ANALYSIS OF THE STABILITY OF TIME-DOMAIN SOURCE SEPARATION ALGORITHMS FOR CONVOLUTIVELY MIXED SIGNALS

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ABSTRACT

In this paper, we investigate the self-adaptive source separation problem for convolutively mixed signals. The proposed approach uses a recurrent structure adapted by a generic rule involving arbitrary separating functions. We first analyze the stability of this class of algorithms. We then apply these results to some classical rules for instantaneous and convolutive mixtures that were proposed in the literature but only partly analyzed. This provides a better understanding of the conditions of operation of these rules. Eventually, we define and analyze a normalized version of the proposed type of algorithms, which yields several attractive features.

1. PROBLEM STATEMENT AND CLASSICAL RESULTS

Blind source separation is a generic signal processing problem which concerns e.g. antenna or microphone array processing [1]. In the model commonly used to represent it [2]-[4], two sensors provide measured signals $y_1(n)$ and $y_2(n)$, which are unknown convolutive mixtures of two unknown source signals $x_1(n)$ and $x_2(n)$, i.e. in the Z domain:

$$Y_1(z) = X_1(z) + A_{12}(z).X_2(z) \qquad (1)$$

$$Y_2(z) = A_{21}(z).X_1(z) + X_2(z), \qquad (2)$$

where $A_{ij}(z)$ is the unknown transfer function of the channel that links source j to sensor i. The impulse response of this channel is denoted $(a_{ij}(k))_{k \geq 0}$ hereafter. In this paper, both mixing filters A_{ij} are assumed to have a causal moving average (MA) structure with the same order M. Moreover, the sources $x_1(n)$ and $x_2(n)$ are assumed to be stationary, zero-mean and statistically independent.

The source separation problem consists in estimating the source signals $x_j(n)$ from the measured signals $y_i(n)$ up to a permutation factor and a filter. This problem was initially investigated in the case of instantaneous mixtures, i.e. mixtures for which each MA filter A_{ij} is restricted to the single coefficient $a_{ij}(0)$. It was recognized that in this case, if no assumptions are made on the color of the source signals, these signals cannot be separated by resorting only to second-order statistics [1]. The first solution to this problem was proposed by Hérault and Jutten [1]. It is based on the recurrent structure shown in Figure 1, with each MA separating filter C_{ij} restricted to the single coefficient $c_{ij}(0)$.
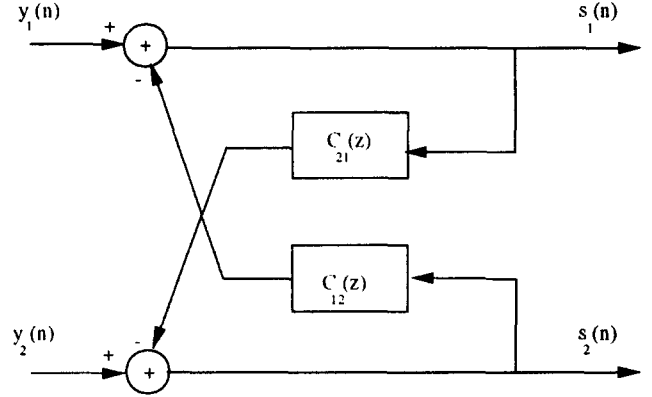

Figure 1. Recurrent source separation structure.

These coefficients are adapted with a rule that reads:

$$c_{ij}(n+1,0) = c_{ij}(n,0) + \mu f(s_i(n))g(s_j(n))$$
$$i \neq j \in \{1,2\}, \qquad (3)$$

where μ is a positive constant adaptation gain and f and/or g are odd nonlinear functions which allow to resort to the higher-order statistics of the signals.

