# NUMERICAL OPTIMIZATION OF NON-ADAPTIVE MICROPHONE ARRAYS

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## ABSTRACT

The paper describes an application of the numerical optimization methods for the design of non-adaptive multi-sensor arrays. The parameters and the geometry of such arrays do not change with changes in the input signals, and must be chosen in advance, depending on the array application. Generally, the goal of a non-adaptive multi-sensor array may be numerically expressed through its pattern function which shows the gain for a signal coming from a particular direction in space. The real pattern function depends on the geometry of the array and on the processing which signals from every sensor undergo. The array pattern function is non-linear and it is frequency dependent. The difference between the goal pattern function and the real pattern function is the essence of numerical optimization. The geometry and the processing parameters of the multi-sensor array are optimized to provide the minimum difference over a specified frequency range. Such minimization is a difficult non-linear and multi-parameter problem . Optimization results for several goal functions for multi-microphone arrays are provided and discussed.

## 1. INTRODUCTION

If we concentrate on linear processing of the received signals, then the output from a multi-sensor array is obtained as a sum of linearly processed sensor inputs. The purpose of such processing is to attenuate the interfering signals (noise) while preserving the signal of interest. Adaptive beamforming algorithms provide the best possible signal to noise ratio since they place nulls only in the directions where the interference really exists, thus optimizing the array for a particular case. On the other hand, non-adaptive arrays are designed to attenuate the signal coming from all directions but the direction of interest. The advantage of non-adaptive arrays lies in their simplicity and ability to work in very fast changing environ-

ments. The classical design of multi-sensor arrays is mostly concerned with optimization of the relative gains of sensor elements, whereas the array geometry and the relative distances between the sensors are fixed. It has much in common with filter design and it is generally done with the assumption that the sensor elements are spaced uniformly. It seems natural that if we loosen the restriction and were able to design sensor arrays of arbitrary geometry and optimal inter-element spacing with different criteria of optimality, we could design an array which better meets our needs. Unfortunately, extension of the theory to optimizing non-uniform or arbitrary spacing of elements is generally not possible. In this work, attempts are made to apply a brute force numerical optimization technique for optimizing the distances between the linear sensor array elements and the parameters of FIR filters applied to signals from every sensor. Although it is possible to apply the technique to sensor arrays of any kind, in this work we have restricted ourselves to microphone arrays and to the range of frequencies typical for speech signal processing applications.

## 2. OPTIMIZATION TECHNIQUE

For the optimization technique we use non-linear least squares optimization of the combined pattern function of the microphone array over a range of frequencies. If the array is restricted to be linear (one dimensional) then only one coordinate is optimized. For this case and when the processing for each sensor is restricted to a FIR filtering the pattern function for a specific frequency is defined by the equation

$$G(f,\theta) = \frac{\left| \sum_{m=0}^{N-1} \sum_{k=0}^{L-1} g_{mk} e^{-j\frac{2\pi f}{c}(d_m \sin(\theta) + \frac{c}{f_s}k)} \right|}{N} \quad (1)$$

where $N$ is the number of microphones, $L$ is the FIR filters length, $\theta$ is the direction of arrival, $f$ is the frequency of the signal, $f_s$ is the sampling rate, $c$ is the sound ve-

locity, $g_{mk}$ is the $k^{th}$ FIR filter coefficient for $m^{th}$ microphone and $d_m$ is the coordinate of the microphone. The purpose of optimization is to reduce the difference between some goal array pattern function $\hat{G}(f,\theta)$ and the actual pattern function defined by Equation (1). If the difference D is defined as the $L_2$ norm over some angle and frequency ranges then the purpose of the optimization is to find $\{g_{mk}\}$ and $\{d_m\}$ which provide the minimum

$$D = \min( \int_{\theta_{min}}^{\theta_{max}} ( \int_{f_{min}}^{f_{max}} (G(f,\theta) - \hat{G}(f,\theta))^2 \, df d\theta \quad )$$

In the real computations the integrals are replaced with sums over finite sets of angles and frequencies $\{\theta_i\}$, $\{f_j\}$ and a non-linear least squares fitting algorithm is used to find the coefficients.

The number of parameters to be optimized in the non-linear model (1) may be rather large, making the numerical technique used for optimization be a key factor for success of the approach. The optimization algorithms used to achieve the results provided below were developed by the author for different applications and have been adapted recently for multi-sensor arrays optimization.

