EFFICIENT ALGORITHM TO COMPUTE LSP PARAMETERS FROM 10TH-ORDER LPC COEFFICIENTS

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ABSTRACT

Line Spectrum Pair (LSP) representation of Linear Predictive Coding (LPC) parameters is widely used in speech coding applications. An efficient method for LPC to LSP conversion is Kabal's method. In this method the LSPs are the roots of two polynomials $P'_p(x)$ and $Q'_p(x)$, and are found by a zero crossing search followed by successive bisections and interpolation. The precision of the obtained LSPs is higher than required by most applications, but the number of bisections cannot be decreased without compromising the zero crossing search. In this paper, it is shown that, in the case of 10^{th}-order LPC, five intervals containing each only one zero crossing of $P'_{10}(x)$ and one zero crossing of $Q'_{10}(x)$ can be calculated, avoiding the zero crossing search. This allows a trade-off between LSP precision and computational complexity resulting in considerable computational saving.

1. INTRODUCTION

LSP representation of LPC parameters is widely used in the domain of speech coding [1] due to its desirable properties such as bounded range, intra- and inter-frame correlation, and ordering (which allows simple checking of filter stability). Additionally, LSP representation allows frame to frame interpolation with smooth spectral changes. The LSP representation of 10^{th}-order LPC coefficients is used in several low-to-medium bit-rate narrowband speech coders such as the DoD FS1016 4.8 kbps CELP coder [2] and the ITU-T G.729 8 kbps CS-ACELP coder [3].

In Section 2 of this paper, the definition of LSP parameters is given. In Section 3, Kabal's method for computing LSP parameters from LPC coefficients is recalled [1], giving the definition of the polynomials $P'_p(x)$ and $Q'_p(x)$ whose roots correspond to the LSPs. A new derivation of these polynomials is given in Section 4. It is shown that in case of a 10^{th}-order LPC system, five intervals, each containing only one zero crossing of $P'_{10}(x)$ and one zero crossing of $Q'_{10}(x)$, can be calculated resulting in a new algorithm without zero crossing search. Experimental evaluation of this algorithm is shown in Section 5. In Section 6, the new algorithm is compared with other methods in terms of computational complexity. Conclusions and direction of future work are given in Section 7.

2. DEFINITION OF LSP PARAMETERS

The starting point for deriving the LSPs is the LPC analysis filter of order p:

$$A_p(z) = 1 + \sum_{k=1}^{p} a_k \cdot z^{-k}$$

A symmetrical polynomial $P_p(z)$ and an antisymmetrical polynomial $Q_p(z)$ are formed by adding and subtracting to $A_p(z)$ its time reversed system function $z^{-(p+1)}A_p(z^{-1})$. If p is even, $P_p(z)$ and $Q_p(z)$ have a zero at $z=-1$ and $z=+1$, respectively:

$$P_p(z) = A_p(z) + z^{-(p+1)}A_p(z^{-1}) = (1+z^{-1}) \cdot P'_p(z)$$
$$Q_p(z) = A_p(z) - z^{-(p+1)}A_p(z^{-1}) = (1-z^{-1}) \cdot Q'_p(z)$$

The polynomials $P'_p(z)$ and $Q'_p(z)$ are symmetrical and their zeros are on the unit circle and interlaced. These zeros are complex conjugates and their angles (upper semicircle of the z-plane only) are the LSP parameters [1].

3. KABAL'S METHOD

In Kabal's method [1], the polynomials $P'_p(x)$ and $Q'_p(x)$, of order p/2, are obtained by evaluating $P_p(z)$ and $Q_p(z)$ on the unit circle ($z=e^{j\theta}$), and applying the mapping $x=\cos(\theta)$ together with Chebyshev polynomials of first kind. The roots of $P'_p(x)$ and $Q'_p(x)$ lie in the real interval $(-1,+1)$, with the root corresponding to the lowest frequency LSP being nearest to +1. In the case of 10^{th}-order LPC, $P'_{10}(x)$ and $Q'_{10}(x)$ are 5^{th}-order polynomials and their zeros cannot be calculated in a closed form. In the numerical solution proposed in [1], zero crossings are searched starting at $x=+1$, with decrements of $\Delta=0.02$. Once a zero crossing is found, its position is refined by four successive bisections and a final linear interpolation. The total number

