COMPARISON OF PROBABILISTIC LEAST SQUARES AND PROBABILISTIC MULTI-HYPOTHESIS TRACKING ALGORITHMS FOR MULTI-SENSOR TRACKING

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ABSTRACT

A key element for successful tracking is knowing from which target each measurement originates. These measurement-to-target associations are generally unavailable, and the tracking problem becomes one of estimating both the assignments and the target states. We present the Probabilistic Least Squares Tracking (msPLST) algorithm for estimating the measurement-to-target assignments and the track trajectories of multiple targets, using measurements from multiple sensors. This is a different approach to that used in Probabilistic Multi-Hypothesis Tracking (PMHT), although both algorithms employ the concept of an extended observer containing both the target states and the measurement-to-target assignments. A comparison of both algorithms is made, and their performance is evaluated using simulated data.

1. INTRODUCTION

The tracking performance of a system may be enhanced by combining measurements from different sensors such as radar and optical. A variable update rate Kalman filter, with a common state space model of the system dynamics, is a particularly useful tool for fusing asynchronous sensor outputs. However, the gains achieved by fusing data can be severely degraded in the presence of noise and other interfering targets. This is evident from our previous work in fusing radar and optical measurements [1], where other interfering targets readily seduced the optical tracking gate.

One way to overcome this problem is to run two trackers and associate the measurements with each target. This association of measurements to targets is referred to as the data association problem, and is particularly important in multi-sensor, multi-target environments where the likelihood of incorrect assignments increases with the number of sensors and targets.

Probabilistic Multi-Hypothesis Tracking (PMHT) has recently been introduced as a promising technique for associating multiple measurements with multiple targets [2, 3]. In this algorithm, the observer is extended to include the unknown measurement-to-target assignments. Soft or probabilistic assignments are then estimated in conjunction with the target states using maximum likelihood techniques. PMHT is intended for tracking multiple targets using measurements from a single sensor. By generalising PMHT for a multi-sensor environment, we have developed a multi-sensor PMHT (msPMHT) algorithm for tracking multiple targets using measurements from multiple dissimilar sensors [4].

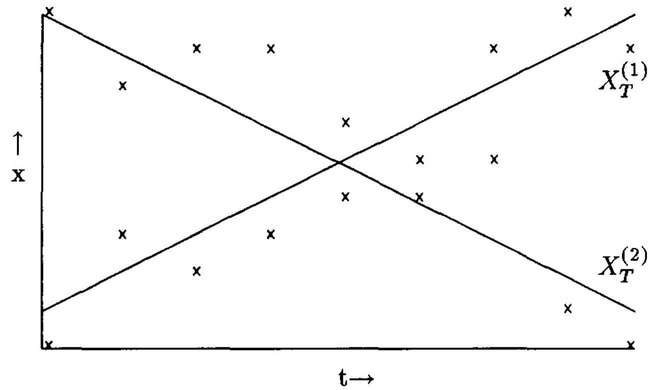

Figure 1. Fitting points to straight lines.

2. LEAST SQUARES FOR MIXED MODELS

Consider the tracking problem in figure 1, where the sets of noisy measurements, $Z_T^{(1)} = (z_{t_1}^{(1)}, \ldots, z_{t_T}^{(1)})$ and $Z_T^{(2)} = (z_{t_1}^{(2)}, \ldots, z_{t_T}^{(2)})$, are to be fitted to the unknown straight line target trajectories, $X_T^{(1)}$ and $X_T^{(2)}$, respectively. Assuming that $Z_T^{(1)}$ contains the measurements associated with $X_T^{(1)}$, and $Z_T^{(2)}$ contains those associated with $X_T^{(2)}$ (ie the measurement-to-target assignments are known), the unknown parameters (m_1, c_1, m_2, c_2) of the two straight line trajectories are determined by minimising the cost function

$$\sum_{i \in Z_T^{(1)}} \left(\epsilon_{t_i}^{(1,1)}\right)^2 + \sum_{i \in Z_T^{(2)}} \left(\epsilon_{t_i}^{(2,2)}\right)^2$$

where $\epsilon_{t_i}^{(m,r)}$ is the usual least squares error between the measurement $z_{t_i}^{(r)}$ and the trajectory $X_T^{(m)}$. If the noise on the measurements varies, for example if the measurements originate from different sensors, each error term in the sum may be weighted by the inverse of its noise. This gives a *weighted least squares* solution.

