as

$$W(k)_{new} = W(k)_{old} - \alpha_k \hat{G}(k)^* X(k)^* E(k) , \tag{4}$$

where $\hat{G}$ is an estimated transfer function of the plant g. X and E are the frequency domain reference and error signals respectively, k is frequency bin and * denotes conjugation. The convergence coefficient for the k-th bin can be normalized so that each bin converges at the same rate by defining

$$\alpha_k = \alpha [|G(k)|^2 \mathrm{E}\{X(k)^* X(k)\}]^{-1} , \tag{5}$$

where α is an overall convergence factor, and $\mathrm{E}\{X(k)^* X(k)\}$ is estimated by using

$$\mathrm{E}\{X(k)^* X(k)\}] = P + \gamma , \tag{6}$$

$$P_{new} = P_{old} \beta + |X(k)|^2 (1-\beta) , \quad 0 < \beta < 1 , \tag{7}$$

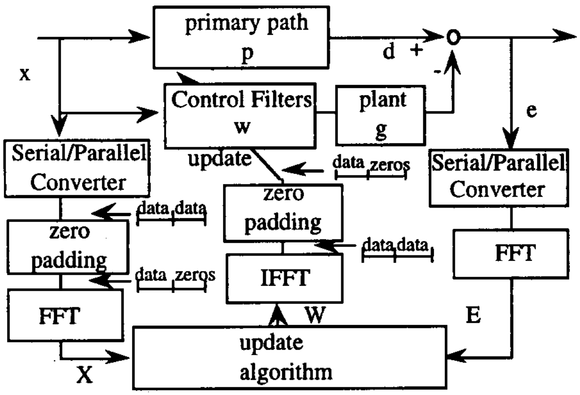


Fig. 5 The implementation of single channel frequency domain filtered-X LMS

where P is the power estimation, and the factor β is introduced to assure an unbiased estimation, and γ is a small positive constant, that prevents $E\{X^*X\}$ being zero. $X(k)^*X(k)$ is calculated for each block of data.

In the simulation described here, the primary path p has several resonances, and the plant g has a single resonance. The performance of the frequency and time domain filtered-X LMS Algorithms in reducing the error signal are compared in Fig. 6 in which the reference signal was either white noise or pink noise. It is clear that improved convergence is obtained with the frequency domain algorithm even if the reference signal is white. This is because the convergence of the time domain filtered-X algorithm is limited by the eigenvalue spread of the autocorrelation matrix of the reference signal filtered by the plant, which is resonant, rather than that of the reference signal itself.

4. MULTIPLE CHANNEL FREQUENCY DOMAIN FILTERED-X ALGORITHM

In a general multiple channel frequency domain system in one frequency bin, the error signal can be denoted by $\underline{\boldsymbol{e}} = \underline{\boldsymbol{d}} + \boldsymbol{G}\boldsymbol{W}\underline{\boldsymbol{x}}$, where the notation $\underline{\boldsymbol{d}}$, $\boldsymbol{G}$, $\boldsymbol{W}$, and $\underline{\boldsymbol{x}}$ are the desired signal, plant transfer function, control filter transfer function and reference signal respectively.

$$\underline{\boldsymbol{e}} = (E_1, E_2, \cdots, E_L)^T,\quad \underline{\boldsymbol{d}} = (D_1, D_2, \cdots, D_L)^T,$$

$$\boldsymbol{G} = \begin{pmatrix} G_{11}, G_{12}, \cdots, G_{1M} \\ G_{21}, G_{22}, \cdots, G_{2M} \\ \vdots \\ G_{L1}, G_{L2}, \cdots, G_{LM} \end{pmatrix} \quad \boldsymbol{W} = \begin{pmatrix} W_{11}, W_{12}, \cdots, W_{1K} \\ W_{21}, W_{22}, \cdots, W_{2K} \\ \vdots \\ W_{M1}, W_{M2}, \cdots, W_{MK} \end{pmatrix},$$

$$\underline{\boldsymbol{x}} = (X_1, X_2, \cdots, X_k)^T \tag{8}$$

Two Frequency Domain adaptation algorithms were compared for a system with a single reference signal, two secondary sources and two error sensors. The multichannel LMS algorithm in the frequency domain can be written as

$$\boldsymbol{W}_{new} = \boldsymbol{W}_{old} - \alpha\, \boldsymbol{G}^H\, \underline{\boldsymbol{e}}\, \underline{\boldsymbol{x}}^H \tag{9}$$

