DESIGN OF AN OPTIMUM WIDEBAND ACTIVE SONAR ARRAY WITH ROBUSTNESS

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***ABSTRACT**: The use of wideband active sonar array processing to estimate the range, velocity and bearing of a target has received much interest in the literature recently. Although increased attention has been focused on wideband correlation processing for estimating range and velocity, array directivity patterns are almost always computed and interpreted under the narrowband signal assumption. This paper considers the target bearing estimation problem using the wideband correlation approach. Via this approach, it will be shown how an optimum set of array weights can be selected for a known transmitted signal. The optimization procedure also provides robustness against errors in the array structure.*

1. INTRODUCTION

Wideband transmissions are usually used in active sonar systems for the detection of targets and the estimation of their parameters in the presence of noise and clutter. Target velocities and ranges can be estimated from the echo signal at the receiver using matched filtering techniques [1]. Also, target bearing can be estimated using array processing methods [7]. Although increased attention in the published literature has been focused on wideband processing of the sonar signal for range-velocity estimation, array directivity patterns are almost always computed and interpreted under the narrowband signal assumption [2][3]. It is the intention of this paper to formulate the problem of target bearing estimate using the approach of wideband correlation processing. In the paper we assume uniform linear arrays. In addition, we will only present results using Gaussian shaped Linear Frequency Modulated signals as the transmitted wideband signal. The technique can be extended, however, to any transmitted waveform and array geometry.

It was noted in [3] that the sidelobe levels of an array receiver directivity pattern can be decreased by proper selection of the array weights. However, techniques for the design of such weights are not discussed in the literature. This paper presents a methodology for designing optimum array weights for reducing the sidelobe levels. The optimization procedure also provides robustness against uncertainties in the actual array sensor positions. This is essential as the directivity pattern can be very sensitive to such errors. For example, in towed array applications, the actual sensor positions are seldom known exactly.

2. WIDEBAND CORRELATION PROCESSING

In active sonar, the transmission is usually of short duration and has a large time-bandwidth product because of the frequency modulation used to achieve 'pulse compression'. Therefore, the narrowband signal assumption is not valid and processing at the receiver must be performed via wideband correlation. The use of wideband signals also improves the directional properties of the array processor.

Consider a single discrete scatterer (target) moving at a constant velocity v towards a single receiver. Suppose the transmitted signal is described by $a(t)$, the time taken by the back-scattered signal to reach the sensor is given by τ_o, and c is the velocity of sound in the medium under observation. The output of the wideband correlator in the delay-scale (τ, α) domain is given by,

$$\rho(\tau, \alpha) = \left(\frac{S_o}{\sqrt{\alpha_o}}\right)\chi\left(\alpha_o[\tau - \tau_o], \frac{\alpha}{\alpha_o}\right) \quad , \qquad (1)$$

where $\alpha_o = (c+v)/(c-v)$ and the factor S_o describes the reflectivity of the scatterer [1]. S_o also includes the attenuation in the medium as well as the receiver gain. In equation 1, the wideband auto ambiguity function of the transmitted signal $a(t)$ is defined as

$$\chi(\tau, \alpha) = \sqrt{\alpha}\int_{-\infty}^{\infty}(a(t)a^*(\alpha[t-\tau])dt) \quad . \qquad (2)$$

In the case of array processing, consider a linear array of $(M+1)$ sensors with uniform spacing d. Let the array weighting coefficient corresponding to the m^{th} element be denoted by w_m and the output of the array be processed using a wideband correlator. Suppose the target is at an angle θ measured with respect to the normal of the array axis. Because of the weight and sum operation of the array processor, equation 1 can be now extended as follows.

$$\rho(\tau, \alpha, \theta) = \sum_{m=-M/2}^{M/2} \left(\frac{S_o}{\sqrt{\alpha_0}}\right) w_m \chi\left(\alpha_0\left[\tau - \tau_o + \frac{md\sin\theta}{c}\right], \frac{\alpha}{\alpha_0}\right) \quad (3)$$

Evaluating the above equation at $\tau = \tau_o$ and $\alpha = \alpha_0$ provides the signal energy of the detection process as a function of θ, i.e.

$$|\rho(\tau_o, \alpha_o, \theta)|^2 = \frac{|S_o|^2}{\alpha_o}\left|\sum_{m=-M/2}^{M/2} w_m R(m\Delta\sin\theta)\right|^2 \quad . \quad (4)$$

$$= \frac{|S_o|^2}{\alpha_0} B(\theta) \qquad \text{where} \qquad \Delta = \frac{\alpha_0 d}{c}$$

In equation 4, $B(\theta)$ defines an array directivity pattern which depends on the autocorrelation function, $R(\tau)$, of the transmitted signal $a(t)$.