In practice, the measurement-to-target assignments may not be known, ie $Z_T^{(1)}$ and $Z_T^{(2)}$ each contain measurements associated with both trajectories. The tracking problem then becomes one of estimating both the target trajectories and the measurement-to-target assignments from the observed measurements. The observer therefore contains both the target states and the assignments. As shown in figure 1, it is not always obvious which target each measurement belongs to, particularly if the targets are close or crossing. For the above problem, a set of unknown assignment weights, $\alpha_{t_i}^{(m,r)}$, are introduced, one for each possible measurement to target assignment, representing a (normalised) confidence that the r^{th} measurement at time t_i is associated with the target m. A weight of one implies that the measurement definitely originated from the target, and zero implies that it didn't. It is these weights, or *soft* assignments, that are estimated.

For the above simplified tracking problem, the least squares criterion can be generalised to

$$\min \sum_{m=1}^{M} \sum_{r=1}^{R} \sum_{i=1}^{T} \alpha_{t_i}^{(m,r)^2} \epsilon_{t_i}^{(m,r)^2}$$

with respect to both the unknown $\alpha_{t_i}^{(m,r)}$'s and the trajectory parameters (m_1, c_1, m_2, c_2). The requirement that $\sum_m \alpha_{t_i}^{(m,r)} = 1$ for every data point is imposed, ensuring all measurements are assigned to exactly one target trajectory.

During each iteration of the algorithm, new target state and assignment weight estimates are determined from the above criterion, using the estimates from the previous iteration. These estimates gradually converge to their true values.

In the following, we present a generalised approach to this problem.

3. MULTI-SENSOR PROBABILISTIC LEAST SQUARES TRACKING (msPLST)

Consider a system of $S \geqslant 1$ sensors ($S \geqslant 2$ for multi-sensor tracking) monitoring $M \geqslant 2$ targets. Assume that at each time, $t_1, \ldots, t_T$, all S sensors produce a single measurement, each of which must be assigned to a target. This does not restrict the problem, as target models can be defined for noise, false alarms, etc.

Assume that an initial state estimate at time t_0, $\bar{x}_{t_0}^{(m)}$, and its covariance, $\bar{\Sigma}_{t_0}^{(m)}$, is available for each target model m. Then each unknown target trajectory can be defined by the stochastic process model

$$x_{t_i}^{(m)} = F_{t_{i-1}}^{(m)} x_{t_{i-1}}^{(m)} + w_{t_{i-1}}^{(m)} \qquad i = 1, \ldots, T.$$

For model $m \in \{1, \ldots, M\}$, $x_{t_i}^{(m)}$ is the state at time t_i, $F_{t_{i-1}}^{(m)}$ is the state transition matrix describing the target dynamics from time t_{i-1} to t_i, and $w_{t_{i-1}}^{(m)}$ is additive zero mean process noise with known covariance $Q_{t_{i-1}}^{(m)}$.

Each measurement in the set $Z_T = (Z_T^{(1)}, \ldots, Z_T^{(R)})$, $Z_T^{(r)} = (z_{t_1}^{(r)}, \ldots, z_{t_T}^{(r)})$, can be represented by one of the measurement models in the set

$$z_{t_i}^{(r)} \in \{H_{t_i}^{(m,r)} x_{t_i}^{(m)} + v_{t_i}^{(m,r)} \qquad m = 1, \ldots, M\}.$$

$z_{t_i}^{(r)}$ is the measurement originating from the r^{th} sensor during the scan at time t_i. $H_{t_i}^{(m,r)}$ is the measurement matrix that maps the state space of target m onto the measurement $z_{t_i}^{(r)}$, and $v_{t_i}^{(m,r)}$ is additive zero mean measurement noise with known covariance $R_{t_i}^{(m,r)}$.

