A SHORT-TIME WIENER FILTER FOR NOISE REMOVAL IN UNDERWATER ACOUSTIC DATA

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ABSTRACT

A noise removal algorithm based on short-time Wiener filtering is described. An analysis of the performance of the filter in terms of processing gain, mean square error, and signal distortion is presented. A generalized form of the filter is also discussed and results of applying the algorithm to some typical underwater acoustic data are presented.

1. INTRODUCTION

Data from passive sonar is generally accompanied by ambient noise arising from shipping traffic, marine life, wave motion, moving and cracking ice (in the Arctic), and numerous other sources. The statistical properties of the noise are variable, even direction-dependent, and have been the source of many studies and analyses, e.g. [1, 2, 3]. Noise hampers sonar data collection and related processing of the data to extract information since many of the signals of interest are of short duration and of relatively low energy. This paper describes an algorithm based on the Wiener filter and a generalization of the Wiener filter and illustrates how the algorithm can be applied to remove additive noise and therefore enhance further processing of the data.

2. BASIC ALGORITHM

Since for low level signals the ocean behaves like a linear transmission medium, a significant component of the noise encountered in data collection is additive. The situation can be modeled as

$$x(n) = s(n) + \eta(n) \tag{1}$$

where $x(n)$ is the received noise-corrupted signal, $s(n)$ is the uncorrupted signal, and $\eta(n)$ is the additive noise. If the signal $s(n)$ is treated as a random process, then considering the above generation conditions for the noise, the signal and noise are independent. Further, since the noise has zero mean, the relation between the correlation functions is also additive. Although the signal is not usually stationary over a long observation time, for a short time interval we can write

$$R_x(l) = R_s(l) + R_\eta(l) \tag{2}$$

The estimate of the signal is a classical problem in statistical signal processing [4], and the vector of optimal FIR (Wiener) filter coefficients $\mathbf{h}$ is the solution to the Wiener-Hopf equation

$$\mathbf{R}_x\mathbf{h} = \mathbf{r}_s \tag{3}$$

where $\mathbf{R}_x$ is the correlation matrix for the observed noisy signal, and $\mathbf{r}_s$ is a vector of terms from the correlation function $R_s(l)$ of the uncorrupted signal.

Since none of the correlation functions is known *a priori*, they must be estimated from the data at hand. Since only $x(n)$ is observed, however, only $R_x(l)$ can be estimated directly. Nevertheless the nature of the problem provides a way to compute the needed statistics. Since the signal is very short (on the order of seconds or milliseconds) compared to the time over which the noise statistics are likely to change, an estimate for $R_\eta(l)$ can be made from the received data prior to the onset of $s(n)$. This estimate can in theory be subtracted from $R_x(l)$ to produce an estimate of $R_s(l)$. Thus, in principle, all of the quantities to perform the optimal filtering over a short time interval can be computed, and this can be repeated over successive blocks of data.

The estimate of $R_s(l)$ by subtraction of the estimated correlation functions is not well formed because there is no guarantee that such an estimate will be positive (semi)definite. This problem is mitigated if the noise is white because the procedure then involves subtraction of only a single parameter, the white noise variance, from the estimated R_x at lag zero. In fact, if the subtraction is done in the spectral domain the positive definite property can be tested as part of the procedure. Further, the estimate of this single parameter has lower variance than the estimate of the correlation function as a whole. Therefore before any further steps, the entire data set is processed by a linear predictive filter that whitens the noise. After noise removal, the data is processed by the inverse filter as shown in Fig. 1.

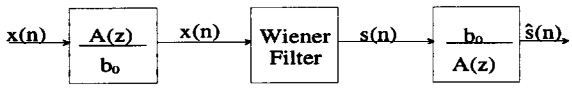

Figure 1. Prewhitening in short time Wiener filter algorithm.

