FAST MAXIMUM LIKELIHOOD ESTIMATION WITH MULTIPLE SIGNAL INITIALIZATION

Robert B. MacLeod

Naval Undersea Warfare Center
Newport R.I. 02841 usa
macleod@fs811.npt.nuwc.navy.mil

Richard J. Vaccaro

University of Rhode Island
Kingston R.I. 02881 usa
vaccaro@ele.uri.edu

ABSTRACT

In this paper we are concerned with signal processing of acoustic signals resulting from active transmissions by high frequency sonar systems. These signals consist of structured interference related to propagation effects in the media, reflections from targets, and measurement noise. The methods herein model these signals as replicas of the transmitted signal, scaled in amplitude and time, and delayed. Furthermore, we are interested in signals with 'simple' time frequency profiles, such as linear frequency modulated (LFM) or hyperbolic frequency modulated (HFM) signals. These signals have the underlying property that the principle ridge of the autoambiguity function crosses the mid point of the time-frequency plane in a smooth manner, with a simple relationship between time delay and time scaling (frequency shifting). This paper describes a method for estimating the delay and time scale of signal components using fast maximum likelihood, while preserving the high resolution property of related time delay estimation techniques.

1. INTRODUCTION

This paper will be organized as follows. First, we will present a parameter estimation method which will simultaneously estimate the signal and interference components of a signal. This method will be shown to be an extension of the Fast Maximum Likelihood method of parameter estimation, allowing for more accurate analysis of closely spaced signal or interference components. Next, simulation results will be presented, to demonstrate the features of these methods.

1.1. Signal Model

The returned signal will be assumed to be of the form

$$\begin{aligned} x &= \sum_{i=1}^{K_S} a_i s(c_i t - \tau_i) + \sum_{i=K_S+1}^{K_S+K_I} a_i s(c_i t - \tau_i) + n \\ &= x_S + x_I + n \; . \end{aligned} \tag{1}$$

The signal x_S, and the interference x_I, are both modeled as linear combinations of time delayed (τ_i) and time scaled (c_i) replicas of the transmit, s. The signal and interference amplitudes, delays, and time scaling factors will be considered to be deterministic but unknown parameters. As is usual for sonar signal processing systems, these signals will complex valued in general, being derived from real-valued pressure signals which have been shifted in frequency and downsampled. The noise will therefore be assumed to be complex Gaussian, i.e.

$$n \sim \mathcal{N}\left(0, (\sigma^2/2)I\right) + j\mathcal{N}\left(0, (\sigma^2/2)I\right) \; . \tag{2}$$

2. DISCUSSION

For the case of the measurement noise of equation 2, the maximum likelihood estimate of the signal parameters can be made by maximizing the compressed likelihood function (CLF)

$$\begin{aligned} \mathsf{L}_{\text{clf}} &= x^t * \mathbf{H}(\mathbf{H}^t\mathbf{H})^{-1}\mathbf{H}^t * x \\ &= \|\mathbf{P_H} x\|^2 \; , \end{aligned} \tag{3}$$

searching over the dimension $2(K_S + K_I)$ space of the vector model parameters c and τ. A complete treatment of the topic is offered in reference [1]. Here, $\mathbf{H}$ is the combined signal and interference model matrix, formed by stacking together in columns time delayed and scaled transmit replicas.

$$\begin{aligned} \mathbf{H} = [&s(c_1 t - \tau_1) \ldots s(c_{K_S} t - \tau_{K_S}) \\ &\ldots s(c_{K_S+K_I} t - \tau_{K_S+K_I})] \end{aligned} \tag{4}$$

Since both the signal and the interference are modeled in exactly the same way, it will not be necessary to distinguish the components, and we will refer to the model matrix as simply $\mathbf{H}$. The problem of maximizing the CLF has been treated extensively in the literature in many guises, see for example [1, 2]. The fundamental problem all have in common is to estimate the parameters of a new component

$$h_i = s(c_i t - \tau_i) \ , \tag{5}$$

having estimated $i-1$ other components. To illuminate the problem, assume a new component h_i is sought, and the previous $i-1$ components are available in a matrix $\mathbf{H}_{i-1}$.