The process and measurement noises are assumed to be uncorrelated, and independent between models. The assignment weights are assumed to be independent between scans.

This leads to the cost function

$$J = \sum_{m=1}^{M} \left(\sum_{r=1}^{R} \sum_{i=1}^{T} \alpha_{t_i}^{(m,r)} \epsilon_{t_i}^{(m,r)T} R_{t_i}^{(m,r)-1} \epsilon_{t_i}^{(m,r)} \alpha_{t_i}^{(m,r)} + \varepsilon_{t_0}^{(m)T} \bar{\Sigma}_{t_0}^{(m)-1} \varepsilon_{t_0}^{(m)} + \sum_{i=1}^{T} \varepsilon_{t_i}^{(m)T} Q_{t_i}^{(m)-1} \varepsilon_{t_i}^{(m)} \right) \quad (1)$$

$$\text{where} \quad \epsilon_{t_i}^{(m,r)} = \left(z_{t_i}^{(r)} - H_{t_i}^{(m,r)} x_{t_i}^{(m)} \right) \quad (2)$$

$$\varepsilon_{t_0}^{(m)} = \left(x_{t_0}^{(m)} - \bar{x}_{t_0}^{(m)} \right) \quad (3)$$

$$\varepsilon_{t_i}^{(m)} = \left(x_{t_i}^{(m)} - F_{t_{i-1}}^{(m)} x_{t_{i-1}}^{(m)} \right). \quad (4)$$

It contains the standard least squares error terms (2), representing the differences between the actual measurements and those that would have resulted in the absence of noise. It also includes the errors between the actual target states and their initial estimates (3), and the errors between the actual target states and those predicted by the target process model (4).

Minimising J with respect to the unknown assignment weights, subject to $\sum_{m=1}^{M} \alpha_{t_i}^{(m,r)} = 1$, yields

$$\alpha_{t_i}^{(m,r)} = \frac{\left(\epsilon_{t_i}^{(m,r)T} R_{t_i}^{(m,r)-1} \epsilon_{t_i}^{(m,r)} \right)^{-1}}{\sum_n^M \left(\epsilon_{t_i}^{(n,r)T} R_{t_i}^{(n,r)-1} \epsilon_{t_i}^{(n,r)} \right)^{-1}}. \quad (5)$$

for $m = 1, \ldots, M$ and $r = 1, \ldots, R$.

Minimising the cost function with respect to the target states produces M independent sets of $T+1$ simultaneous equations. The state estimates at times t_0 to t_T for a particular target model are obtained by solving the appropriate set of equations. These equations can be solved using M fixed interval Kalman smoothers, one for each target model. For this, we define the M composite measurement models

$$\tilde{z}_{t_i}^{(m)} = \tilde{H}_{t_i}^{(m)} x_{t_i}^{(m)} + \tilde{v}_{t_i}^{(m)} \qquad m = 1, \ldots, M.$$

For target m, $\tilde{H}_{t_i}^{(m)}$ is the composite measurement matrix that maps the state vector onto each possible measurement type (ie bearing, range, range rate, etc) and $\tilde{v}_{t_i}^{(m)}$ is the composite zero mean measurement noise with covariance $\tilde{R}_{t_i}^{(m)}$. The composite measurement and covariance for each target model are defined as

$$\tilde{R}_{t_i}^{(m)} = \left(\sum_{r=1}^{R} \alpha_{t_i}^{(m,r)^2} \mathcal{H}_{t_i}^{(m,r)T} R_{t_i}^{(m,r)-1} \mathcal{H}_{t_i}^{(m,r)} \right)^{-1}$$

$$\tilde{z}_{t_i}^{(m)} = \tilde{R}_{t_i}^{(m)} \sum_{r=1}^{R} \alpha_{t_i}^{(m,r)^2} \mathcal{H}_{t_i}^{(m,r)T} R_{t_i}^{(m,r)-1} z_{t_i}^{(r)} \qquad (6)$$

$$\text{where} \qquad \mathcal{H}_{t_i}^{(m,r)} = H_{t_i}^{(m,r)} \tilde{H}_{t_i}^{(m)T} \left(\tilde{H}_{t_i}^{(m)} \tilde{H}_{t_i}^{(m)T} \right)^{-1}.$$

These composite values are substituted for H, R and z in the the Rauch-Tung-Striebel form of the fixed interval Kalman smoother equations [5].

