MULTITARGET DETECTION/TRACKING OF ECHOES WITH KNOWN WAVEFORM: ALGORITHM AND APPLICATIONS

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ABSTRACT

The Time of Delay (TOD) estimation of multiple echoes is here solved with an iterative multitarget detection/tracking algorithm. The evaluation of the TODs is based on their a-posteriori probability, while a first-order Markov model is used for a-priori probability estimation. The effectiveness of the algorithm (low false-alarm rate and robustness) is also experimentally proven. Moreover the algorithm exhibits a better noise rejection and an improved target resolution with respect to algorithms that perform separate detection and tracking.

1. INTRODUCTION

Many remote-sensing applications involve detection and tracking of the time of delay (TOD) of echoes of known waveform. When a moving active sensor is used to perform several measurements, the same target medium is illuminated from different locations. The TODs of backscattered echoes are closely related to the physical properties of the propagating medium and form a profile (also indicated as *interface* to underline the physical meaning) that we propose to identify. Detection and TOD estimation are considered to be the major performance issues as detection of erroneous echoes (false alarms) introduces errors. In addition, these errors accumulate and bias the physical model estimation.

The TODs of the echoes, when present, are spatially continuous across the scans. We propose to exploit this continuity for both the detection of multiple interfaces and the tracking of their TODs. The multitarget detection/tracking algorithm is obtained by applying iteratively the single target detection/tracking one. The basic idea is based on the framework presented in references [1] [2]. But we estimate the TOD of each interface after detection/tracking of interfaces instead of estimating only TODs of isolated echoes after echo detection. This is obtained by evaluating the a-posteriori probability density function (pdf) for a set of hypotheses each of them corresponding to a different TOD. Lateral continuity of echoes, that pertain to the same interface, is based on a first-order Markov model for the evaluation of a-priori pdf.

The algorithm has been applied to a monostatic pulse radar system (Ground Penetrating Radar - GPR) that has been designed for the estimation of layer thicknesses and of structural features in road and highway pavements from the TODs of backscattered echoes [4]. The GPR system is mounted on a vehicle that performs a continuous survey at the speed higher than 50 Km/h. The amount of measurements is approx. 15-20 Mbytes/Km (or 12.5-16.7 Mbytes/min) and multitarget detection/tracking is the only feasible approach that can be realistically considered for real-time implementation of GPR in pavement profiling.

2. DETECTION/TRACKING ALGORITHM

In active sensor systems, the measured signal at space location $x_i = i\Delta x$ (or the *i-th* scan) is modelled as a superposition of L_i echoes that correspond to L_i interfaces (or targets):

$$s(x_i, t) = \sum_{l=1}^{L_i} a_l(x_i)\, w(t - \tau_l(x_i)) + n(x_i, t); \qquad (1)$$

$w(t)$ is the known pulse waveform having time resolution T_w; $a_l(x_i)$ and $\tau_l(x_i)$ are the amplitude and the TOD, respectively, of the echo due to the l-th target; $n(x_i, t)$ is the zero mean uncorrelated Gaussian noise with variance σ_n^2. The TODs that correspond to the *l-th* interface are laterally continuous (when present), and slowly varying across scans: $|\tau_l(x_i) - \tau_l(x_{i+1})| \leq T_w$. The number of the interfaces L_i may change from one location to an adjacent one up to one interface, $|L_{i+1} - L_i| \leq 1$, but it is bounded by the maximum number of interfaces L ($0 \leq L_i \leq L$). According to model (1), these assumptions involve multiple targets (up to L) that may be continuous or discontinuous across scans.

For one echo only, the likelihood ratio test is useful to discriminate between two alternative hypotheses: $H_0(x_i)$ and $H_1(x_i)$. $H_1(x_i)$ denotes the presence of an echo within the observation window $T \gg T_w$, regardless of its TOD $\tau_1(x_i)$, while $H_0(x_i)$ indicates absence of echo [i.e. $s(x_i, t) = n(x_i, t)$]. When the pulse waveform $w(t)$ is known, the correlation filter represents the optimum approach for the estimation of echo amplitude and TOD [6].