In this case α was normalized to be

$$\alpha = \alpha' \left[(\Sigma\Sigma |G_{lm}|^2)(E\{\underline{\boldsymbol{x}}^H \underline{\boldsymbol{x}}\}) \right]^{-1} \tag{10}$$

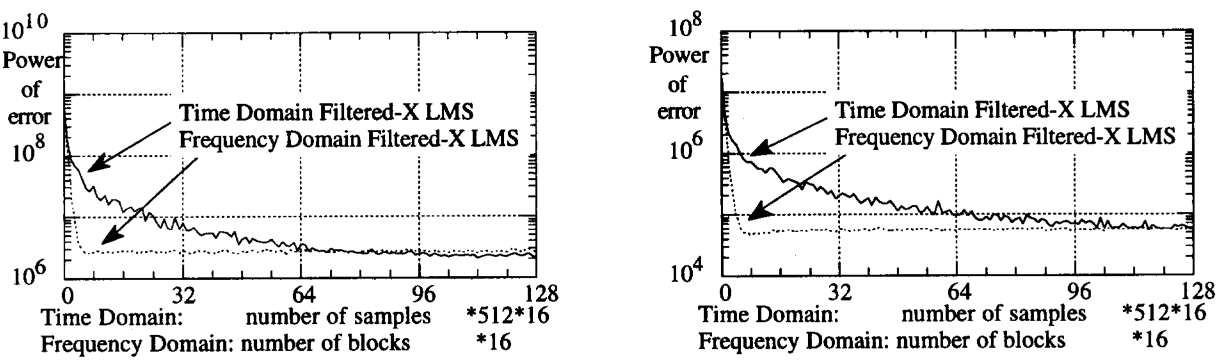
The effects of the plant coupling and the autocorrelation structure of the reference signals can be accounted for in a frequency domain version of Newton's algorithm [6], which can be written in each frequency bin as

$$\boldsymbol{W}_{new} = \boldsymbol{W}_{old} - \alpha\, [\boldsymbol{G}^H \boldsymbol{G} + \gamma \boldsymbol{I}]^{-1} \boldsymbol{G}^H\, \underline{\boldsymbol{e}}\, \underline{\boldsymbol{x}}^H E[\underline{\boldsymbol{x}}\, \underline{\boldsymbol{x}}^H]^{-1}, \tag{11}$$

where the parameter γ is introduced to avoid singularities in the $\boldsymbol{G}^H\boldsymbol{G}$ matrix, and given by

$$\gamma = \begin{cases} 0 & \lambda_1 / \lambda_2 > r \\ \lambda_1 r & \lambda_1 / \lambda_2 \le r \end{cases}, \tag{12}$$

where λ_1, λ_2 ($\lambda_1 < \lambda_2$) are min and max eigenvalues of $\boldsymbol{G}^H\boldsymbol{G}$, and r is a threshold value. In the case when $\gamma = 0$, equation (11) is a pure Newton's Algorithm, and when γ is large the equation is similar to steepest descent [7], therefore



(a) white noise reference

(b) colored(pink) noise reference

Fig. 6 Power of error curves in the frequency and time domain filtered-X LMS algorithms
Each curve is ensemble average of 8 learning curves. time domain filtered-X algorithm is normalized by input power, while frequency domain filtered-X LMS algorithm is normalized by input power within each frequency bin. α=0.0001 was used.

