

BISPECTRAL RECONSTRUCTION USING INCOMPLETE PHASE KNOWLEDGE: A NEUROELECTRIC SIGNAL ESTIMATION APPLICATION

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ABSTRACT

The bispectral averaging technique is often used in order to analyze signal with variable signal delay, in presence of noise. Unfortunately, as the bispectrum is time-shift invariant, the initial phase of the signal can't be recovered. When studying somatosensory evoked potentials (neuroelectric signals) this phase is generally the major information, especially when it characterizes pathologies. We show that some informations about this phase can be extracted from the averaged signal. An attempt to include this knowledge in the magnitude and phase recovery algorithms is made. We illustrate the benefits of this approach on a simulation and a real application leading to a details enhancement of the analyzed signal.

1. INTRODUCTION

In this communication, we assume that a large number N of realizations of a recurrent signal $s(t)$ are observed with a random delay and in a noisy environment. In our application, the recurrent signal will be the somatosensory evoked potentials (SEP). The random behaviour of these realizations is assumed to be due to the random delay and the additive noise.

Each realization $x_i(t)$ is modeled using the following expression:

$$x_i(t) = s(t - d_i) + n_i(t) \quad (i = 1 \dots N; 0 \leq t \leq T) \quad (1)$$

where $s(t)$ is the deterministic signal (SEP) not symmetrically distributed and $n_i(t)$ the background noise EEG (electroencephalographic) assumed to be symmetrically distributed without the assumption of whiteness.

The classical technique for the deterministic signal retrieval is the synchronous averaging which is the best

estimator when delays d_i are known. On the contrary, when d_i is a random variable whose probability density p_d is unknown, this technique is no more efficient. So, the averaged signal $\bar{x}_N(t)$ defined by $\bar{x}_N(t) = \frac{1}{N} \sum_{i=1}^N x_i(t)$ can be approximated by the following convolution product, when N is large enough:

$$\bar{x}_N(t) \approx (p_d * s)(t) + \bar{n}_N(t) \quad (2)$$

In our application domain, the SNR ($\approx 0dB$) and the large number of records (> 600) permits to neglect the residual noise $\bar{n}_N(t)$ in regard to the filtered recurrent signal. When the probability density function (pdf) p_d of d_i is assumed to be gaussian or uniform, the signal $s(t)$ is low-pass filtered.

Attempts to enhance the averaged signal has been proposed. Some are based on deconvolution procedures when the filter p_d is assumed to be known. To avoid the deconvolution problems, we can make use of time delay estimators in order to perform time alignment of signals before the averaging process. Due to the very low SNR and the lack of template, this approach is limited. One recent domain where such an application problem find some suitable solution is the high order statistics domain, especially the bispectrum.

Some previous works based on the bispectrum averaging has been proposed [2][4], leading to a waveform estimation. The bispectrum being not influenced by a linear phase, the initial phase of the signal $s(t)$ cannot be recovered from the bispectrum. This information is important for some pathologies characterizations where the latency of the evoked potentials is the required value.

We will show that some knowledge on the signal phase can be advantageously extracted from the averaged signal using some realistic assumptions. We will propose an algorithm for the amplitude reconstruction based on the real and imaginary parts of the bispectrum, using this knowledge. We will compare its performances with

The author would like to thank J. Leroux and C. Huet for their help.

classical algorithms in simulation. The phase reconstruction will be based on a recursive algorithm after proving that it is well adapted to our approach. We will finish this communication with a real application on SEP, where the advantages of this method will be highlighted.

2. MATHEMATICAL DEVELOPMENT

When neglecting the residual noise, the convolution equation (2) can be expressed in the Fourier domain $\bar{X}_N(\omega) = P_d(\omega)S(\omega)$. One assumption commonly encountered is that the random delay is zero mean and symmetrically distributed, i.e its Fourier transform $P_d(\omega)$ is real. Its positive and negative parts can be distinguished in order to express $\bar{X}_N(\omega)$ in polar representation. The values of ω corresponding to a strictly positive $P_d(\omega)$ are defined by the interval Ω^+ , and Ω^- for the strictly negative values of $P_d(\omega)$. The modulus and phase of $\bar{X}_N(\omega)$ is then expressed by:

$$\begin{aligned} |\bar{X}_N(\omega)| &= |P_d(\omega)| \cdot |S(\omega)| & (3) \\ \arg(\bar{X}_N(\omega)) &= \begin{cases} \arg(S(\omega)) & \text{if } \omega \in \Omega^+ \\ \arg(S(\omega)) \pm \pi & \text{if } \omega \in \Omega^- \end{cases} & (4) \end{aligned}$$

The first difficulty is to determine Ω^+ . The second one is to estimate the phase of $\bar{X}_N(\omega)$ when the modulus is closed to zero.