$$\mathbf{H}_i = [\mathbf{H}_{i-1} \mid h_i] \tag{6}$$

It is not hard to show that the CLF can be rewritten as

$$\mathsf{L}_{\text{clf}} = \left\|\mathbf{P}_{\mathbf{H}_{i-1}} x\right\|^2 + \left\|\mathbf{P}_{\tilde{h}_i} x\right\|^2 \ , \tag{7}$$

where the projection involving the new component h_i has apparently been modified by substituting $\tilde{h}_i$,

$$\tilde{h}_i = \mathbf{P}_{\underset{\mathbf{H}_{i-1}}{\perp}} h_i \ . \tag{8}$$

Since the first term in this relationship does not depend on the new component h_i, the maximization of L_{clf} can be accomplished by maximizing the second term alone. After rewriting, this can be shown to be equivalent to maximizing

$$\mathsf{L} = \|\frac{h_i^t}{|\tilde{h}_i|}(x - \mathbf{P}_{\mathbf{H}_{i-1}} x)\|^2 \ . \tag{9}$$

Equation 9 has the following interpretation. To maximize the Compressed Likelihood Function L_{clf} over the complete set of model vectors $[\mathbf{H}_{i-1} \mid h_i]$, first project the data onto the subspace spanned by the known components, and find the residual $x - \mathbf{P}_{\mathbf{H}_{i-1}} x$. Next, take the inner product of the residual with the modified matched filter $h_i^t/|\tilde{h}_i|$, searching over the parameters τ_i and c_i, where $h_i = s(c_i t - \tau_i)$. The i^{th} component will be $h_i = s(\hat{c}t - \hat{\tau})$, where $\hat{c}$ and $\hat{\tau}$ maximize L.

Our method of estimation draws on two recently published results, the method of Fast Maximum Likelihood Estimation [1], and the method of Complex to Real Least Squares Time Delay Estimation, 'CRALS' [3, 4].

2.1. Fast Maximum Likelihood Estimation

In Fast Maximum Likelihood Estimation, the $c_i = 0$ line in the c-τ plane is first searched, followed by a second search along the known ambiguity ridge on the plane. To visualize this, imagine a signal such as figure 1 has been transmitted, and it is known that the time scaling c simply shifts the return vertically. The projection of delayed versions of the transmit signal along the $c = 1$ axis will match up well with the return, just at the wrong time delay. The ridges of the delayed transmit and the return will however line up perfectly. If it is known how to shift the delay and time scaling of the replica along this ridge, the correct maximum can be found, without resorting to a two dimensional search. The Fast Maximum Likelihood procedure does just this, evaluating equation 9 along the $c_i = 1$ axis, and then the ambiguity ridge, building up the model matrix $\mathbf{H}$ one vector at a time.

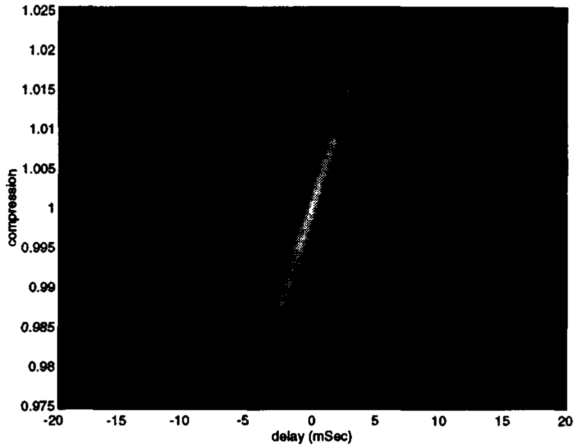

Figure 1: Wideband Auto Ambiguity Function of an FM Transmit

2.2. Complex to Real Least Squares Time Delay Estimation

The method of Complex to Real Least Squares time delay estimation, 'CRALS' [3, 4], solves a similar problem, but focuses on a different aspect of performance. In this method, the signal model is restricted to the $c_i = 1$ case, which indicates a one dimensional search, component by component, in the spirit of equations 8 and 9. CRALS has two enhanced features which distinguish it from other maximum likelihood methods. First, the problem of overlapping signal components is addressed, and second, a means for constraining the amplitudes of the signal components to be real-valued is incorporated. We will be interested in the first feature, which can resolve signal components closely spaced on the time time-scale plane.