3. OPTIMIZATION OF ARRAY WEIGHTS

In this section, we consider the selection of an optimum set of array weights for a known transmitted signal. The optimization can be obtained by minimizing the energy of the beam pattern outside the main lobe (defined as $|\theta| < \theta_b$ [4]), i.e.,

$$\min_w \left(Q = \int_{\theta_b}^{\pi/2} B(\theta) d\theta \right) \qquad \text{subjected to} \qquad w^T U = 1 \quad (5)$$

where w is a column vector containing the array weights and U is column vector with unit elements. Note that the constraint $w^T U = 1$ is necessary to ensure unity gain in the look direction $\theta = 0$. Substituting for $B(\theta)$, the cost function Q of equation 5 can be expressed as, $Q = w^T C w$, where C is an $(M+1) \times (M+1)$ matrix with its $(p, q)^{th}$ element given by,

$$c_{p,q} = \int_{\theta_b}^{\pi/2} R(p\Delta\sin\theta) R(q\Delta\sin\theta) d\theta \quad . \quad (6)$$

The solution to the above minimization problem is given by $w_{opt} = C^{-1} U^T (U C^{-1} U^T)^{-1}$ [5].

4. ROBUSTNESS CONSTRAINT

In this section we provide a robustness analysis of the array processor. Suppose $B^\varepsilon(\theta)$ is defined as the directivity pattern when the system parameters differ from the ideal, i.e.,

$$B^\varepsilon(\theta) = \left|\sum_{m=-M/2}^{M/2} w_m^\varepsilon R(m\Delta^\varepsilon \sin\theta^\varepsilon)\right|^2 , \quad (7)$$

where the superscript ε is used to denote the non-ideal parameters. Defining the error directivity pattern,

$$E(\theta) = \left|\sum_{m=-M/2}^{M/2} w_m^\varepsilon R(m\Delta^\varepsilon \sin\theta^\varepsilon) - \sum_{m=-M/2}^{M/2} w_m R(m\Delta\sin\theta)\right|^2 , \quad (8)$$

we obtain,

$$B^\varepsilon(\theta) = B(\theta) + 2\sqrt{B(\theta)E(\theta)} + E(\theta) \quad . \quad (9)$$

It follows therefore, that the smaller the value of $E(\theta)$ the more robust the array processor. In the following sections we investigate the behaviour of $E(\theta)$ for sensor position errors.

Suppose δ_m^x and δ_m^y are the position errors of the m^{th} sensor along and normal to the array axis, respectively. Assuming that errors have zero mean and independent identically distributed, equation 8 now results in,

$$E(\theta) = \left|\sum_{m=-M/2}^{M/2} w_m R\left(m\Delta\sin\theta + \frac{\delta_m^x \sin\theta}{c} + \frac{\delta_m^y \cos\theta}{c}\right) - \sum_{m=-M/2}^{M/2} w_m R(m\Delta\sin\theta)\right|^2 \quad (10)$$

Using a Taylor series expansion, and taking the expected value, the above can be expressed as,

$$\overline{E(\theta)} = \left(\frac{\sigma^2}{c^2}\right) \sum_{m=-M/2}^{M/2} w_m^2 R'^2(m\Delta\sin\theta) + \frac{\sigma^4}{4c^4}\left(\sum_{m=-M/2}^{M/2} w_m R''(m\Delta\sin\theta)\right)^2 + \frac{\kappa(\cos 4\theta + 3) - \sigma^4(3\cos 4\theta + 1)}{16c^4} \sum_{m=-M/2}^{M/2} w_m^2 R''^2(m\Delta\sin\theta) + \ldots \quad (11)$$

where $R'(\tau)$ and $R''(\tau)$ denote the first and second derivatives of $R(\tau)$, and σ^2 and κ are the second and fourth order moments of the position error distribution. Note that in deriving equation 11, it is also assumed that the element position errors have zero third order moments.

We now investigate $\overline{E(\theta)}$ in the vicinity of the look direction. As the first term in the RHS of equation 11 is zero when $\theta = 0$, neglecting higher order terms, we get,

$$\overline{E(\theta)}\Big|_{\theta \to 0} = \frac{1}{4}\left(\frac{\sigma}{c}\right)^4 R''^2(0)\left(1 + \left(\frac{\kappa}{\sigma^4} - 1\right) \sum_{m=-M/2}^{M/2} w_m^2\right) \quad (12)$$

Therefore, to keep $E(\theta)$ small, it is necessary to impose a robustness constraint to the optimization procedure such that $w^T w < \xi$. The choice of ξ depends on the anticipated error characteristics of the array processor.