2.1. Single-target detection/tracking algorithm

Let us assume, at first, that there is only one interface in the model (1): $0 \leq L_i \leq L = 1$. For the i-th scan, the signal $s(x_i, t)$ is uniformly sampled in time (time sampling is Δt with M samples or bins/scan) and arranged in column vector $\mathbf{s}_i = [s(x_i, t_1), \ldots, s(x_i, t_M)]^T$. Echoes may have any of M TODs within the time sampling bins. Thus, a sequence of $M+1$ disjoint sub-hypotheses $h_m(x_i)$ ($m = 0, 1, ..., M$) is defined as a partition of the whole hypotheses space $[\bigcup_{j=1}^{M} h_j(x_i) \equiv H_1(x_i)]$. Hypothesis $h_m(x_i)$ (with $m = 1, ..., M$) indicates that the echo is present with TOD $t_m = m\Delta t$, hypothesis $h_0(x_i)$ holds true when the echo is absent $[h_0(x_i) \equiv H_0(x_i)]$.

The a-posteriori pdf of the $M+1$ sub-hypotheses $p[h_m(x_i) \mid \mathbf{S}_i]$, evaluated for all the data up to the i-*th* scan (i.e. $\mathbf{S}_i = [\mathbf{s}_i, ..., \mathbf{s}_1]$), allows us the maximum a–posteriori (MAP) or the minimum mean square error (MMSE) estimation of echo TODs as the maximum or the mean value of a-posteriori pdf (evaluated for $m = 1, ..., M$) respectively. Similarly, the interface detection depends on the a-posteriori pdf $\sum_{m=1}^{M} p[h_m(x_i) \mid \mathbf{S}_i]$. From the Bayes' theorem, the a–posteriori pdf $p[h_m(x_i) \mid \mathbf{S}_i]$ is described by:

$$p\left[h_m(x_i) \left| \mathbf{S}_i\right.\right] = \Gamma_i \, p\left[\mathbf{s}_i \middle| h_m(x_i)\right] \, p\left[h_m(x_i) \left| \mathbf{S}_{i-1}\right.\right], \quad (2)$$

for $m = 0, 1, \ldots, M$. Here $p[h_m(x_i) \mid \mathbf{S}_{i-1}]$ denotes the a-priori pdf that hypothesis $h_m(x_i)$ holds true in the i-th scan; $p[\mathbf{s}_i \mid h_m(x_i)]$ is the conditional pdf of observations conditioned to the presence of an echo with TOD $m\Delta t$ and under the assumption that the observations $\mathbf{s}_i$ are independent of all other observations up to the *(i-1)-th* scan; Γ_i represent a normalization term. The a-posteriori pdf of detection (here denoted with superscript d) is derived from the union of all sub-hypotheses $h_m(x_i)$:

$$p^{(d)}\left[H_1(x_i) \left| \mathbf{S}_i\right.\right] \equiv \sum_{m=1}^{M} p\left[h_m(x_i) \left| \mathbf{S}_i\right.\right]. \quad (3)$$

The a-posteriori pdf of tracking (superscript t)

$$p^{(t)}\left[h_m(x_i) \left| \mathbf{S}_i\right.\right] = \frac{p\left[h_m(x_i) \left| \mathbf{S}_i\right.\right]}{\sum_{m=1}^{M} p\left[h_m(x_i) \left| \mathbf{S}_i\right.\right]}, \quad (4)$$

for $m \geq 1$, is the probability that hypothesis $h_m(x_i)$ holds true conditioned to the assumption that an echo has been detected in the i-th scan, regardless of its TOD. For each scan an interface is assumed to be present if the a-posteriori pdf of detection is $p^{(d)}[H_1(x_i) | \mathbf{S}_i] > 1/2$; the estimation of interface TOD $\hat{\tau}(x_i)$ is then performed based on the a-posteriori pdf of tracking $p^{(t)}[h_m(x_i) | \mathbf{S}_i]$ exploiting the MAP or MMSE criteria. Both interface detection and interface tracking pdfs need to be evaluated to assess the presence and the TOD of an interface. At the i-*th* scan, the pdf value of sub-hypothesis $h_0(x_i)$ allows us to check if the hypothesis $H_0(x_i)$ [or, equivalently, $H_1(x_i)$] holds true. Therefore, at the i-th scan, the algorithm can be either in tracking-state (i.e., it is tracking the TOD corresponding to an interface) or in not-tracking-state (i.e., there is no-interface).