The pdf p_d of the latency, in the SEP case, is unknown. Nevertheless, it is reasonable to model it by the normal or uniform law. For the first one, the solution of finding Ω^+ is obvious because $P_d(\omega)$ is strictly positive, i.e $\Omega^+ =]-\infty, +\infty[$. For the second one, the solution is not so obvious since $P_d(\omega)$ is not strictly positive. When p_d is the rectangular function $(1/2t_r)r(t; -t_r, t_r)$, its Fourier transform is $P_d(\omega) = \text{sinc}(\omega t_r)$. So, $P_d(\omega)$ is strictly positive for $\omega \in \Theta = [0, \pi/t_r]$. This interval is only a part of Ω^+ but it will be sufficient in this case. It has been shown in [1] that for the SEP, the interval Θ is large in regard to the $S(\omega)$ spread.

The second difficulty will be solved empirically choosing a threshold equal to 5% of the $\bar{X}_N(\omega)$ maximum. In the following, we will show how to include the knowledge of $\arg(S(\omega))$ for $\omega \in \Theta$ in the amplitude and phase reconstruction using the averaged bispectrum.

3. BISPECTRAL RECONSTRUCTION: AMPLITUDE

The use of the discrete averaged bispectrum [4] defined by:

$$\bar{B}_N(k, l) = \frac{1}{N} \sum_{i=1}^N B_i(k, l) \quad (5)$$

with

$$B_i(k, l) = X_i(k)X_i(l)X_i^*(k+l) \quad (6)$$

is justified by its time shift invariance and its properties with gaussian noise. We will use the principal region of the bispectrum plane excepted the $l = 0$ axis.

The phase of $\bar{X}_N(\omega)$ for $\omega \in \Theta$ will be noted $\varphi(m)$ for $m \in [0, M]$, in the discrete case. The unknown spectrum $S(\omega)$ will be defined in complex notation $S(\omega) = S_r(\omega) + j \cdot S_i(\omega)$. In order to clarify the development, the residual noise in (5) will be omitted. So, we can show that the real and imaginary part of $\bar{B}_N(k, l)$ can be expressed by:

$$\Re[\bar{B}_N(k, l)] = S_r(k)S_r(l)S_r(k+l)F(\psi; k, l) \quad (7)$$

$$\Im[\bar{B}_N(k, l)] = S_i(k)S_i(l)S_i(k+l)F(\Psi; k, l) \quad (8)$$

for $1 \leq k \leq M, 1 \leq l \leq k$ and $k+l \leq M$ and using the following definition of F :

$$\begin{aligned} F(\Psi; k, l) &= 1 + \Psi(k)\Psi(k+l) + \Psi(l)\Psi(k+l) - \Psi(k)\Psi(l) & (9) \\ \text{with } \Psi(k) &= (\psi(k))^{-1} = (\tan(\varphi(k)))^{-1} \end{aligned}$$

The spectrum of $s(t)$ being mainly concentrated in the low-frequency domain, we will use a recursive technique similar to those given in [5], using the relations:

$$S_r(k+1) = \frac{\Re[\bar{B}_N(k, 1)]}{S_r(1)S_r(k)F(\psi; k, 1)} \quad (10)$$

$$S_i(k+1) = \frac{\Im[\bar{B}_N(k, 1)]}{S_i(1)S_i(k)F(\Psi; k, 1)} \quad (11)$$

for $1 \leq k \leq M$ and with the initial values $S_r(1)$ and $S_i(1)$ given by:

$$\begin{aligned} |S_r(1)| &= & (12) \\ &= \left| \frac{\Re[\bar{B}_N(1, 1)]^3 \Re[\bar{B}_N(3, 1)] F(\psi; 2, 1) F(\psi; 2, 2)}{(\Re[\bar{B}_N(2, 1)] \Re[\bar{B}_N(2, 2)] F(\psi; 1, 1)^3 F(\psi; 3, 1))} \right|^{\frac{1}{5}} \end{aligned}$$