Equation (5) and the Kalman smoother are used recursively to estimate the measurement-to-target assignments and target states.

The algorithm is easily extended for the case of asynchronous measurements by allowing the number of measurements to vary with time. The measurement number, r, now takes the values $1, \ldots, n_{t_i}$ (instead of $1, \ldots, R$), where n_{t_i} denotes the number of measurements occuring at time t_i.

4. COMPARISON WITH msPMHT

The likelihood function for the msPMHT algorithm [6] for the linear-Gaussian case is given as

$$Q = -\sum_{m=1}^{M} \left(\sum_{r=1}^{R} \sum_{i=1}^{T} \omega_{t_i}^{(m,r)'} \epsilon_{t_i}^{(m,r)T} R_{t_i}^{(m,r)-1} \epsilon_{t_i}^{(m,r)} + \varepsilon_{t_0}^{(m)T} \bar{\Sigma}_{t_0}^{(m)-1} \varepsilon_{t_0}^{(m)} + \sum_{i=1}^{T} \varepsilon_{t_i}^{(m)T} Q_{t_i}^{(m)-1} \varepsilon_{t_i}^{(m)} \right) + \sum_{m=1}^{M} \sum_{i=1}^{T} \log \pi_{t_i}^{(m)} \sum_{r=1}^{R} \omega_{(t_i)}^{(m,r)'}$$

where $'$ denotes the value from the previous iteration of the algorithm. The first term constitutes the negated msPLST cost function (1), with the measurement assignment probability, $\omega_{t_i}^{(m,r)'}$, replacing the squared measurement weight, $\alpha_{t_i}^{(m,r)^2}$. The msPMHT estimates the measurement probabilities $\pi_{t_i}^{(m)}$, calculating the assignment probabilities from these estimates. The measurement probabilities, $\pi_{t_i}^{(m)}$, represent the fraction of measurements assigned to each target model, and produce the final summation in the likelihood function.

The target states in both algorithms are estimated with a fixed interval Kalman smoother. The composite measurement and covariance expressions for the msPMHT [6] are equivalent to the expressions in (6), with $\alpha_{t_i}^{(m,r)^2}$ replaced by $\omega_{t_i}^{(m,r)'}$.

The msPMHT likelihood function is maximised subject to the constraint that all measurement probabilities at any time must sum to unity. The msPLST is constrained by the more stringent requirement that the measurement weights associated with each measurement must sum to unity.

To compare performance, we consider a system of two sensors tracking a single constant velocity target. The two target models in the tracker are both initialised using this target. After 15 seconds, a second target with a lower velocity seduces the second sensor, the first continuing to track the original target. The first sensor has a measurement noise covariance ten times that of the second. The target tracks obtained from the msPMHT algorithm are shown in fig. 2. The msPLST tracks about the point where the second target is introduced are shown in the inset for comparison.

It can be seen that both algorithms maintain track on both targets. During the first 15 seconds, measurements from both sensors are fused, and during the later stages, each target is assigned measurements from only one sensor. The tracking performance (track error covariance) is similar for both algorithms under these conditions. However, the introduction of the second target causes the msPMHT tracks to deviate significantly from the true target position for several hundred samples. The msPLST assignment weights (5) are more sensitive to instantaneous changes in error between the measurements and predicted target position than the msPMHT assignment probabilities, which from [4] are given by