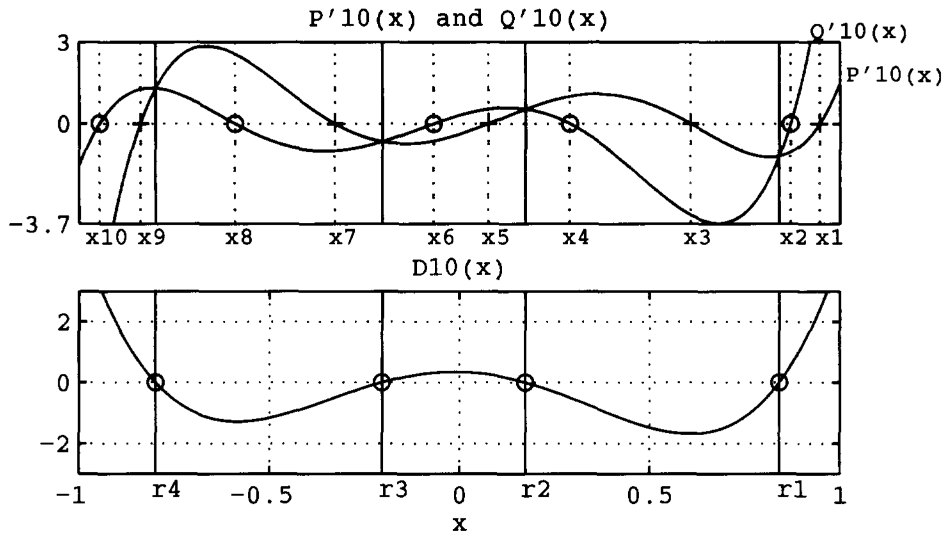

Figure 1: Behavior of the functions $P'_{10}(x)$, $Q'_{10}(x)$ and $D_{10}(x)$ (x_1 to x_{10} are the LSPs in the $x=\cos(\theta)$ domain, and r_1 to r_4 are the roots of $D_{10}(x)$).

of needed polynomial evaluations is less than 150. An efficient recursive evaluation requiring only 4 multiplications and 9 additions is also proposed in [1].

4. NEW METHOD: MIXED-LSP

Derivation of $P'_p(x)$ and $Q'_p(x)$

In the case of even p, starting from the auxiliary function $C_p(z)=z^{(p+1)/2}A_p(z)$ [4], a different derivation of $P'_p(x)$ and $Q'_p(x)$ is done, using the mapping $x=\cos(\theta)$ and Chebyshev polynomials of first and second kind. The derivation is given in Appendix A. The obtained polynomials are expressed as:

$$P'_p(x) = C_p(x) + D_p(x)$$
$$Q'_p(x) = C_p(x) - D_p(x)$$

where $C_p(x)$ is of order p/2 and $D_p(x)$ is of order (p/2−1). In the case of 10^{th}-order LPC, $D_{10}(x)$ is a 4^{th}-order polynomial and hence its roots can be calculated in a closed form. The behavior of the functions $P'_{10}(x)$, $Q'_{10}(x)$, and $D_{10}(x)$ can be observed in Figure 1, where x_1 to x_{10} are the LSPs in the $x=\cos(\theta)$ domain, and r_1 to r_4 are roots of $D_{10}(x)$. It can be proved that the roots of $D_{10}(x)$ are real, different, inside the interval (−1,+1) and correspond to the intersections of $P'_{10}(x)$ with $Q'_{10}(x)$. If the LPC filter is stable, the LSPs are ordered such that $x_1>x_2>\ldots>x_{10}$ [1]. The odd-indexed LSPs correspond to the roots of $P'_{10}(x)$ and the even-indexed LSPs correspond to the roots of $Q'_{10}(x)$ [1]. When going from $x=+1$ to $x=-1$ (θ going from 0 to π), $P'_{10}(x)$ is crossing the x-axis first at x_1, then $Q'_{10}(x)$ has its first zero-crossing at x_2. As the next LSP is x_3, $P'_{10}(x)$ and $Q'_{10}(x)$ intersect each other before crossing the x-axis at x_3 and x_4, respectively, then they intersect again before x_5, x_6, before x_7, x_8 and before x_9, x_{10}. Thus the roots of $D_{10}(x)$ divide the interval (−1,+1) into five sections, *each section containing only one zero-crossing of $P'_{10}(x)$ and one zero crossing of $Q'_{10}(x)$.*

Description of the proposed algorithm (Mixed-LSP)

The roots of $D_{10}(x)$ are calculated and ordered to obtain the five intervals containing each only one zero crossing of $P'_{10}(x)$ and one zero crossing of $Q'_{10}(x)$. The position of these zero crossings is refined by five successive bisections and a final interpolation, giving a total of 60 polynomial evaluations. Particular attention was paid to the optimization of the root calculation of $D_{10}(x)$, which finally needs the following operations: 20 multiplications, 34 add/sub, 2 divisions, 5 square roots, and also 3 comparison/swapping operations for root ordering.