5. OPTIMIZATION WITH LINEAR AND QUADRATIC CONSTRAINT

Conventionally, the minimization of a quadratic cost function $Q = w^T C w$ subject to the linear equality constraint $w^T U = 1$ and the quadratic inequality constraint $w^T w < \xi$, is obtained using root finding algorithms [5]. Such algorithms can be either computationally expensive or highly sensitive to accumulation of round-off errors [6]. In the following we present a novel technique for solving the quadratically constrained optimization problem using a transformed weight vector, γ.

Suppose F is a matrix of size $(M+1)\times(M+1)$, obtained from a set of orthonormal vectors. For example, F can be formed using the Fourier basis functions. We now define the transformed weight vector γ as, $\gamma = F^T w$ where $w = F\gamma$. The optimization problem can be now posed as,

$$\min_{\gamma}(Q=\gamma^T F^T C F \gamma) \qquad \text{subjected to} \qquad \begin{matrix}\gamma^T F^T U = 1 \\ \gamma^T \gamma \le \xi\end{matrix} \qquad (13)$$

We now choose F such that $F^T U = [1, 0, 0 \ldots\ldots, 0]^T$ and use the following partition relations.

$$F^T C F = \begin{bmatrix} r_o & r_b^T \\ r_b & C_1 \end{bmatrix} \quad ; \quad \gamma = \begin{bmatrix} \gamma_o \\ \beta \end{bmatrix} \quad ; \quad . \qquad (14)$$

$$(r_o, \gamma_o \in \Re), (r_b, \beta \in \Re^M), (C_1 \in \Re^{M \times M})$$

The cost function and the constraints in Equation 8 then simplify to,

$$Q = r_o + 2 r_b^T \beta + \beta^T C_1 \beta \quad ; \quad (\gamma_o = 1), (\beta^T \beta \le \xi - 1) \quad (15)$$

The cost function Q of equation 10 now needs to be minimized with respect to the vector β subject only to a single quadratic constraint. Using a Lagrange multiplier λ, the solution to this minimization problem is given by,

$$\beta = -(C_1 + \lambda I)^{-1} r_b \quad ; \quad r_b^T (C_1 + \lambda I)^{-2} r_b \le \xi - 1 \quad (16)$$

It is easy to show that the LHS of the quadratic constraint in equation 16 is a monotonically decreasing function of $\lambda \in \Re^+$. Therefore, with a simple search procedure, it is possible to find the value of λ that satisfies the inequality constraint and hence yield the solution to the minimization problem posed in equation 8.

6. AN EXAMPLE USING GAUSSIAN SHAPED LFM SIGNAL

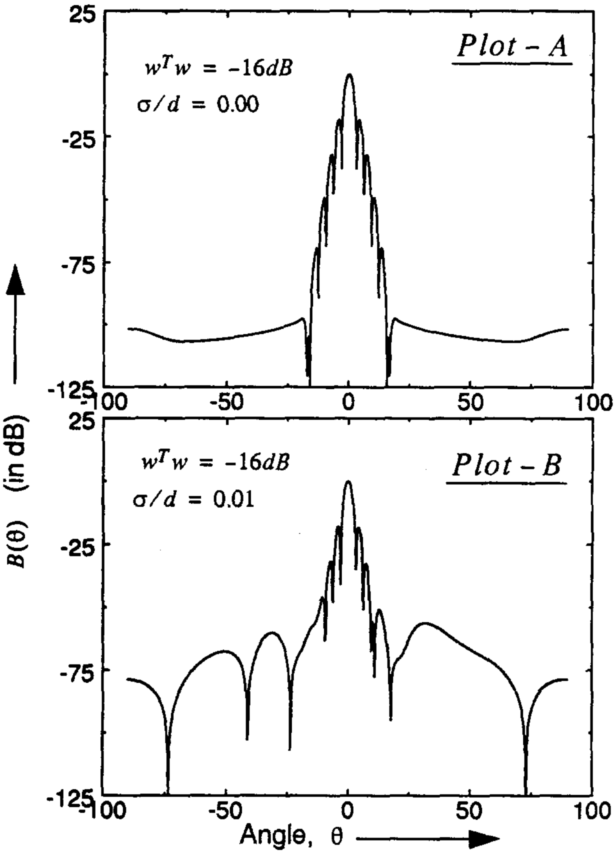

Figure 1: Directivity Pattern Resulting from Gaussian LFM Signals with Uniform Array Weights (A) No Position Errors (B) with Errors.