The prewhitened data is segmented into blocks where an estimate of the local correlation function $R_x(l)$ is formed for each segment. Optimal filtering is then performed for each segment using a Wiener filter designed for the segment and the data is processed by the inverse filter to undo the effects of the prewhitening. In performing the Wiener filtering, the data is processed both forward and backward through the optimal filter which gives an approximation to a symmetric noncausal Wiener filter of approximately twice the length. Since the optimal filters are different for each block, discontinuities at the boundaries can arise. The effect of such discontinuities can be minimized by using points from the adjacent segment to filter the early points of the current segment. In the algorithm, the data is actually processed twice. The data is first segmented and filtered and the resulting frames are weighted by a triangular window (see Fig. 2). The data is then resegmented using frames

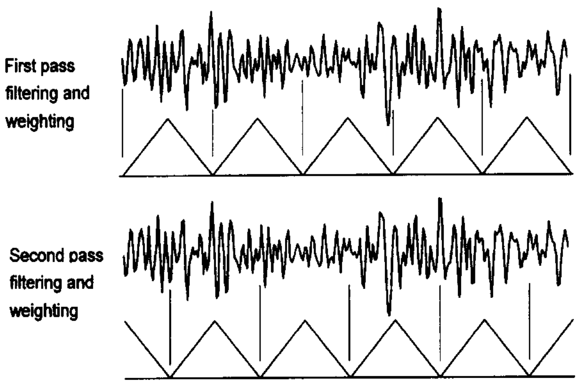

Figure 2. Overlap averaging technique used in noise removal.

shifted by half of the frame length, filtered again and weighted by a triangular window. The two weighted sets of data are then added to produce the final result and minimize any effects that may occur at the boundaries between frames.

3. WIENER FILTER PERFORMANCE

While the Wiener filter produces the best mean-square estimate of the signal, other criteria may also be important in processing acoustic data. Figure 3 depicts the linear filtering of a signal in additive noise and indicates the two portions of the output: $y_s(n)$ the result of processing the signal alone, and $y_\eta(n)$ the part due to processing the noise alone, which can

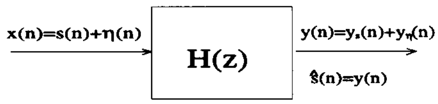

Figure 3. Definition of signals in optimal filtering.

be thought of as the residual noise left after processing. If the input and output signal-to-noise ratio (SNR) are defined by $E\left\{s^2(n)\right\}/E\left\{\eta^2(n)\right\}$ and $E\left\{y_s^2(n)\right\}/E\left\{y_\eta^2(n)\right\}$ then the filter *Processing Gain* is defined as the ratio of output SNR to input SNR in dB. A measure of signal distortion introduced by the filter can be defined as

$$\text{SD} \ = 1 - \frac{\left(E\left\{s(n)y_s(n)\right\}\right)^2}{E\left\{s^2(n)\right\}\cdot E\left\{y_s^2(n)\right\}} \tag{4}$$

A normalized form of mean-square error will be used which is defined as

$$\text{MSE} = \frac{E\left\{(s(n)-y(n))^2\right\}}{E\left\{s^2(n)\right\}} \tag{5}$$

Figure 4 shows the performance of the FIR Wiener filter for a signal with exponential correlation function $R_s(l) = \sigma_s^2\alpha^{|l|}$ in white noise as a function of the correlation parameter α. Note that as the correlation increases, MSE decreases and processing gain increases up to a finite value corresponding to finite time integration. Signal distortion, on the other hand, is zero for $\alpha = 0$ (this is the case where the signal is also white noise and the filter is all-pass) and $\alpha = 1$ (finite time integration). In between, distortion reaches a peak in a region of moderately high correlation corresponding many typical signals of interest.

4. ALGORITHM GENERALIZATION

Ephraim and Van Trees [5] have suggested a more general filter for minimising the distortion subject to constraining the residual noise power $E\left\{y_\eta^2(n)\right\} \leq \sigma_r^2$. We consider a variation on their approach where the signal distortion is measured by (4). We can write (4) as

$$\text{SD} \ = 1 - \frac{(\mathbf{h}^T\mathbf{r}_s)^2}{\sigma_s^2\mathbf{h}^T\mathbf{R}_s\mathbf{h}} \tag{6}$$

and

$$E\left\{y_\eta^2\right\} = \mathbf{h}^T R_\eta \mathbf{h} \leq \sigma_r^2 \tag{7}$$

We can formulate a problem to minimize (6) subject to the constraint (7) by considering the Lagragian

$$\mathcal{L} = -(\mathbf{h}^T\mathbf{r}_s)^2 + \mu(\mathbf{h}^T\mathbf{R}_s\mathbf{h} - C) + \lambda(\mathbf{h}^T R_\eta\mathbf{h} - \sigma_r^2) \tag{8}$$

where μ and λ are Lagrange multipliers and C is an arbitrary constant to be determined. By setting the

