AN ALGORITHM FOR DETECTING CLOSELY SPACED DELAY/DOPPLER COMPONENTS

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ABSTRACT

This paper considers a method for estimating time delays, amplitudes, and Doppler scales of a multipath signal. The method is an extension of work previously reported by Manickam and Vaccaro [1] which dealt solely with time delays and amplitudes, and extended by Habboosh and Vaccaro [2] to include Doppler scale. In this paper, an algorithm is presented for determining the size of the indicator set to reduce ill-conditioning of the signal subspace matrix. Simulation results are shown and comparisons to the Cramer-Rao lower bound provided; these results show that significant reduction in estimate variances can be achieved using the deconvolution approach with a properly selected indicator set.

1. INTRODUCTION

This paper considers a method for estimating time delays, amplitudes, and Doppler scales of a multipath signal. The method is an extension of work previously reported by Manickam and Vaccaro [1] which dealt solely with time delays and amplitudes, and extended by Habboosh and Vaccaro [2] to include Doppler scale.

The sampled received signal model is described mathematically as

$$r(nT_s) = \sum_{m=1}^{M} a_m s(d_m(nT_s - \tau_m)) + w(nT_s), 0 \le n \le N-1 \tag{1}$$

where, $s(t)$ is the transmitted signal, a_m, τ_m, and d_m are the attenuation value (amplitude), time delay, and Doppler scale factor, respectively, for path m, N is the number of samples in r, T_s is the sample period, and $w(nT_s)$ is zero mean white Gaussian noise with variance σ_w^2. The problem is to estimate the unknown amplitudes, delays, and Doppler scale factors. This signal model, unlike those generally used, does not assume a narrowband signal and therefore does not assume the Doppler scale to be approximated by a frequency shift. Also, (1) allows for time delays which are non-integer multiples of T_s.

The properties of the transmitted signal play an important role in the performance of any delay/Doppler estimation method. The most critical property is a narrow ambiguity function in both the delay and Doppler axes. The ambiguity function using sampled data is defined as the magnitude squared of the two dimensional autocorrelation function and is described mathematically as

$$c(\tau, d) = \left| \sum_{n=-\infty}^{\infty} s(nT_s)s^*(d(nT_s + \tau)) \right|^2. \tag{2}$$

One signal which has the desired property is the frequency-hopped *Costas signal* described in [3], which is mathematically described as

$$\mu(t) = \sum_{h=0}^{H-1} p_h(t - hT_p) \tag{3}$$

where,

$$p_h(t) = e^{j2\pi f_h t}, \quad 0 \le t \le T_p$$
$$p_h(t) = 0, \quad elsewhere$$

and,

$$\begin{aligned} f_h &= \frac{a_h}{T_p} \\ T_p &= \frac{T}{H} \\ a_h &= \{a_0 a_1 \dots a_{H-1}\} \end{aligned}$$

and the a_h are selected according to [3] to have the appropriate property.

The parameters of the transmitted signal, used throughout this discussion, are given by

$$\begin{aligned} T_s &= 25\mu S \\ T &= 15.0mS \\ a &= \{7\ 1\ 3\ 6\ 4\ 5\ 2\} \\ H &= 7. \end{aligned} \tag{4}$$

This paper considers the generalized delay/Doppler estimation problem and therefore assumes nothing about the bandwidth of the signal. Tufts [4] presents an approach which uses the shape of the compressed likelihood function for a specific signal.

A deconvolution method for the problem without Doppler (i.e., $d_m = 1$) was given by Manickam and Vaccaro [1]. It was shown in [2] that this algorithm does not perform well when $d_m \ne 1$, providing the impetus for the development of the two-dimensional delay/Doppler estimation algorithm of [2], described briefly in the following section.