Given that all the roots of the polynomials $P'_{10}(x)$, $Q'_{10}(x)$ and $D_{10}(x)$ are inside the interval (−1,+1) and their leading coefficients are positive, then $P'_{10}(x=+1)$, $Q'_{10}(x=+1)$ and $D_{10}(x=+1)$ are positive. Therefore the directions of the sign changes at every zero crossing are known. This property is used for improving reliability of the algorithm.

5. EXPERIMENTAL EVALUATION

The final version of the Mixed-LSP algorithm was tested using the whole TIMIT database (6300 speech files) [5]. The speech files were downsampled to 8 kHz and the LPC vectors were calculated as in [2], using high-pass filtering of the speech input, 30 ms Hamming windowing, autocorrelation method, and 15 Hz bandwidth expansion

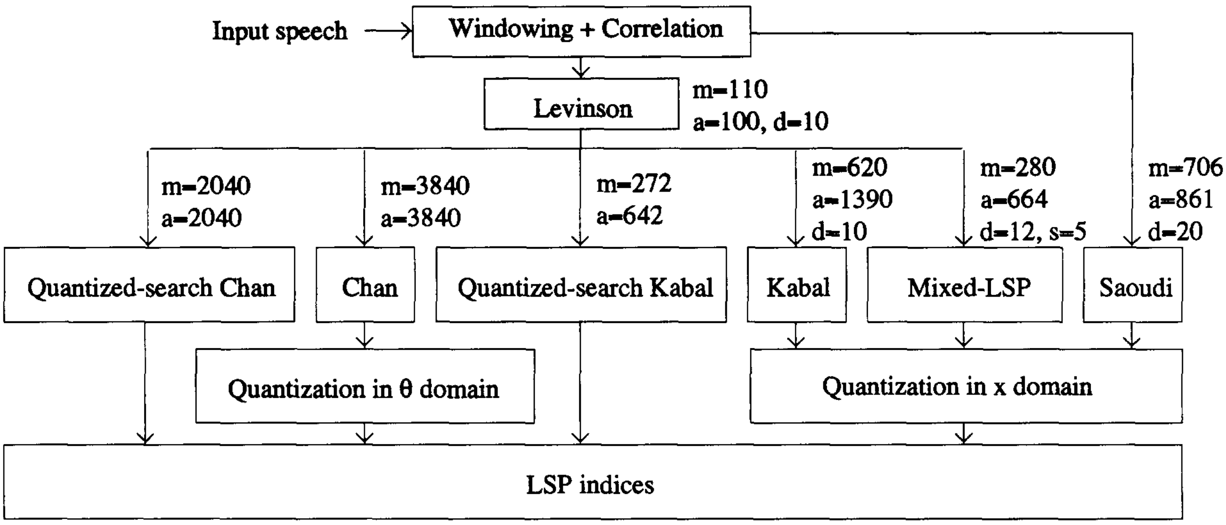

Figure 3: Computational complexity of different LSP calculation methods (10^{th}-order LPC system) (M=multiplications, A=add/sub, D=divisions, S=square roots).

($\gamma = 0.994$). For every speech file, two sets of LSP vectors were calculated, one using the Mixed-LSP algorithm, and the other with a high accuracy method. The maximum absolute difference found is 0.0092 (LSPs are bounded, with $|x_i| < 1$). The histogram of the absolute differences found on the whole TIMIT database is given in Figure 2.