Nguyen and Jutten [2] then proposed a natural extension of the above approach for causal convolutive mixtures. The recurrent structure of Figure 1 then contains causal MA separating filters C_{ij}, whose coefficients are adapted with the rule:

$$c_{ij}(n+1,k) = c_{ij}(n,k) + \mu f(s_i(n))g(s_j(n-k))$$
$$i \neq j \in \{1,2\}, k \in [0,M]. \qquad (4)$$

This approach was mainly studied in the case $f(x) = x^3$ and $g(x) = x$, for which it achieves a stochastic cancellation of the $(3,1)$ cross-moments of the output signals s_i and s_j, and for which the rule (4) reads:

$$c_{ij}(n+1,k) = c_{ij}(n,k) + \mu s_i^3(n)s_j(n-k)$$
$$i \neq j \in \{1,2\}, k \in [0,M]. \qquad (5)$$

A similar approach was independently developed by Al-Kindi et al. [3] and Van Gerven et al. [4] for the case of strictly causal mixing filters, i.e. filters A_{ij} such that $a_{ij}(0) = 0$. The justification provided for not considering the instantaneous part of the mixtures (corresponding to

$a_{ij}(0))$ was the nonzero propagation delay between sensors in the considered configuration [3]. This approach is also based on the recurrent structure [3],[4] (or on the corresponding feedforward structure [4]). The coefficients of its strictly causal MA separating filters C_{ij} are adapted by a rule based on the second-order statistics of the signals, i.e:

$$c_{ij}(n+1,k) = c_{ij}(n,k) + \mu s_i(n)s_j(n-k)$$
$$i \neq j \in \{1,2\}, k \in [1,M], \quad (6)$$

which achieves a decorrelation of the output signals.

While the experimental performance obtained with the above algorithms has been reported in detail, their theoretical properties have only been partly analyzed. Especially, stability analyses were mainly provided for a restricted case of the rule (3) corresponding to $f(x) = x^m$ and $g(x) = x^n$ (see e.g. [5]). On the contrary, the stability of the extended rules (4) or (5) has not been analyzed, and the rule (6) has only been studied in the case when the mixing and separating filters contain a single time-delay coefficient [4].

In Sections 2. and 3. below, we present new results about these rules, and more generally speaking we provide an analysis of the stability of the recurrent structure of Fig. 1 for a larger class of non-linear adaptation rules that reads:

$$c_{ij}(n+1,k) = c_{ij}(n,k) + \mu f_i(s_i(n))g_i(s_j(n-k))$$
$$i \neq j \in \{1,2\}, k \in [0,M], \quad (7)$$

where f_i and g_i are arbitrary functions at this stage. This analysis is performed for the desired convergence point of the adaptation rule, called the "separating point". This point corresponds to $C_{ij}(z) = A_{ij}(z)$ and yields $S_i(z) = X_i(z)$. The only condition set on the functions f_i and g_i is: $E[g_i(x_j)] = 0$, which ensures that the separating point is an equilibrium point of the rule (7).

An extension of this generic rule is then considered in Section 4. and conclusions are drawn from the overall investigation in Section 5..

2. GENERIC STABILITY ANALYSIS

All the coefficients $c_{ij}(n,k)$ of both separating filters C_{ij} involved in the above rule (7) may be gathered in a vector

$$\theta_n = [c_{12}(n,0),\dots,c_{12}(n,M),$$
$$c_{21}(n,0),\dots,c_{21}(n,M)]^T. \quad (8)$$

The rule (7) may thus be expressed in vector form as

$$\theta_{n+1} = \theta_n + \mu H(\theta_n,\xi_{n+1}). \quad (9)$$

where ξ_{n+1} and $H(\theta_n,\xi_{n+1})$ are column vectors defined as:

$$\xi_{n+1} = [y_1(n),y_2(n),$$
$$s_1(n-1),\dots,s_1(n-M),$$
$$s_2(n-1),\dots,s_2(n-M)]^T, \quad (10)$$
$$H(\theta_n,\xi_{n+1}) = [f_1(s_1(n))g_1(s_2(n)),\dots,$$
$$f_1(s_1(n))g_1(s_2(n-M)),$$
$$f_2(s_2(n))g_2(s_1(n)),\dots,$$
$$f_2(s_2(n))g_2(s_1(n-M))]^T. \quad (11)$$