2.2. Conditional pdf with known waveform $w(t)$

The conditional pdf $p[\mathbf{s}_i | h_m(x_i)]$, for $m \geq 1$, is evaluated by assuming that the waveform $w(t)$ is known. Equation (2) is more conveniently rewritten in term of the likelihood ratio function of the data samples $\Lambda[\mathbf{s}_i | h_m(x_i)] \equiv p[\mathbf{s}_i | h_m(x_i)] / p[\mathbf{s}_i | h_0(x_i)]$: $p[h_m(x_i) | \mathbf{S}_i] = \Gamma_i' \, \Lambda[\mathbf{s}_i | h_m(x_i)] \, p[h_m(x_i) | \mathbf{S}_{i-1}]$ by changing the normalization term Γ_i only. If the time interval is long enough to contain the waveform without edge effects, the likelihood ratio is simply given by ($m \geq 1$):

$$\Lambda[\mathbf{s}_i | h_m(x_i)] = \exp\left\{-\frac{E_w - 2g(x_i, t_m)}{2\sigma_n^2}\right\} \quad (5)$$

where E_w is the energy of the waveform and $g(x_i, t_m) = \sum_{k=1}^{M} s(x_i, t_k) \, w(t_k - m\Delta t)$ denotes the correlation between the measured data $s(x_i, t)$ and the waveform $w(t)$ delayed by $t_m = m\Delta t$ (note that $\Lambda[\mathbf{s}_i | h_0(x_i)] \equiv 1$).

A simple way to compensate for waveform distortion, that frequently is considered in applications, is by using analytic signals both for $s(x_i, t)$ and $w(t)$. In this case, the likelihood ratio (5) needs to be modified accordingly [3].

2.3. First-order Markov model

The lateral interface continuity constraint is applied in the form of a relationship between the a-priori pdf of the i-th scan and the a-posteriori pdf of the *(i-1)*-th scan. This is obtained assuming a first order Markov model for the evaluation of the a-priori pdfs [2]. The a-priori pdf for the i-th scan are thus obtained by using different transition probabilities for detection pdf $p^{(d)}\left[H_1(x_i)|\mathbf{S}_{i-1}\right]$ and tracking pdf $p^{(t)}\left[h_m\left(x_i\right)|\mathbf{S}_{i-1}\right]$:

$$p^{(d)}\left[H_1(x_i)|\mathbf{S}_{i-1}\right] = (1-q)\,p^{(d)}\left[H_1(x_{i-1})|\mathbf{S}_{i-1}\right] + +q\,p^{(d)}\left[H_0(x_{i-1})|\mathbf{S}_{i-1}\right] \quad (6a)$$

$$p^{(t)}\left[h_m\left(x_i\right)|\mathbf{S}_{i-1}\right] = \sum_{k=-N}^{N} b_k\, p^{(t)}\left[h_{m-k}\left(x_{i-1}\right)|\mathbf{S}_{i-1}\right]. \quad (6b)$$

The transition probability for detection q is the probability to change from the state of interface absent to interface present or viceversa. The transition probabilities for tracking b_k have an even symmetry with respect to b_0 ($b_0 \geq b_k$) and their sum is normalized to one. The choice of the shape of the transition probabilities b_k as well as the width N depends on the specific application; in any case the value N is bounded by the pulse resolution $N < T_w/\Delta t$. We preferred here a triangular shape: $b_k = \left(1-|k|/N\right)/N$. To preserve uniform stationary pdf across the scans, mirror boundary conditions should be adopted in equation (6b). Transition of a-posteriori detection probability from interface absent to interface present (lock transition) or from interface present to interface absent (unlock transition) are mostly dominated by the SNR term and by the transition probabilities q and b_k. A general analysis of the lock/unlock transition is rather cumbersome, however analytical results may be derived using horizontal interfaces and $N=1$. Steady state values of $p^{(d)}\left[H_1\left(x_i\right)|\mathbf{S}_i\right]$ when the algorithm is both in tracking and in non-tracking mode is symmetrical with respect to the threshold. During the initialization phase, the a-priori pdf of detection and tracking are chosen to be uniformly distributed.