$$\omega_{t_i}^{(m,r)} = \frac{\pi_{t_i}^{(m)'} \exp\left(-\frac{1}{2} \epsilon_{t_i}^{(m,r)T'} R_{t_i}^{(m,r)-1} \epsilon_{t_i}^{(m,r)'}\right)}{\sum_{n=1}^{M} \pi_{t_i}^{(n)'} \exp\left(-\frac{1}{2} \epsilon_{t_i}^{(n,r)T'} R_{t_i}^{(n,r)-1} \epsilon_{t_i}^{(n,r)'}\right)}$$

$$\text{where} \qquad \pi_{t_i}^{(m)'} = \frac{1}{R} \sum_{r=1}^{R} \omega_{t_i}^{(m,r)'}.$$

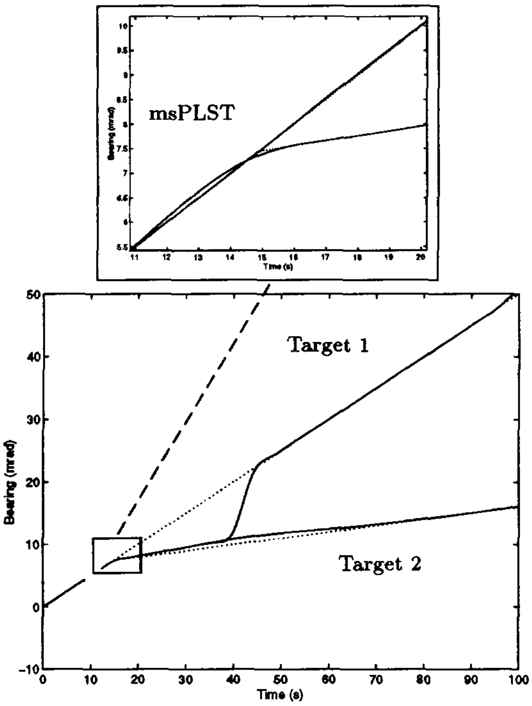

Figure 2. msPMHT tracks.

The effect of this on the assignment variables for measurements $Z_T^{(1)}$ and target m is shown in figure 3. While the targets remain sufficiently close, this lower sensitivity, and the tendency of the algorithm to favour the measurements with lower noise ($Z_T^{(2)}$), causes a fall in $\pi_{t_i}^{(1)}$ as the targets separate. This produces the drop in assignment probability shown in figure 3.

In both algorithms, initialisation is extremely important. This is because neither guarantees convergence to a global maximum or minimum; the msPMHT converging to the nearest local maximum and the msPLST converging to the nearest local minimum. Therefore initialisation is critical to obtaining the correct solution. As both algorithms employ different cost functions, one possible approach could be to repetitively use both algorithms, the estimates obtained from the msPMHT initialising the msPLST and vice-versa.

5. CONCLUSIONS

We have presented the multi-sensor Probabilistic Least Squares Tracking (msPLST) algorithm to estimate the unknown target states and measurement-to-target assignments for multiple targets using measurements from multiple sensors.

We have compared its performance with the multi-sensor PMHT algorithm, and, although both have similarities, the msPLST has superior performance in the presence of interfering targets (provided it is correctly initialised). Neither algorithm guarantees convergence to the true target states, but their complementary performance may be useful in obtaining the correct solution.

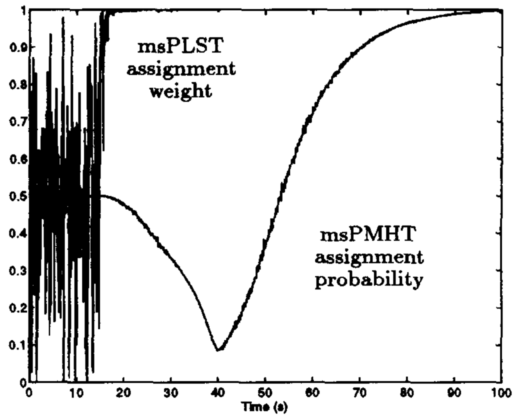

Figure 3. $\mathbf{Z_T^{(1)}}$ to target 1 assignments.

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