The approach used in this paper to analyze stability is the so-called Ordinary Differential Equation technique (ODE) [6], which approximates the discrete recurrence (9), for a small adaptation gain μ, by a continuous differential system that reads:

$$\frac{d\theta}{dt} = E_\theta[H(\theta,\xi_{n+1})]. \quad (12)$$

The differential system (12) is locally stable in the vicinity of any given equilibrium point θ^* if and only if (iff) the associated tangent linear system:

$$\frac{d\theta}{dt} = J(\theta^*)(\theta - \theta^*) \quad (13)$$

is stable, i.e. iff all the eigenvalues of $J(\theta^*)$ have negative real parts. For any point θ, $J(\theta)$ denotes the Jacobian matrix of the system, i.e. the matrix of partial derivatives with entries:

$$J_{ij}(\theta) = \frac{\partial(E_\theta[H(\theta,\xi_{n+1})]^{(i)})}{\partial\theta^{(j)}}, \quad (14)$$

where $E_\theta[H(\theta,\xi_{n+1})]^{(i)}$ is the ith component of $E_\theta[H(\theta,\xi_{n+1})]$ and $\theta^{(j)}$ is the jth component of vector θ.

For the rule (7) considered in this paper it can be shown [7] that, if $x_1(n)$ and $x_2(n)$ are independent identically distributed random variables, the eigenvalues of the Jacobian matrix at the separating point, $J(\theta^s)$, are[1]

$$1) \quad -\alpha_1 w_{eq}(0) \quad (15)$$
$$2) \quad -\alpha_2 w_{eq}(0) \quad (16)$$
$$3) \quad [-\delta \pm \sqrt{\Delta}]\frac{w_{eq}(0)}{2} \quad if \ \Delta > 0 \quad (17)$$
$$[-\delta \pm i\sqrt{-\Delta}]\frac{w_{eq}(0)}{2} \quad if \ \Delta < 0 \quad (18)$$
$$-\delta w_{eq}(0)/2 \quad if \ \Delta = 0 \quad (19)$$

with the following notations (in which j is chosen so that $i \neq j \in \{1,2\}$):

$$\alpha_i = E[f_i'(x_i)]E[x_j g_i(x_j)] \quad (20)$$
$$\beta_i = E[x_i f_i(x_i)]E[g_i'(x_j)] \quad (21)$$
$$\varphi_i = a_{ji}(0)E[f_i(x_i)]E[x_j g_i'(x_j)] \quad (22)$$
$$\delta = \alpha_1 - \varphi_1 + \alpha_2 - \varphi_2 \quad (23)$$
$$\Delta = [(\alpha_1 - \varphi_1) - (\alpha_2 - \varphi_2)]^2 + 4\beta_1\beta_2 \quad (24)$$
$$w_{eq}(0) = \frac{1}{1 - a_{12}(0)a_{21}(0)}. \quad (25)$$

The stability condition (i.e. all eigenvalues having negative real parts) therefore reads as follows depending on the sign of Δ:

• if $\Delta > 0$

$$\begin{cases} \alpha_1 w_{eq}(0) > 0 \\ \alpha_2 w_{eq}(0) > 0 \\ (\alpha_1 - \varphi_1)(\alpha_2 - \varphi_2) > \beta_1\beta_2 \\ (\alpha_1 + \alpha_2)w_{eq}(0) > (\varphi_1 + \varphi_2)w_{eq}(0) \end{cases} \quad (26)$$

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[1] In (18), i represents the complex square root of -1.

- if $\Delta \leq 0$

$$\begin{cases} \alpha_1 w_{eq}(0) > 0 \\ \alpha_2 w_{eq}(0) > 0 \\ (\alpha_1 + \alpha_2) w_{eq}(0) > (\varphi_1 + \varphi_2) w_{eq}(0) \end{cases} \quad (27)$$

In the case of strictly causal filters, it can be shown that the eigenvalues become restricted to $-\alpha_1$ and $-\alpha_2$. The stability condition then reads:

$$\begin{cases} \alpha_1 > 0 \\ \alpha_2 > 0 \end{cases} \quad (28)$$

For instantaneous mixture and separation, the eigenvalues consist of the expressions (17)-(19). The stability condition then becomes:

- if $\Delta > 0$

$$\begin{cases} (\alpha_1 - \varphi_1)(\alpha_2 - \varphi_2) > \beta_1\beta_2 \\ (\alpha_1 + \alpha_2) w_{eq}(0) > (\varphi_1 + \varphi_2) w_{eq}(0) \end{cases} \quad (29)$$

- if $\Delta \leq 0$

$$(\alpha_1 + \alpha_2) w_{eq}(0) > (\varphi_1 + \varphi_2) w_{eq}(0). \quad (30)$$

All these stability conditions are detailed for specific algorithms in the next section.