Referring to equation 9, the search for the maximum of L by substituting $h_i = s(t - \tau_i)$ will by design 'run over' the $i-1$ components already estimated, each producing a singularity in L. This is because the projection of h_i onto the complement of $\mathbf{H}_{i-1}$ will yield $\tilde{h}_i \approx 0$. Conversely, in regions where h_i is orthogonal to the other components, $\tilde{h}_i = h_i$, and the search reduces to a matched filter search of the signal residual. In the singular regions, CRALS substitutes the vector

$$\dot{h}_i = \frac{\partial}{\partial \tau_i} h_i \tag{10}$$

for h_i. In [3] it is shown that the projection onto $[\mathbf{H}_{i-1} \mid \dot{h}_i]$ is approximately equal to the projection onto $[\mathbf{H}_{i-1} \mid h_i]$ over a small range of τ. This range corresponds to the range over which a Taylor series expansion in τ, truncated to the first two terms, is valid.

2.3. Summary of the Method

We are researching the ramifications of doing Fast Maximum Likelihood searches of L, that is searches along known ambiguity ridges, starting from a set of delay estimates found by the CRALS algorithm. The signals we will be interested in are signals with components shifted only moderately off the reference Doppler time scale axis. This will be true for nearly static systems making underwater parameter measurements in calm seas, or moving platforms interrogating low Doppler targets. More radical cases would run the risk of shifting signal components completely off the reference axis, defeating the method.

We will call this method of parameter estimation the CRALS-FML Hybrid method, summarized as follows

1. Do a CRALS estimation of signal component delay, assuming that all components are centered at a known Doppler time scaling, usually $c_i = 1$.
2. For signal components isolated in delay from neighboring components by more than half an autoambiguity width, proceed with Fast Maximum Likelihood estimation of that component's Doppler time scale.
3. For groupings of two or more signal components, do a modified Fast Maximum Likelihood estimation of component Doppler time scale as follows.
   (a) Individually constrain each component's time delay and Doppler time scale to follow its known ambiguity ridge.
   (b) Do a dimension K maximization of the CLF along the K ridges, where K is the number of components grouped together.
4. Perform final maximization of $\mathsf{L}_{\mathsf{clf}}$ using gradient search, etc.

Our analysis and evaluation of performance will focus on the initialization stage (steps 1 through 3), since all of the methods presume a final optimization stage. A simulation therefore will be considered successful if the components are resolved to the approximate location on the time delay - time scale plane. Simple gradient methods will then converge on the proper estimate.

3. SIMULATION RESULTS

To illustrate the method, a simple simulation was written with three replicas of an FM signal placed in a length $N = 256$ data record. The FM signals were linear FM, 50 samples long, cosine weighted at the ends, with a digital frequency ranging from $-0.2 \leq f \leq +0.2$. The auto ambiguity ridge of this signal is roughly 4 samples wide, and it was presumed to have been generated from a downsampling operation such that time scalings of $0.95 \leq c \leq 1.05$ spanned the range of digital frequencies in the system. The parameters selected were such that a time scaling of $c = 1.0025$, a compression, would advance the apparent zero crossing of the signal by $\tau = 1.32$ samples. The noise power is -12dB, and the signals were amplitude 0, 6, and 9.5dB proceeding from the first to the last.

Three cases were run,

case I fully orthogonal case

case II pictured in figure 2, has the second two components at an identical delay of 80 samples, but with different time scalings, still approximately orthogonal. The $c_i = 1$ crossings of the second two components are approximately 8 samples apart.

case III is the most difficult case, with the $c_i = 1$ crossings within two samples of each other for the second two components.

case	actuals	fml results
I	(20 , 1)	(21.24 , 1.0025)
	(80 , 1.0075)	(80.09 , 1.0075)
	(140 , 0.9925)	(139.90 , 0.9925)
II	(20 , 1)	(19.92 , 1)
	(80 , 1.0075)	(79.89 , 1.0075)
	(80 , 0.9925)	(81.30 , 0.995)
III	(20 , 0.9925)	(19.98 , 0.9925)
	(80 , 1)	(- , -)
	(82 , 1.0075)	(83.91 , 1.01)