The a-priori pdfs for the $M+1$ hypotheses to be used in Bayes' formula (2) are derived from the a-priori pdfs for detection and tracking as described in equations (3) and (4).

2.4. Multitarget detection/tracking algorithm

The approach discussed so far is for detecting and tracking one interface only. In presence of more than one interface, the detection/tracking algorithm tracks the TOD of the interface that has the largest SNR. This tracking ambiguity could be avoided, in principle, by processing sub-sequences of time samples. In practice, errors are difficult to avoid when two interfaces are close to each other (because of limits in echo resolution), when one interface splits into two interfaces, or when several interfaces merge into one.

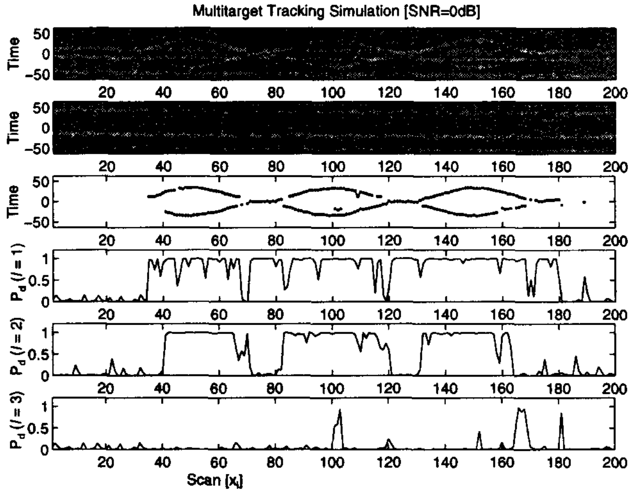

Figure 1: Multitarget tracking of simulated data (from the top): synthetic data with SNR=0 dB [the waveform is: $w(t) = (1-\tilde{t}^2)\exp(-\tilde{t}^2/2)$, where $\tilde{t} = t/T_w$; $T_w = 10\Delta t$]; MAP estimation of interface TODs; residual $s^{(4)}\left(t, x_i\right)$ after tracking (tracking parameters: $q = 0.01$, $N = 4$); $p^{(d)}\left[H_1\left(x_i\right)|R_i\right]$ for various iterations from $l = 1$ to $l = 3$.

Tracking of multiple interfaces is performed here iteratively, each iteration corresponds to detection and tracking of one interface only. Let $\widehat{\tau}_l(x_i)$ be the estimated TOD for the l-th interface at the l-th iteration and let $s^{(l)}(x_i,t)$ be the corresponding data (superscript denotes that all the interfaces, up to the *(l-1)*-th, have been detected/tracked). The maximum likelihood estimate of echo amplitude, if present, simply follows from correlation filter output evaluated at the estimated TOD $\widehat{\tau}_l(x_i)$: $\widehat{a}_l(x_i) = g(x_i, \widehat{\tau}_l(x_i))/E_w$. Both TOD and amplitude of the echo that corresponds to the l-th interface are known. To avoid multiple tracking of the same interface, echoes of the l-th tracked interface are removed from $s^{(l)}(x_i,t)$. Therefore the data to be used for the *(l+1)*-th interface detection/tracking (or *(l+1)*-th iteration) is: $s^{(l+1)}(x_i,t) = s^{(l)}(x_i,t) - \widehat{a}_l(x_i)\,w(t-\widehat{\tau}_l(x_i))$. For the first iteration it is: $s^{(1)}(x_i,t) \equiv s(x_i,t)$. Detection/tracking iterations are performed until the residual $s^{(l+1)}(x_i,t)$ achieves a reasonable level or the detection probability $p^{(d)}[H_1\left(x_i\right)|\mathbf{S}_i^{(l+1)}]$ is mostly below threshold and/or by limiting the maximum number of iterations $l \leq L$.