## 3. EXAMPLES OF OPTIMIZATION

Figure 1 shows the results of optimization of an 8 microphones broad band array for operation in the frequency range from 1000 Hz to 4000 Hz. No FIR filtering for the sensor outputs is applied so that only the distances $\{d_m\}$ and the gains $\{g_{mo}\}$ are optimized. The angular range is from -90 to 90 degrees and the purpose of optimization is to have the unit response in the range [-20,20] degrees and to suppress everything else. The pattern functions for four frequencies inside the optimized frequency range are shown by the solid line. For comparison, the pattern function of the uniformly spaced array with the critical distance between the microphones (half of the shortest wave length) is shown by the dashed line.
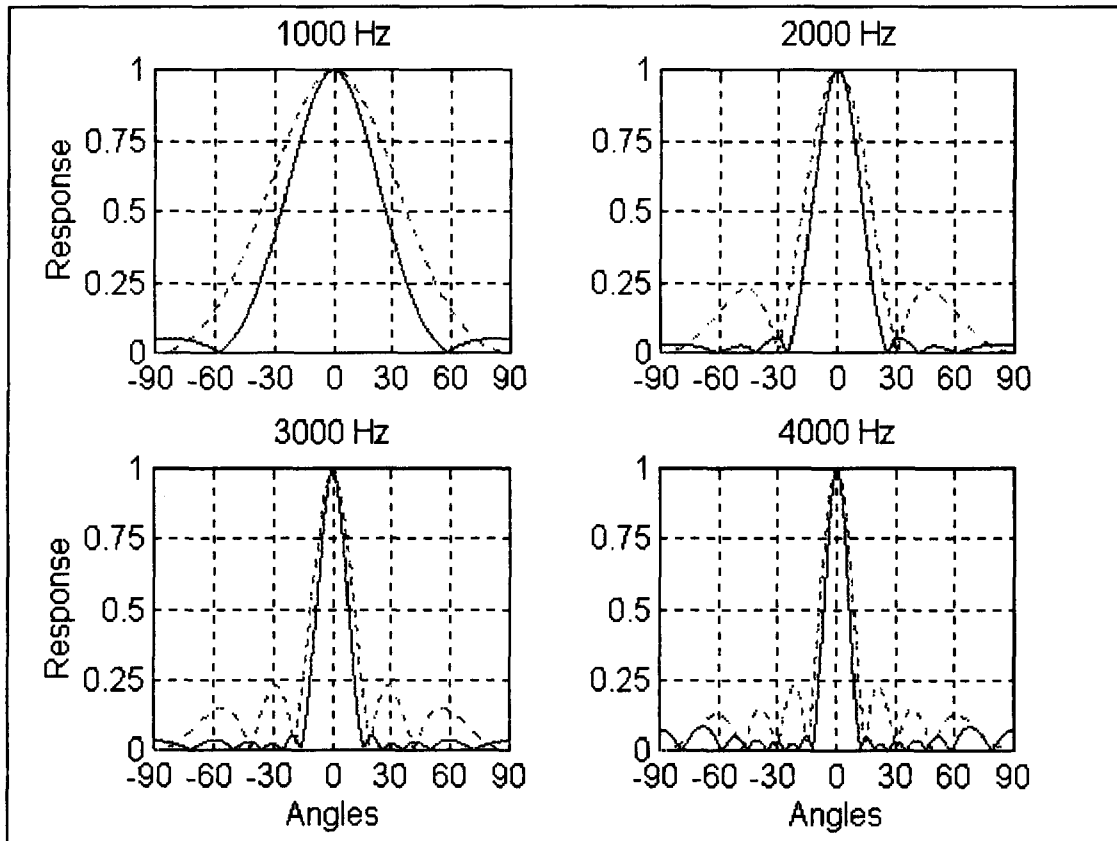


**Figure 1** Pattern functions of optimized and not-optimized 8 microphones arrays for a broad band signal. Solid line - optimized array, dashed line - not-optimized with critical distance and unit gains.

It is seen that significant improvement may be achieved although for low frequencies its still not enough.

Figure 2 shows the characteristics of an 8 microphone array optimized to operate in a narrow frequency range centered at 4 kHz frequency. The total angular range where the optimization is done and the range of interest where the signal is to be preserved are the same as for Figure 1 ([-90,90] and [-20,20] degrees). The pattern function of the optimized array is shown as a solid line and the pattern function of uniformly spaced array with the critical distance between the microphones is shown as a dashed line. This result is very interesting. It shows that for narrow band signals there is practically no need for adaptive beamforming. Very good interference canceling may be obtained for all angles by a proper array design.