3. APPLICATION TO CLASSICAL ADAPTATION RULES

The generic approach presented in Section 2. especially applies to the classical adaptation rules defined in Section 1.. This allows one to derive stability conditions for these algorithms, which have not been reported up to now. This method is applied hereafter, with main emphasis on the most unexplored field, i.e. convolutive mixtures.

3.1. Analysis of two classical algorithms for convolutive mixtures

3.1.1. Analysis of the decorrelation algorithm

As a first step, we consider an extension of the decorrelation approach described in Section 1. to the case of (non strictly) causal mixing and separating filters. The rule (6) then becomes

$$\begin{aligned} c_{ij}(n+1, k) &= c_{ij}(n, k) + \mu s_i(n) s_j(n-k) \\ & i \neq j \in \{1, 2\}, k \in [0, M]. \end{aligned} \quad (31)$$

This rule is a particular case of the general framework considered in Section 2., corresponding to

$$\begin{cases} f_i(x) = x & i \in \{1, 2\} \\ g_i(x) = x & i \in \{1, 2\} \end{cases} \quad (32)$$

Note that the assumption $E[g_i(x_j)] = 0$ made above on the separating functions g_i is valid here, since it corresponds to the zero-mean hypothesis made on the sources, i.e:

$$E[x_i] = 0, \quad i \in \{1, 2\}. \quad (33)$$

The eigenvalues of the Jacobian matrix at the separating point are derived by applying (15)-(25) to the functions defined in (32), thus yielding:

$$\begin{cases} -E[x_1^2] w_{eq}(0) \\ -E[x_2^2] w_{eq}(0) \\ -(E[x_1^2] + E[x_2^2]) w_{eq}(0) \\ 0 \end{cases} \quad (34)$$

Since these eigenvalues are real, the stability condition corresponds to their negativeness which requires that:

$$w_{eq}(0) > 0 \quad (35)$$

or equivalently:

$$1 - a_{12}(0) a_{21}(0) > 0. \quad (36)$$

Nevertheless, the algorithm always yields a null eigenvalue. This implies that the algorithm is not asymptotically stable, but only globally stable with fluctuations. This means also that there exists a one-dimensional subspace, the Kernel of $J(\theta^s)$, associated to eigenvectors corresponding to the null eigenvalue, in which asymptotic convergence cannot be reached. In fact, from a computational point of view, the estimation associated to the null eigenvalue may take small but non null values that may be positive, leading then to instability.

Let us now consider the case of strictly causal filters. The eigenvalues of the Jacobian matrix are then $-E[x_1^2]$ and $-E[x_2^2]$ that are always negative. Hence, the decorrelation scheme yields an asymptotically stable separating point in this case and becomes then a potentially attractive separation procedure.

3.1.2. Analysis of the algorithm based on (3,1) cross-moment cancellation

We have shown above that the decorrelation criterion may lead to a numerically unstable algorithm in the general case of causal filters. The cause of this problem is that the first-order lag coefficients $c_{12}(n, 0)$ and $c_{21}(n, 0)$ are updated by the same correcting term, i.e. $\mu s_1(n) s_2(n)$. There are different strategies to overcome this problem. A well-known one consists in using the algorithm (4) with at least one non-linear separating function f or g. Especially, the specific case which has been considered in the literature is the rule (5). This algorithm is also a particular case of (7), obtained for:

$$\begin{cases} f_i(x) = x^3 & i \in \{1, 2\} \\ g_i(x) = x & i \in \{1, 2\} \end{cases} \quad (37)$$

By applying (15)-(25) to the algorithm (5), the eigenvalues of the Jacobian matrix at the separating point become:

$$\begin{cases} -3E[x_1^2] E[x_2^2] w_{eq}(0) \\ -\left(3E[x_1^2] E[x_2^2] \pm \sqrt{E[x_1^4] E[x_2^4]} \right) w_{eq}(0) \end{cases} \quad (38)$$

and the stability condition is then:

$$\begin{cases} w_{eq}(0) > 0 \\ E[x_1^4] E[x_2^4] < 9 E^2[x_1^2] E^2[x_2^2] \end{cases} \quad (39)$$

The second condition in (39) corresponds to the global sub-gaussianity of the sources. It should be noted that (39) is the same stability condition as with the (3.1) cross-moment cancellation algorithm for instantaneous mixtures [5].