Fig.1 shows a simulation of the multitarget detection/tracking algorithm for two interfering sinusoidal interfaces (SNR=0dB) where it has been assumed $L = 3$. After $l = 2$ the a-posteriori probability of detection is mostly below threshold; this corresponds to the case when no more interfaces are to be tracked. In addition, $s^{(4)}(x_i, t)$ after tracking $L = 3$ (potential) interfaces does not show any residual interface to be tracked.

The iterative approach followed here for multitarget tracking is similar, in practice, to the inverse probability used in ref.[1] to avoid tracking the same target several times. Both methods are compared in the next section.

3. APPLICATION TO MONOSTATIC GPR

In monostatic GPR applications (see ref.[4]), layer-stripping inversion is a well known technique to estimate the permittivity profile in heterogeneous media from the estimation of the TODs $\{\tau_l(x_i)\}$ and amplitudes $\{a_l(x_i)\}$. Up to now, the multitarget detection/tracking appears to be the only feasible approach for layer-stripping inversion in real-time application of GPR. Fig.2 (a) shows an example of experimental GPR data (T_w =1ns); the first echo (time sample $m \simeq 50$) corresponds to air/ground interface, later echoes are asphalt/concrete ($m \simeq 300$) and concrete/ground ($m \simeq 500$) interfaces. Echoes are discontinuous along depeer interfaces because of low SNR (mostly due to signal attenuation) or for complex structural features. Comparisons between the multitarget tracking algorithm applied after echo detection [1] and the multitarget detection/tracking one proposed in this paper are shown in Fig.2 (b) and (c), respectively. TODs have been estimated with MAP criterium and the tracking parameters have been optimized for both methods. According to comparative analyses shown in ref.[5], it may also be concluded that, with respect to multitarget tracking after echo detection, our multitarget detection/tracking algorithm shows: i) better time-resolution of interfaces in multilayered structure, ii) less false alarms and iii) better continuity of the reconstructed interfaces. Results of other applications of multitarget detection/tracking algorithm to geophysics validate these results.

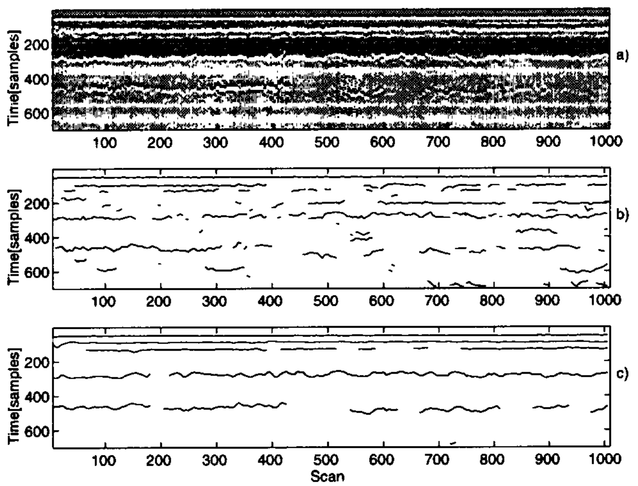

Figure 2: Comparison between multitarget tracking algorithms. (a): data from monostatic GPR measurements; (b): multitarget tracking algorithm after echo detection (parameters according to [1]: $P_d = 0.4$, $P_f = 0.01$, $q = 0.01, L = 7$); (c) multitarget detection/tracking algorithm ($N = 4$, $SNR = 5dB$, $q = 0.01$, $L = 7$).

4. CONCLUSIONS

Iterative technique for detection/ tracking of multiple interfaces have been proposed as a two-step approach: first the interface is detected and echoes are spatially tracked (either using MAP or MMSE criteria), then the echoes that belong to the detected interface are removed from the data. Compared to methods that track the TOD after echo detection, the detection/tracking algorithm shows good capability (tradeoff between detection and false alarms) to detect and track the TODs of interfaces that are discontinuous, overlapping, and closely spaced (compared to the waveform resolution).

5. REFERENCES

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