2. DELAY/DOPPLER DECONVOLUTION

This section briefly describes the two dimensional delay/Doppler estimation problem. The discrete time signal model is defined by

$$\mathbf{r} = \mathbf{Sh} + \mathbf{w} \tag{5}$$

where, $\mathbf{r}$ and $\mathbf{w}$ are the $N \times 1$ vectors containing $r(nT_s)$ and $w(nT_s)$ in (1), $\mathbf{S}$ is the $N \times M$ matrix with columns containing $s(d_m(nT_s - \tau_m))$ in (1), and $\mathbf{h}$ is the $M \times 1$ vector containing the coefficients a_m. The matrix $\mathbf{S}$ is defined by

$$\mathbf{S} = \begin{bmatrix} \mathbf{S_s} & \mathbf{0^T} & \cdots & \mathbf{0^T} \\ \mathbf{0^T} & \mathbf{S_s} & \cdots & \mathbf{0^T} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{0^T} & \mathbf{0^T} & \cdots & \mathbf{S_s} \end{bmatrix}. \tag{6}$$

The submatrices appearing in $\mathbf{S}$ are denoted

$$\mathbf{S_s} = \begin{bmatrix} s(0) & \cdots & s(0) \\ s(d_1 T_s) & \cdots & s(d_Q T_s) \\ \vdots & \vdots & \vdots \\ s(d_1(P-1)T_s) & \cdots & s(d_Q(P-1)T_s) \end{bmatrix} \quad (7)$$

and the vector $\mathbf{0}$ is defined as a $Q \times 1$ vector of zeros. Each column of the matrix $\mathbf{S_s}$ consists of the transmitted signal at some value of Doppler scale. The expected range of Doppler scales is quantized into Q different values $d_1, \cdots, d_Q$, and the entire $\mathbf{S_s}$ matrix is delayed to form the columns of $\mathbf{S}$. The vector $\mathbf{h}$ is partitioned into subvectors of length Q

$$\mathbf{h} = \begin{bmatrix} \mathbf{h_0} \\ \mathbf{h_1} \\ \vdots \\ \mathbf{h_{N-P}} \end{bmatrix}. \quad (8)$$

Each subvector corresponds to a delay value and entries within a subvector correspond to a Doppler value.

If the received signal consists of components which are delayed by integer multiples of T_s and Doppler scaled by one of the Q quantized values, then, in the absence of noise, the subvectors $\mathbf{h_i}$ of (8) will contain at most a single nonzero entry. If a nonzero entry occurs in the subvector $\mathbf{h_k}$, then the corresponding time delay is $\tau_k = kT_s$ and the amplitude a_k is the value of the nonzero entry. Also, if the nonzero entry occurs in the jth element of $\mathbf{h_k}$, then the Doppler scale for that component is d_j. In general, the delays and Doppler scales are non-integer multiples of their corresponding sampling intervals and the vector $\mathbf{h}$ must be interpolated. This two-dimensional interpolation is currently being addressed.

In the presence of noise, a least squares solution must be found for $\mathbf{h}$ and the matrix $\mathbf{S}$ will be ill-conditioned. Under these conditions, a two-dimensional indicator set must be defined, which is used to select columns of the matrix $\mathbf{S}$. The reduction in the size of the matrix $\mathbf{S}$ will be more significant than in the one dimensional case, because of the addition of the Doppler dimension to the problem.

The maximum of $|\sum \mathbf{rs^*}|^2$, under conditions of white Gaussian noise and single path, provides the Maximum Likelihood Estimate (MLE) of both delay and Doppler [5]. However, if the spacing between paths (in both delay and Doppler) is smaller than the width of the signal autocorrelation (ambiguity for delay and Doppler) function, then the matched filter will only reveal one path. Under these conditions, deconvolution will provide estimates of both paths.

The two dimensional indicator set is determined by finding the delay and Doppler samples corresponding to the matched filter peaks and defining a rectangular window around each one. The window is 11 samples in the delay direction and 7 samples in the Doppler direction resulting in 77 delay-Doppler pairs (the selection for the size of the indicator set will be discussed in the next section), corresponding to columns of the $\mathbf{S}$ matrix which are used to form $\mathbf{S_E}$ in

$$\mathbf{r} = \mathbf{S_E h_E} + \mathbf{w}. \quad (9)$$

Although $\mathbf{S_E}$ is significantly better conditioned than $\mathbf{S}$ because of the removal of all columns not in E, there is still a possible ill-conditioning problem in the presence of noise. This problem is eliminated with proper indicator set selection, as shown in the next section.

3. INDICATOR SET SELECTION

In the previous section, it was shown that if the indicator set includes the correct paths, a single least squares deconvolution will provide some results as to the true path delay

and Doppler along with an estimate of the amplitude. However, when the indicator set is too large, ill-conditioning of the $\mathbf{S}$ matrix can result because significant noise is allowed into the estimate, providing poor estimates of delay and Doppler.