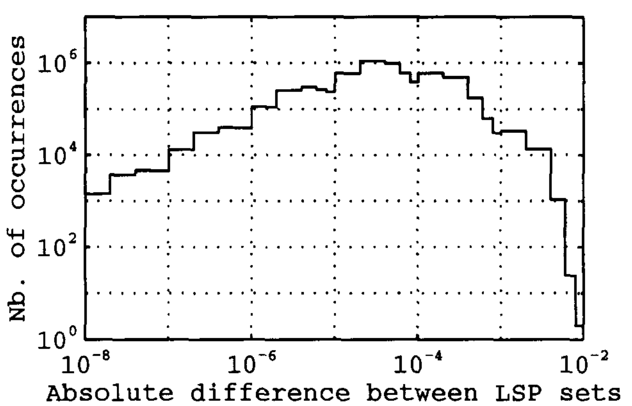

Figure 2: Histogram of the absolute difference between LSP sets calculated with Mixed-LSP on one side, and high precision on the other side.

6. COMPUTATIONAL COMPLEXITY

The proposed Mixed-LSP algorithm was compared in complexity with the methods given in [1], [4], and [6]. The required number of operations is shown in Figure 3. The algorithms denoted as "Quantized-search Kabal" and "Quantized-search Chan" are modified versions of the algorithms of Kabal [1] and Chan [4] in which the search for zero crossings is done directly on the grid of values of the 34-bit non-uniform scalar quantizer of the CELP FS1016 [2]. The accuracy of the LSPs obtained with Mixed-LSP is lower than in Kabal's method, but sufficient for speech coding applications. The accuracy in Mixed-LSP can be improved using more bisections, at the cost of 10*(4·Mult+10·Add) operations per bisection. The accuracy can also be decreased, trading precision with computational complexity. In Kabal's method, accuracy can be increased at the cost of more bisections, but it cannot be decreased, reducing complexity, without affecting the zero-crossing search. "Quantized-search Kabal" is slightly more efficient than Mixed-LSP but is tied to the utilization of the 34-bits non-uniform scalar quantizer of the CELP FS1016.

7. CONCLUSIONS AND FURTHER WORK

In this paper, we have proposed a new method for LSP computation from 10^{th}-order LPC coefficients. In this method, five distinct intervals containing only one odd- and one even-indexed LSP are calculated, avoiding the zero crossing search used in Kabal's method. This allows a trade-off between LSP precision and computational complexity resulting in considerable computational saving.

The proposed algorithm is not tied to the CELP DoD coder and could be used in any other algorithm which makes use of LSP representation of order 10. In addition to the described work, a simulation of the fixed point quantization effects was done, following the methodology explained in [7], in order to determine the minimum word-length and the scaling required at every point of the Mixed-LSP algorithm. It was found that the required wordlength is less than 24 bits. Also, the scaling needed at each point of the algorithms was determined. These results will be used in both a real time implementation on a

DSP56000 processor, and a very low power ASIC implementation for portable applications.

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APPENDIX A

The following derivation is done for p=10, but it can be easily extended to any even p. Starting from the auxiliary function $C_p(z)=z^{(p+1)/2}A_p(z)$ [4], evaluated on the unit circle, $z=e^{j\theta}$:

$$C_{10}(e^{j\theta}) = e^{\frac{j\cdot(10+1)\cdot\theta}{2}} A_{10}(e^{j\theta}) = Cr_{10}(e^{j\theta}) + j\cdot Ci_{10}(e^{j\theta})$$

The symmetrical and antisymmetrical polynomials, can be expressed as a function of the real and imaginary part of $C_{10}(e^{j\theta})$:

$$P_{10}(e^{j\theta}) = A_{10}(e^{j\theta}) + e^{-j11\cdot\theta}A_{10}(e^{-j\theta}) = e^{-j\frac{11}{2}\theta}\cdot 2Cr_{10}(e^{j\theta})$$

$$Q_{10}(e^{j\theta}) = A_{10}(e^{j\theta}) - e^{-j11\cdot\theta}A_{10}(e^{-j\theta}) = e^{-j\frac{11}{2}\theta}\cdot 2jCi_{10}(e^{j\theta})$$

The zero crossings of $Cr_{10}(e^{j\theta})$ and $Ci_{10}(e^{j\theta})$ correspond to the zero crossings of $P_{10}(e^{j\theta})$ and $Q_{10}(e^{j\theta})$, respectively. Grouping terms of $C_{10}(e^{j\theta})$:

$$C_{10}(e^{j\theta}) = e^{\frac{j\cdot 11\cdot\theta}{2}}(1 + a_1\cdot e^{-j\theta} + \ldots + a_{10}\cdot e^{-j\cdot 10\cdot\theta})$$
$$= e^{j\frac{\theta}{2}}(e^{j5\theta} + a_1\cdot e^{j4\theta} + \ldots + a_{10}\cdot e^{-j5\theta})$$