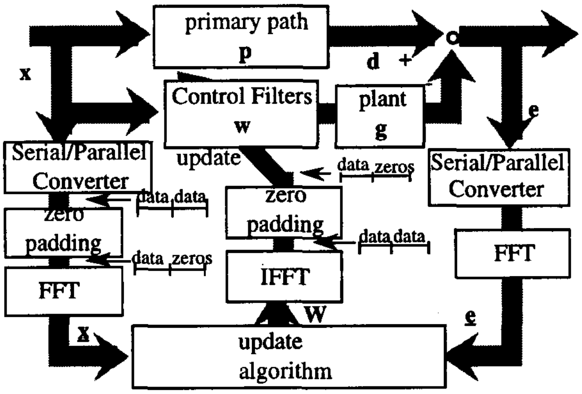


Fig.7 The implementation of multiple channel frequency domain filtered-X LMS

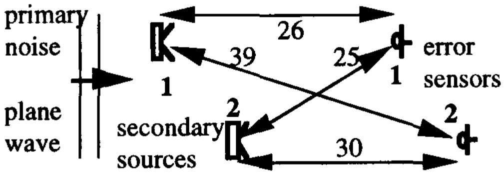


Fig.8 The arrangement of secondary sources and error sensors in one reference, two secondary sources and two error sensors system in the free field

equation (11) is considered as an intermediate form between the pure Newton's and the Steepest Descent algorithms.

In these simulations, $E[\underline{x}\,\underline{x}^H]^{-1}$,that is a scalar in this case, was calculated in the same way as single channel simulation.

As $[G^H G+\gamma I]^{-1} G^H$ is calculated once at the beginning of the adaptation, the real time computation requirements of the two algorithms are comparable.

The arrangement of secondary sources and error sensors used in the simulation are shown in Fig. 8. This arrangement has no ill conditioned frequency bins. Fig. 9 shows that the frequency domain steepest descent algorithm converges at a similar rate to the LMS algorithm implemented in the time domain, but the frequency domain Newton's algorithm converges significantly faster.

5. CONCLUSIONS

The frequency domain filtered-X LMS algorithm requires an unbiased estimation of the cross spectral density, which is provided by Zeropadding + the use of a rectangular window.

A simulation of a single channel frequency domain normalized filtered-X LMS has been presented. This converged more rapidly than a time domain LMS implementation even for a white noise reference signal.

A simulation of a multiple channel frequency domain filtered-X LMS and Newton's Algorithm has also been presented, and show that problems associated with cross-coupling within the plant can also be reduced.

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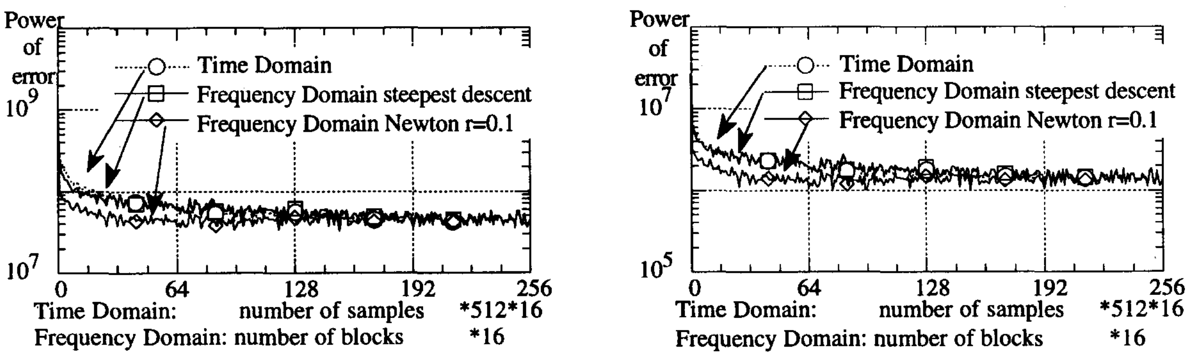

(a) white noise reference (b) colored(pink) noise reference

Fig.9 Power of error curve in frequency and time domain filtered-X LMS algorithms
Each curve is ensemble average of 8 learning curves. α=0.001 was used.