Probably, the most interesting and a little bit disappointing result obtained for the first two cases is that the optimal spacing for both, wide band and narrow band, cases is uniform. The algorithm converges to uniform spacing even when the initial spacing is different. This distance is always slightly larger than the critical distance equal to the half of the shortest wave length.

It is well known that the beamwidth of a linear array decreases as frequency increases [1]. This effect is undesirable in many microphone array applications due to a low-pass filtering effect for outer portions of the main beam [2]. One purpose of using non-uniform, so-called "nested" arrays [3] is to provide a more uniform pattern function over a wide frequency range. The following example shows how this goal may be reached by a proper prefiltering of the input channels. The reported optimization technique was used to find the parameters of the filters.

To achieve a constant beam width for an 8 microphone array, we have computed a set of short, 8-tap FIR filters for every channel. The distance between the microphones is fixed and equal to 4 cm in accordance with the previous results (Fig.2,3). The prefiltering of the signals with the computed filters is supposed to provide a constant beam in the angular range [-25,25] degrees over a frequency rage from 1KHz to 4KHz and to suppress all signals coming from other directions. Due to the symmetry of the desired pattern function the number of optimized filter coefficients is reduced from 64 to 32.
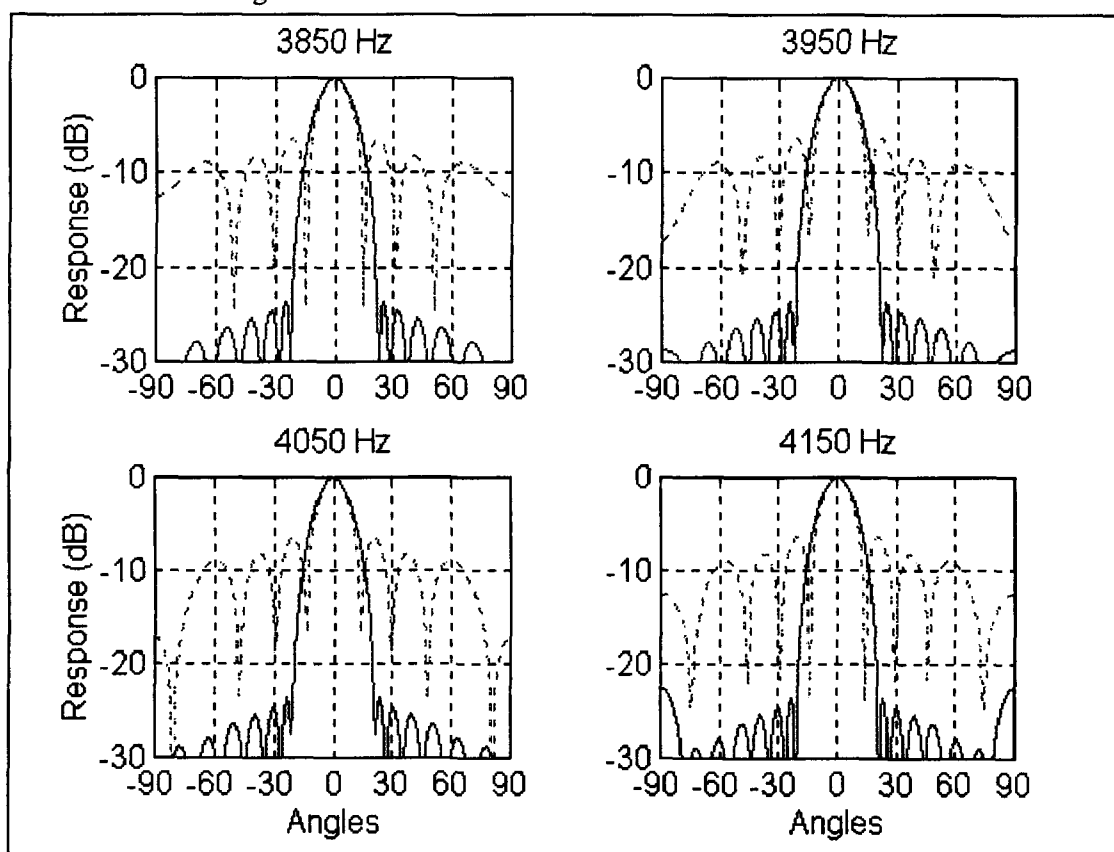