$$C_{10}(e^{j\theta}) = \left[\cos\left(\frac{\theta}{2}\right) + j\cdot\sin\left(\frac{\theta}{2}\right)\right]\cdot\left[R_{10}(e^{j\theta}) + j\cdot I_{10}(e^{j\theta})\right]$$

$$\begin{cases} Cr_{10}(e^{j\theta}) = \cos\left(\frac{\theta}{2}\right)\cdot R_{10}(e^{j\theta}) - \sin\left(\frac{\theta}{2}\right)\cdot I_{10}(e^{j\theta}) \\ Ci_{10}(e^{j\theta}) = \sin\left(\frac{\theta}{2}\right)\cdot R_{10}(e^{j\theta}) + \cos\left(\frac{\theta}{2}\right)\cdot I_{10}(e^{j\theta}) \end{cases}$$

where:

$$R_{10}\left(e^{j\theta}\right) = (1 + a_{10})\cdot\cos(5\theta) + \ldots + (a_4 + a_6)\cdot\cos(\theta) + a_5$$
$$= A_5\cos(5\theta) + A_4\cos(4\theta) + \ldots + A_1\cos(\theta) + A_0$$

$$I_{10}\left(e^{j\theta}\right) = (1 - a_{10})\cdot\sin(5\theta) + \ldots + (a_4 - a_6)\cdot\sin(\theta)$$
$$= E_5\sin(5\theta) + E_4\sin(4\theta) + \ldots + E_1\sin(\theta)$$

Using Chebyshev Polynomials of first and second kind and the mapping $x=\cos(\theta)$:

$$R_{10}(x) = 16A_5x^5 + 8A_4x^4 + \ldots + (A_4 - A_2 + A_0)$$

$$I_{10}(x) = \sqrt{1-x^2}\cdot\left[16E_5x^4 + 8E_4x^3 + \ldots + (E_5 - E_3 + E_1)\right]$$
$$= \sqrt{1-x^2}\cdot I'_{10}(x)$$

$$Cr_{10}(x) = \sqrt{\frac{1+x}{2}}\cdot R_{10}(x) - \sqrt{\frac{1-x}{2}}\cdot\sqrt{1-x^2}\cdot I'_{10}(x)$$
$$= \sqrt{\frac{1+x}{2}}\cdot\left[R_{10}(x) - (1-x)\cdot I'_{10}(x)\right] = \sqrt{\frac{1+x}{2}}\cdot Cr'_{10}(x)$$

$$Ci_{10}(x) = \sqrt{\frac{1-x}{2}}\cdot R_{10}(x) + \sqrt{\frac{1+x}{2}}\cdot\sqrt{1-x^2}\cdot I'_{10}(x)$$
$$= \sqrt{\frac{1-x}{2}}\cdot\left[R_{10}(x) + (1+x)\cdot I'_{10}(x)\right] = \sqrt{\frac{1-x}{2}}\cdot Ci'_{10}(x)$$

In the interval of interest, which is $\theta\in[0,\pi]$, the terms:

$$\sqrt{\frac{1+x}{2}} = \cos\left(\frac{\theta}{2}\right) \quad\text{and}\quad \sqrt{\frac{1-x}{2}} = \sin\left(\frac{\theta}{2}\right)$$

are different from zero, except at $x=-1$ and $x=+1$, respectively. Thus, they can be removed without affecting the position of the other zeros (LSPs). Then the functions:

$$Cr'_{10}(x) = \left[R_{10}(x) - (1-x)\cdot I'_{10}(x)\right] = C_{10}(x) - D_{10}(x)$$
$$Ci'_{10}(x) = \left[R_{10}(x) + (1+x)\cdot I'_{10}(x)\right] = C_{10}(x) + D_{10}(x)$$

have all the zero crossings (LSPs) of Kabal's polynomials $P'_{10}(x)$ and $Q'_{10}(x)$, and also the same leading coefficient (32). Therefore Kabal's polynomials can be expressed as:

$$P'_{10}(x) = Cr'_{10}(x) = C_{10}(x) - D_{10}(x)$$
$$Q'_{10}(x) = Ci'_{10}(x) = C_{10}(x) + D_{10}(x)$$

where $C_{10}(x)$ is a 5^{th}-order polynomial and $D_{10}(x)$ is a 4^{th}-order polynomial.