Table 1: Simulation Results of Estimation of Signal Parameters by Fast Maximum Likelihood. Entries are (delay in samples, Doppler time scale)

case	actuals	hybrid results
	(20 , 1)	(21.30 , 1.0025)
I	(80 , 1.0075)	(80.04 , 1.0075)
	(140 , 0.9925)	(139.90 , 0.9925)
	(20 , 1)	(19.96 , 1)
II	(80 , 1.0075)	(78.47 , 1.0050)
	(80 , 0.9925)	(79.87 , 0.9925)
	(20 , 0.9925)	(19.92 , 0.9925)
III	(80 , 1)	(80.04 , 1.0025)
	(82 , 1.0075)	(83.86 , 1.01)

Table 2: Simulation Results of Estimation of Signal Parameters by Hybrid Method. Entries are (delay in samples, Doppler time scale)

The results for these representative cases, displayed in tables 1, and 2, show that both methods perform well when the signals are approximately orthogonal. As the components move closer, and start to overlap in time, both methods can still resolve the signals, as long as the $c_i = 1$ crossings of the ambiguity ridges are separated. When the $c_i = 1$ crossings are within 1/2 of an autoambiguity ridge of each other, the signals are very much non-orthogonal, and FML can have trouble resolving them. In table 1, case III, estimation of the second component fails, giving a low amplitude false component in the noise (not shown). CRALS however does pickup two signals in the region, which are further resolved when the FML stage of the Hybrid algorithm is executed.

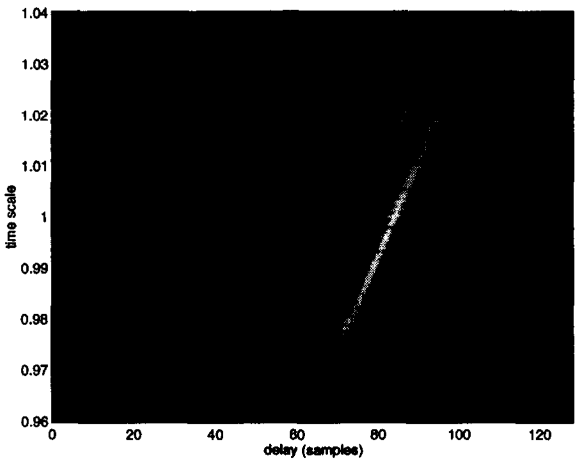

Figure 2: Wideband Cross Ambiguity Function of Simulated Case II

4. SUMMARY

Presented is a hybrid method of parameter estimation, which forces the Fast Maximum Likelihood algorithm to start with the high resolution time delay estimates from CRALS. The FML algorithm can use these delay estimates, but must be modified for components extremely closely spaced in delay. For a subset of K closely spaced components, a brute force maximization of $\mathsf{L}_{\mathsf{clf}}$ would require a dimension $2K$ search, FML would require K one dimensional searches, and the Hybrid method requires a dimension K search. In other situations FML, CRALS, and the Hybrid method are identical. Research will continue to make a more thorough examination of these claims. In particular, performance bounds are sought for CRALS which deal with delay estimation for mis-matched signals, in situations such as these.

5. REFERENCES

[1] D. W. Tufts, H. Ge, and S. Umesh, "Fast maximum liklihood estimation of signal parameters using the shape of the compressed likelihood function," *IEEE Journal of Oceanic Engineering*, vol. 18, no. 4, pp. 388–399, October 1993.

[2] I. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternating projection," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 36, pp. 1553–1560, October 1988.

[3] T. G. Manickam, R. J. Vaccaro, and D. W. Tufts, "A least-squares algorithm for multipath time-delay estimation," *IEEE Transactions on Signal Processing*, vol. 42, pp. 3229–3233, November 1994.

[4] T. G. Manickam, *Estimation in a Multipath Environment*, Ph.D. thesis, University of Rhode Island, 1994.