**Figure 2** Pattern functions of optimized and not-optimized 8 microphones arrays for a narrow band signal. Solid line - optimized array, dashed line - not-optimized with critical distance and unit gains.
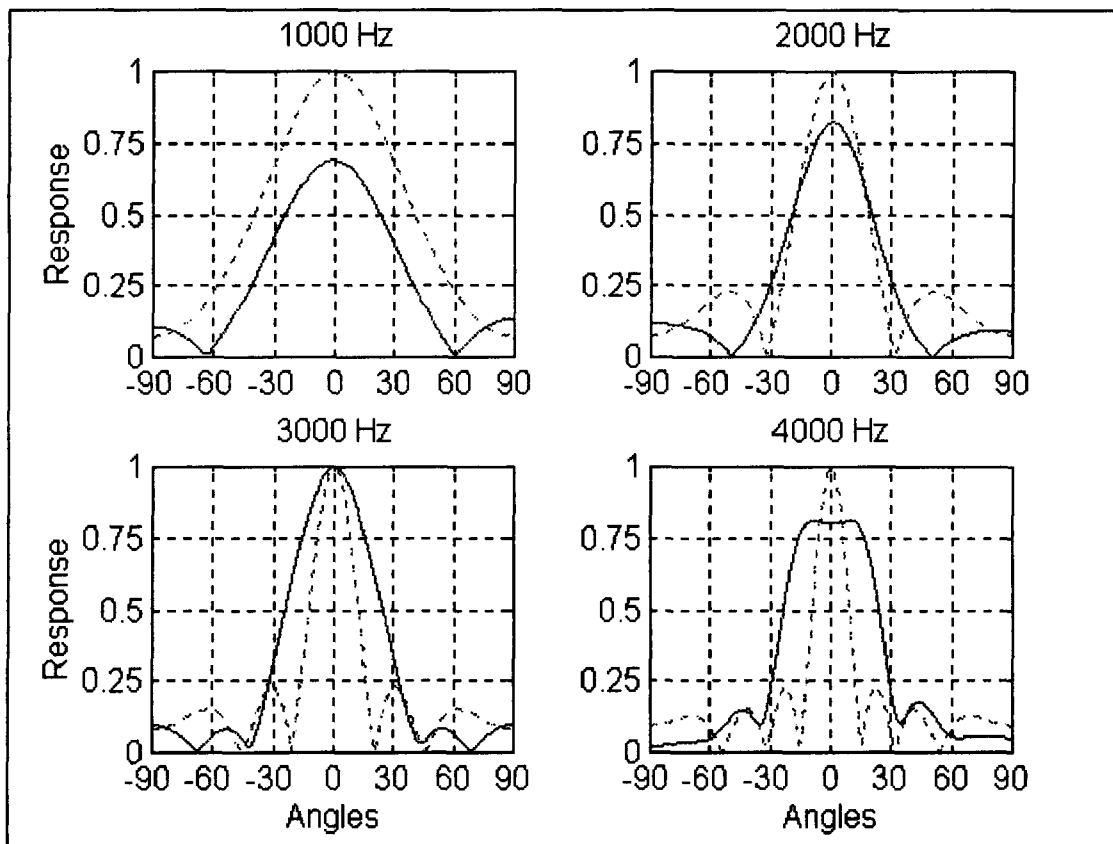
**Figure 3 Pattern functions corresponding 8 microphone arrays. Solid line - 8-taps FIR filtering of array signal channels optimized for a constant beamwidth. Dashed line - not optimized array with critical sampling and unit gains.**

The resulting pattern function for this optimization problem are shown on Figure 3 by the solid line. The pattern function of the corresponding non-optimized array with unit gains is shown by the dashed line as a reference. It is clearly seen that for the array with optimized prefiltering of each channel, the suppression level -6dB (0.5) is reached near the angle 25° over the whole frequency range where optimization is done.

## CONCLUSIONS

Examining the results a conclusion can be made that the brute force numerical optimization may serve as a powerful tool for designing arrays of sensor elements. Array pattern functions with desired characteristics may be successfully approximated by a proper choice of the coefficients for short, non-adaptive FIR filters operating on each input channel. Using non-uniform arrays where all sensors are used simultaneously for the whole frequency

range has not proved itself to give good results. Given the freedom in optimizing the sensor locations, the optimization procedure has always ended up with uniform distances between the elements.

Further research may concern the use of subband schemes where the optimization of FIR filters coefficients is made separately for every subband with different criteria for optimality.

## REFERENCES

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