As an example, consider the Gaussian shaped Linear Frequency Modulated (GLFM) transmitted signal described by, $a(t) = e^{-\eta t^2}\cos(2\pi f_o t + \pi\mu t^2)$, $(-\infty < t < \infty)$, where the instantaneous frequency at $t = 0$ is given by f_o, and μ is the frequency sweep rate. Plot A of Figure 1 shows the directivity pattern for the GLFM signal obtained using uniform weights, in the absence of position errors in the sensors. Plot B of Figure 1 shows the resulting directivity pattern when the position errors are Gaussian distributed with $\sigma/d = 0.01$. (The simulation parameters are $(M+1) = 39$, $f_o = 200$, $\mu = 112$, $\Delta = 0.0025$, $\theta_b = 3°$ and $\eta = \pi/2$.) The norm of the weight array, $w^T w$ is also shown in the plots. To sharpen the directivity pattern we now optimize the weight vector, w. Figure 2 shows the directivity patterns from the optimization using only the linear constraint, $w^T U = 1$. It can be seen from Figure 2 that the optimization has improved the directivity pattern in the absence of position errors. However, the mere presence of any position errors can have a disastrous effect on the directivity pattern. Figure 3 shows the directivity pattern obtained by incorporating the quadratic constraint, $w^T w < \xi = 10$ to the optimization algorithm. Comparing Figures 2 and 3 it can be seen that the quadratic constraint has reduced the norm from $+51dB$ to $+10.0dB$ without affecting the mainlobe characteristics, in the absence of position errors. It can also be noted that the directivity pattern in Figure 3 is robust to element position variations.

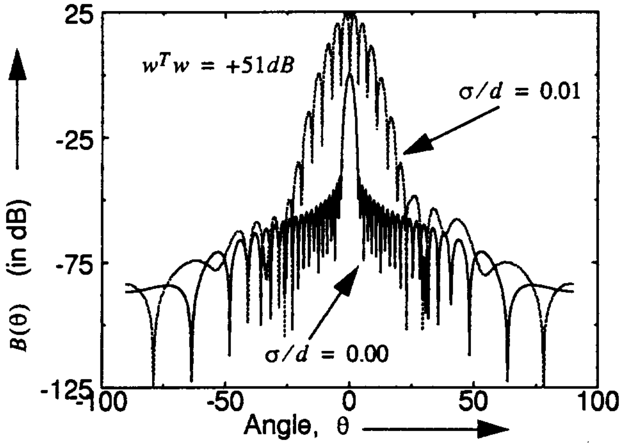

Figure 2: Directivity Pattern Resulting from Gaussian LFM Signals with Optimum Weights - No Quadratic Constraint is used in optimization.

7. CONCLUSIONS

A technique for obtaining an optimum set of array weights for a linear active sonar array is presented in the paper. The optimization algorithms also incorporate a quadratic robustness constraint in addition to linear constraints. The robustness constraint is imposed to reduce the sensitivity of the directivity patterns to errors in the array sensor positions. The paper presents a new method of solving the optimization problem subject to linear and quadratic constraints without resorting to root finding techniques.

8. ACKNOWLEDGEMENTS

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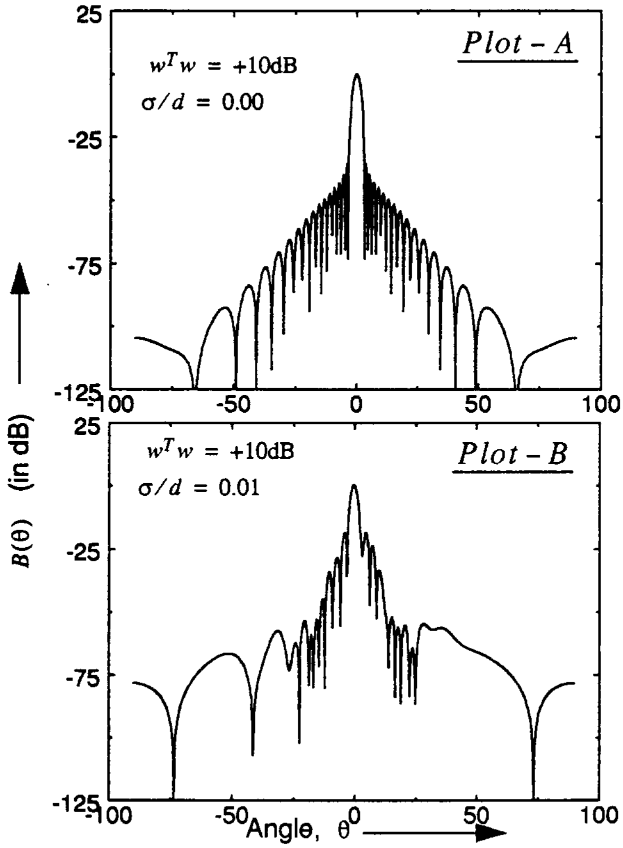

Figure 3: Directivity Pattern Resulting from Gaussian LFM Signals with Optimum Weights - Quadratic Constraint $w^T w = +10dB$ is used in optimization. (A) No Position Errors (B) with Errors.