This section provides an algorithm for determining the indicator set for two closely spaced, equal amplitud paths with delay and Doppler values equal to grid samples. It will be shown that with proper selection of the indicator set, the variance of the estimated delay and Doppler will be much smaller than that achieved when using a rectangular indicator set as described earlier.

3.1. Size of Rectangular Indicator Set

The size of the rectangular indicator set, described in Section 2. was determined such that it encompasses all possible delay/Doppler combinations of two paths which result in a single matched filter peak. Figure 1 provides a graphical representation of these combinations. The circles represent the difference in delay and difference in Doppler (in samples) for the two paths. Since the maximum delay spread is
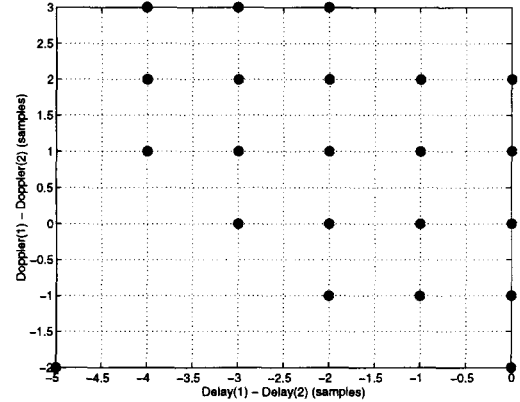

Figure 1. Graphic of path pairs which cause single peak in matched filter

5 samples and maximum Doppler spread is 3 samples, the indicator set is defined as ± 5 delay samples and ± 3 Doppler samples centered at the peak of the matched filter. However, for many of the cases shown in Figure 1, this indicator set is too large and causes ill-conditioning of the $\mathbf{S}$ matrix. The algorithm described in this section takes advantage of the shape of the ambiguity function to generate an indicator set contour which significantly improves the $\mathbf{S}$ matrix conditioning.

3.2. Ambiguity Function Peak Analysis

The ambiguity function, described by (2), can be written in vector form as

$$\mathbf{Z}(\tau, D) = (\mathbf{s}^H \mathbf{r})^* (\mathbf{s}^H \mathbf{r}) = \mathbf{A}^* \mathbf{A} \quad (10)$$

where, $\mathbf{s}$ is the discrete time transmitted signal. This is a nonlinear function of both delay and Doppler scale, making prediction of the peak location difficult. Therefore, a quadratic approximation to this function is used and is given by

$$\begin{aligned} \mathbf{Z}(\tau, D) &\approx \mathbf{Z}(\boldsymbol{\theta}_0) + \nabla_{\boldsymbol{\theta}} \, \mathbf{Z}(\boldsymbol{\theta}_0) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0) \\ &+ \tfrac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}_0)^T H(\boldsymbol{\theta}_0) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0) \end{aligned} \quad (11)$$

where,

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T = \begin{bmatrix} \tau & D \end{bmatrix}$$

$$\boldsymbol{\theta}_0 = \begin{bmatrix} \theta_{01} & \theta_{02} \end{bmatrix}^T = \begin{bmatrix} \tau_0 & D_0 \end{bmatrix}^T,$$

$H(\theta_0)$ is the Hessian matrix evaluated at θ_0 and the point of expansion (τ_0, D_0) is the mean of the true path locations. With a high SNR assumption, noise square terms are ignored resulting in estimates of the mean and covariance of the ambiguity peak location. The covariance matrix defines an elliptical error contour based on the noise variance (i.e., SNR). The center of the ellipse is located at the mean.

3.3. Generating the Scale Table

The error contours defined in the previous section are used in this section to predict the minimum indicator set size and shape which will contain the true path pairs. The results of the previous section have shown that the peak location of the ambiguity function can be within an elliptical contour which is a function of both SNR and the relative location of the true path pairs. Since, in the processing of the signals, the true path locations are not known, only the ambiguity function can be used to make the location estimate. The algorithm described in this section is used to create a look-up table which relates the orientation angle of the ambiguity function to the path pair relative separation in both delay and Doppler. In this section, the path pair delays are (100, 102) samples and the Dopplers are (5, 4) samples.