3.2. Analysis of a classical algorithm for instantaneous mixtures

Here, we consider the Hérault-Jutten algorithm (3) for instantaneous mixtures. As stated in Section 1., the corresponding stability condition was only studied for a limited class of separating functions and symmetrically distributed sources. This paper extends these results by providing a stability condition at the separating point for possibly asymmetrically distributed sources and any separating functions f and g. This is an application of the results of Section 2. about instantaneous mixtures to the case when:

$$f_1 = f_2 = f, \tag{40}$$

$$g_1 = g_2 = g. \tag{41}$$

The stability condition is then directly obtained by applying (29)-(30) to the case defined by (40)-(41).

4. DEFINITION AND ANALYSIS OF A CLASS OF NORMALIZED ADAPTATION RULES

In this section, we introduce an extended version of the class of non-linear adaptation rules (7) considered above. This extended type of rules reads:

$$c_{ij}(n+1,k) = c_{ij}(n,k) + \mu \frac{f_i(s_i(n))}{\sqrt{E[f_i^2(s_i)]}} \frac{g_i(s_j(n-k))}{\sqrt{E[g_i^2(s_j)]}}$$
$$i \neq j \in \{1,2\}, k \in [0,M], \tag{42}$$

where the terms $\sqrt{E[f_i^2(s_i)]}$ and $\sqrt{E[g_i^2(s_j)]}$ are estimated in practical situations, using first-order low-pass filtering. This rule can be seen as a zero-search procedure for the set of correlation coefficients between the random variables $f_i(s_i(n))$ and $g_i(s_j(n-k))$, instead of the classical zero-search procedure for the non-normalized correlation between these two variables used in (7). The main motivation for this rule is that the variance of its correcting term $\frac{f_i(s_i(n))}{\sqrt{E[f_i^2(s_i)]}} \frac{g_i(s_j(n-k))}{\sqrt{E[g_i^2(s_j)]}}$ is equal to one at the separating point, thanks to the normalization performed by the terms $\sqrt{E[f_i^2(s_i)]}$ and $\sqrt{E[g_i^2(s_j)]}$. This value is independent from the scales and statistics of the sources and from the separating functions. The adaptation gain μ can therefore be selected independently from these parameters, so as to achieve the desired trade-off between convergence accuracy and speed. This independence with respect to the source parameters is especially attractive, as these parameters are supposedly unknown. This rule is also well-suited to non-stationary sources, for which short-term estimates of $\sqrt{E[f_i^2(s_i)]}$ and $\sqrt{E[g_i^2(s_j)]}$ make it able to automatically track the fluctuations of the characteristics of these sources.

The results presented above for the non-normalized rule (7) cannot be applied directly to the rule (42) considered here, because the latter rule includes additional estimated parameters (i.e. $\sqrt{E[f_i^2(s_i)]}$ and $\sqrt{E[g_i^2(s_j)]}$). However,

(42) combined with the estimation of the associated parameters $\sqrt{E[f_i^2(s_i)]}$ and $\sqrt{E[g_i^2(s_j)]}$ can be formulated as a relaxation scheme having the form (7) with a second-order perturbation term [7]. It can also be shown that this normalization scheme does not modify fundamentally the results obtained in the previous sections. More precisely, computations show that the stability condition for the algorithm (42) is the same as the condition for the algorithm (7), except that f_i and g_i are resp. replaced by F_i and G_i defined as:

$$F_i(x) = \frac{f_i(x)}{\sqrt{E[f_i^2(x)]}},$$
$$G_i(x) = \frac{g_i(x)}{\sqrt{E[g_i^2(x)]}}. \tag{43}$$

5. CONCLUSIONS

While a few time-domain source separation algorithms for convolutive mixtures have been proposed in the literature and experimentally studied, their convergence properties had almost not been analytically studied up to now. In order to fill this gap, in this paper we have analyzed the stability (at the separating point) of a large class of algorithms, which especially includes the classical approaches. As a by-product, we have thus also extended the theoretical results reported for the Hérault-Jutten algorithm for instantaneous mixtures. Moreover, we have defined and studied a normalized version of the proposed class of algorithms, which yields several attractive features.

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