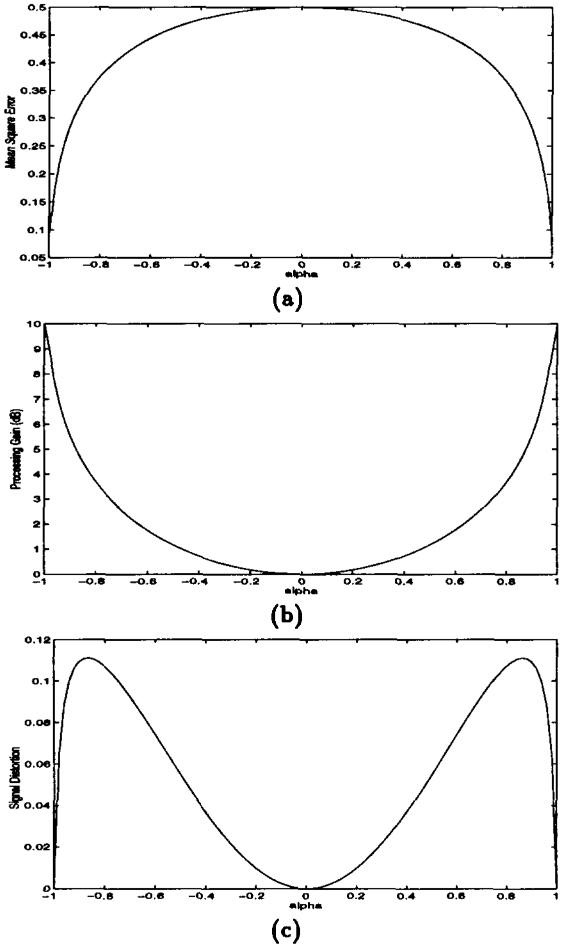

Figure 4. Performance of FIR Wiener filter for exponential signal in white noise for filter length $P = 10$. (a) Mean Square Error. (b) Processing Gain. (c) Signal Distortion.

gradient of (6) with respect to the filter vector equal to 0, we have the necessary condition

$$-2\mathbf{r}_s + 2\mu\mathbf{R}_s\mathbf{h} + 2\lambda\mathbf{R}_\eta\mathbf{h} = 0 \qquad (9)$$

which yields $\mathbf{h} = (\mu\mathbf{R}_s + \lambda\mathbf{R}_\eta)^{-1}\mathbf{r}_s$. Since any value for μ results in satisfying the condition $\mathbf{h}^T\mathbf{R}_s\mathbf{h} = C$ where C is a constant, we choose $\mu = 1$, so when the noise $\mathbf{R}_\eta = 0$, there is no amplification or attenuation of the signal, so that

$$\mathbf{h} = (\mathbf{R}_s + \lambda\mathbf{R}_\eta)^{-1}\mathbf{r}_s \qquad (10)$$

Similarly, by substituting the expression (10) into (7) with the equality and rearranging, we find that the parameter λ satisfies

$$\mathrm{tr}\ \mathbf{r}_s\mathbf{r}_s^T\mathbf{R}_\eta(\mathbf{R}_s + \lambda\mathbf{R}_\eta)^{-2} = \sigma_r^2 \qquad (11)$$

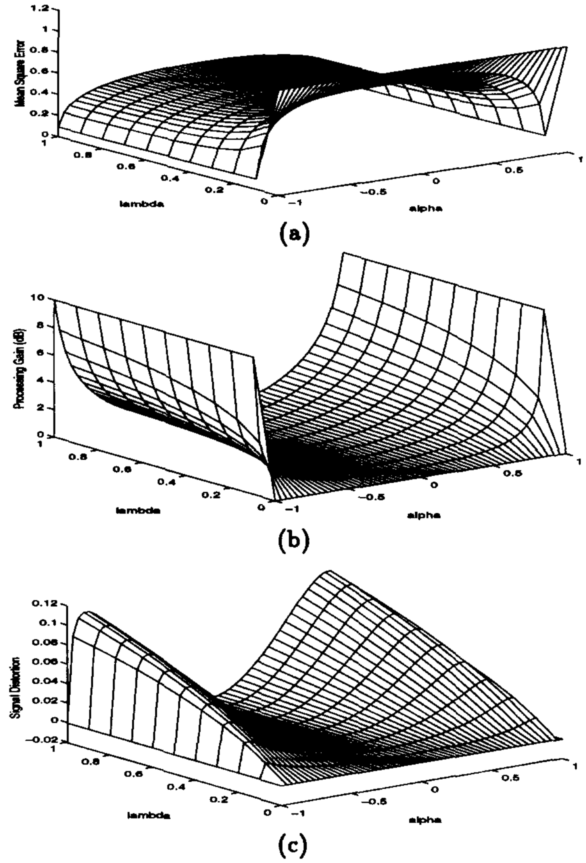

Figure 5. Performance of Generalized Wiener filter for exponential signal in white noise as a function of parameter λ for filter length $P = 10$. (a) Mean Square Error. (b)Processing Gain. (c) Signal Distortion.