The noise-free ambiguity function is thresholded to 95% of its peak value and the resulting contour found using Mat-Lab (solid line in Figure 2). The nearly vertical lines at 99.5 and 102.5 delay samples are due to the narrow width of the ambiguity function in the delay direction. The estimated location of the peak (shown by the $*$ in Figure 2) is found by performing an ellipsoid interpolation through the nine samples near the highest value of the ambiguity function.
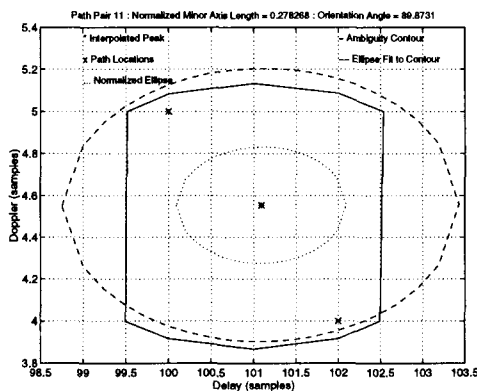

Figure 2. Generating the Ellipse which Normalizes the Ambiguity Contour

An ellipse is fitted to the ambiguity function contour with center at the peak and shown as a dashed line in Figure 2. The fitted ellipse is normalized to have major axis length of 1 (dotted line in Figure 2. Derivations of the elliptical fit and normalization method are provided in [6].

The final stage of the algorithm is to determine the minimum scale factor that, when applied to the normalized ellipse, ensures that the resulting ellipse will include the two paths under investigation. This is done by centering the normalized ellipse at each point along the error ellipse, computing the required scale factor for that center, and reporting the maximum of these values.

A table is generated which contains the SNR, the minor axis length of the normalized ellipse, the orientation angle of the ellipse, and the maximum scale factor required to ensure the ellipse contains both paths.

The following section describes how noise-corrupted signals are processed, using the table described above.

3.4. Processing the Signals

Before processing noise-corrupted signals, the stored table is divided into two tables: (1) containing those pairs corresponding to ambiguity contour orientation angles greater than $90°$ and (2) containing those pairs corresponding to ambiguity contour orientation angles less than $90°$. The two tables are then each sorted by increasing minor axis length.

When noise-corrupted signals are processed, the ambiguity function is first computed. It is then thresholded at 95% of its peak value and the location of the peak found via the ellipsoid interpolation described above. An ellipse is fit to the 95% contour and normalized as described above. The correct table is selected by determining the orientation angle of the normalized ellipse. Then, the correct scale factor is determined by finding the minor axis length, from the table, which is closest to the computed minor axis length. The selected scale factor is used to scale the normalized ellipse, from which the indicator set is selected. Those samples which lie within the scaled ellipse are used as E in (9) to compute the channel impulse response.

Figure 3 shows the elliptically shaped indicator set and a rectangular one. The size of the rectangular one, shown by the combination of unfilled and filled circles in the figure, is selected such that it encompasses all of the delay/Doppler pairs which result in a single ambiguity function peak. The filled circles in the figure represent those samples used for the elliptical indicator set. The resulting channel response
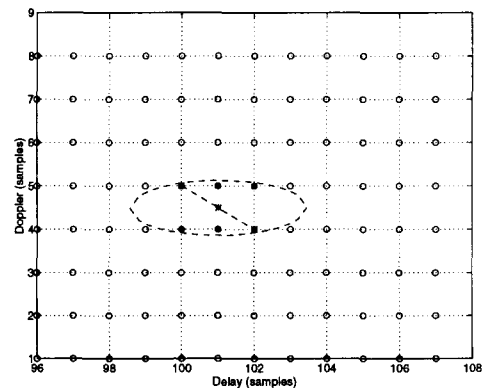

Figure 3. The elliptical and rectangular indicator sets for path pair 11 and 20 dB SNR

estimates, using both indicator sets, are shown in Figures 4 and 5. Since the S matrix using the elliptical indicator set,
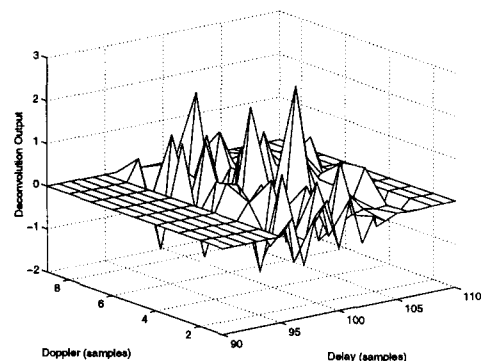

Figure 4. Channel response estimate using rectangular indicator set as shown in Figure 3

is less ill-conditioned and allows less noise than that of the

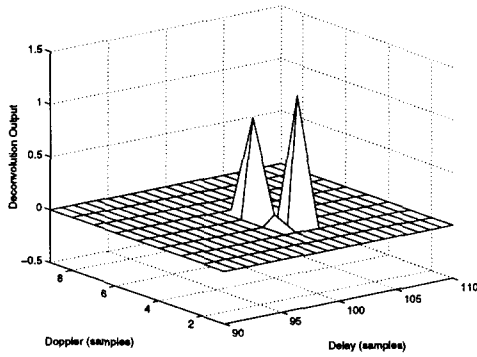rectangular one, a considerable improvement in the channel response estimate results.

Figure 5. Channel response estimate using elliptical indicator set as shown in Figure 3

Simulations were conducted for the different cases in Figure 1 with an SNR of 20 dB and the results are shown in Figure 6. From these figures, it is clear that even at high
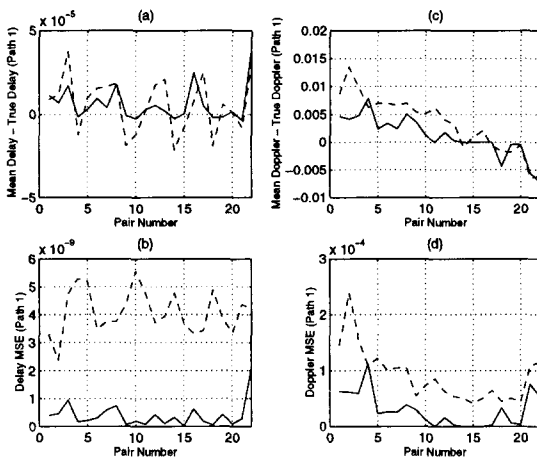

Figure 6. Delay and Doppler estimate comparison for path 1. Elliptical indicator set (solid) and rectangular indicator set (dashed)

SNR, a significant performance improvement results from using a smaller indicator set which encompasses the true paths. This occurs because the **S** matrix becomes less ill-conditioned with reduced size.

Figure 7 shows the results of simulations as a function of SNR for the current example and again verify that the elliptical indicator set provides improved results as a function of SNR, over the rectangular indicator set.

4. SUMMARY

This paper introduced an extension to the deconvolution approach of [1] to the problem with Doppler scale. Unlike [7, 8], this delay/Doppler Deconvolution does not make the narrowband signal assumption, and provides the estimate with a single least squares deconvolution step. The results, in the absence of noise, show that even extremely closely spaced paths can be recovered using this approach, even when the ambiguity surface cannot distinguish these paths.

An algorithm for determining a reduced size indicator set for pairs of closely spaced multipaths was also provided. It was shown that, with proper selection of the indicator set, the conditioning of the **S** matrix can be significantly
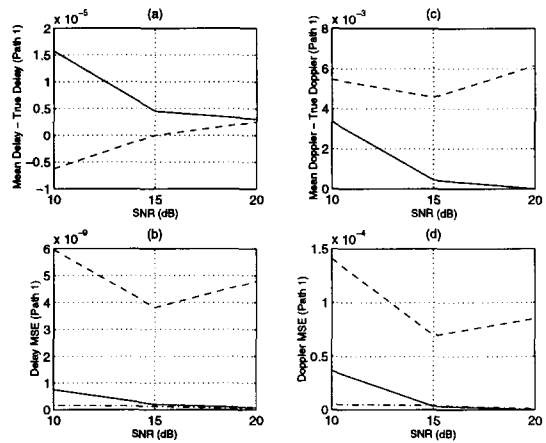

Figure 7. Delay and Doppler estimate comparison for current example vs. SNR. Elliptical indicator set (solid) and rectangular indicator set (dashed) and Cramer-Rao bound (dash-dot)

improved, resulting in considerable performance improvements.

The deconvolution approach can prove useful in cases where accurate delay estimates are required, such as vehicle tracking. The Doppler scale estimate can prove to be a valuable tool in determining the type of signal path, i.e., direct or reflected and can be used to eliminate the effects of vehicle motion on the received signal.

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