At the two extremes, for $\lambda = 0$ we obtain an all-pass filter with $\sigma_r^2 = \sigma_\eta^2$ while for $\lambda = 1$ we obtain the Wiener filter which has minimum residual noise power $\sigma_r^2 = \sigma_w^2$. Figure 5 shows the performance of the filter as a function of both the correlation parameter α and the parameter λ. Note that for values of λ greater than approximately 0.4 the performance of the filter in terms of processing gain and mean-square error does not change significantly, while the distortion decreases significantly over this range of values of λ. This implies that the generalised filter with an appropriately chosen value of λ may be more desirable in practical applications involving noise removal than the standard Wiener filter. Fortunately the algorithm described above can accommodate the generalised filter easily since the generalised filter is merely a Wiener filter designed for a white noise variance reduced by the factor λ.

5. EXPERIMENTAL RESULTS

Figure 6(a) shows an segment of data consisting of

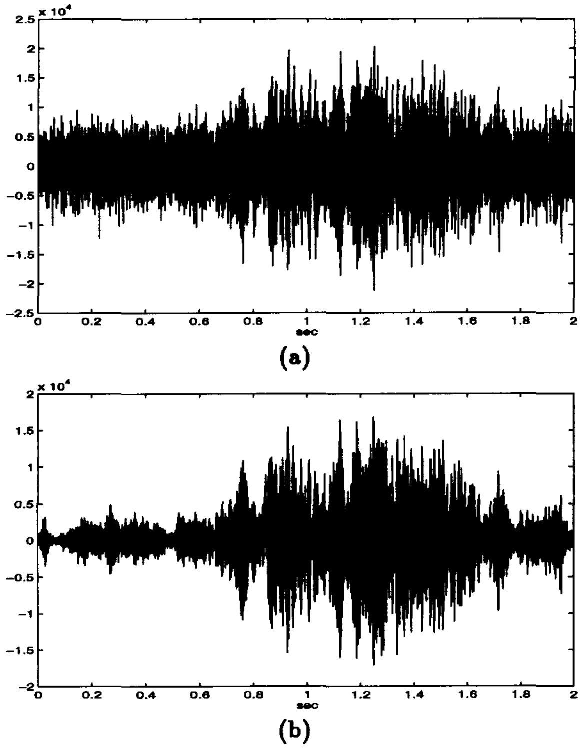

Figure 6. Application of noise removal algorithm to underwater acoustic data. (a) Original noisy data. (b) Data after processing.

a porpoise whistle sound. The data is sampled at approximately 10 kHz and is considerably degraded by noise. Figure 6(b) shows the data after noise removal. On the scale shown the noise statistics were estimated from a segment of the noise about 1000 points (0.1 sec.) in length from the early part of the data where only noise was assumed to be present. The FIR filter had a length of P=50 and λ was chosen to be 0.75. The prewhitening filter had a length of 35 points. In the early portion of the data where only noise is present, the noise power is reduced by approximately 28 dB. The performance of the algorithm in the signal region is difficult to quantify since the true underlying signal is unknown. However, listening to the data before and after noise removal shows a definite improvement due to the processing. Details in the sound of the signal appear much more clearly.

6. CONCLUSIONS

An algorithm for noise removal based on optimal filtering of short segments of the data has been developed. The algorithm was developed for improved processing of underwater acoustic data where the noise is assumed additive and stationary but the underlying signal is highly nonstationary. Noise statistics can be estimated in a region where only noise is assumed to be present, but signal statistics have to be derived from the observation of signal plus noise. A generalized version of the algorithm allows one to trade off signal processing gain and improvements in meassquare error for lower signal distortion. In application to data collected in the open ocean there is a noticeable decrease in the noise background and in aural listening tests there is an improvement in the ability to hear details in the underlying signal.

7. ACKNOWLEDGEMENT

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