$$\begin{aligned} |S_i(1)| &= & (13) \\ &= \left| \frac{\Im[\bar{B}_N(1, 1)]^3 \Im[\bar{B}_N(3, 1)] F(\Psi; 2, 1) F(\Psi; 2, 2)}{(\Im[\bar{B}_N(2, 1)] \Im[\bar{B}_N(2, 2)] F(\Psi; 1, 1)^3 F(\Psi; 3, 1))} \right|^{\frac{1}{5}} \end{aligned}$$

As the phase of $S(1)$ is known, the sign of $S_r(1)$ and $S_i(1)$ is readily obtained.

Several estimations of $S_r(k+1)$ and $S_i(k+1)$ can be readily obtained with $l = 1 \dots M/2$ if M is even or $l = 1 \dots (M-1)/2$ if M is odd. These estimates are commonly averaged in order to reduce the estimation variance. We can show that a variance reduction can be performed rewriting (10) and (11) in matrix form, for the previous values of l (similar equation than (10) and (11) are readily obtained with l different than 1):

$${}_{k,l} \mathbf{V}_r S_r(k+1) = {}_{k,l} \mathbf{U}_r \quad (14)$$

$${}_{k,l} \mathbf{V}_i S_i(k+1) = {}_{k,l} \mathbf{U}_i \quad (15)$$

Where vectors ${}_{k,l}\mathbf{V}_r$, ${}_{k,l}\mathbf{U}_r$, ${}_{k,l}\mathbf{V}_i$, ${}_{k,l}\mathbf{U}_i$ are deduced from (7) and (8). The solutions for $S_r(k+1)$ and $S_i(k+1)$ can be obtained in the least-squares sense from (14) and (15):

$$\hat{S}_r(k+1) = \frac{{}_{k,l}\mathbf{V}_r^T \cdot {}_{k,l}\mathbf{U}_r}{{}_{k,l}\mathbf{V}_r^T \cdot {}_{k,l}\mathbf{V}_r} \quad (16)$$

$$\hat{S}_i(k+1) = \frac{{}_{k,l}\mathbf{V}_i^T \cdot {}_{k,l}\mathbf{U}_i}{{}_{k,l}\mathbf{V}_i^T \cdot {}_{k,l}\mathbf{V}_i} \quad (17)$$

The estimated modulus of $S(k+1)$ is then calculated from the real and imaginary parts previously estimated. Using a simulation example (section 5), we have shown that the proposed method doesn't yield to a performances improvement in comparison to the bispectra recursive algorithm [5] including a least-squares step as in (16) and (17). We can conclude that the knowledge of the phase doesn't necessary lead to an amplitude reconstruction improvement.

4. BISPECTRAL RECONSTRUCTION: PHASE

We are going to show that contrarily to the amplitude reconstruction, the knowledge of the phase for $\omega \in [0, M]$ can be advantageously used in the phase reconstruction.

Assuming that $s(t)$ is a shifted signal, its phase $\varphi(m)$ can be decomposed as following:

$$\varphi(m) = \phi(m) + m.\phi_l \quad (18)$$

where ϕ_l is unknown. As the bispectrum is not influenced by a linear phase, we will note $\phi(m)$ the phase effectively taken into account by the bispectrum equation [3]:

$$\phi_3(k, l) = \varphi(k) + \varphi(l) + \varphi(k+l) \quad (19)$$

using the definition of $\varphi(m)$, (19) becomes:

$$\phi_3(k, l) = \phi(k) + \phi(l) + \phi(k+l) \quad (20)$$

where $\phi_3(k, l) = \arg(\bar{B}_N(k, l)) \bmod 2\pi$. As in the previous section, we will use a recursive method illustrated with a fixed value $l = 1$ and using (19):

$$\varphi(M+1) = \varphi(M) + \varphi(1) - \phi_3(M, 1) \quad (21)$$

assuming that $\varphi(m)$ is known for $\omega \in [0, M]$.

Using (18) and (20), equation (21) becomes:

$$\begin{aligned} \varphi(M+1) &= \phi(M) + M\phi_l + \phi(1) + \phi_l - \phi(M) - \phi(1) \\ &\quad + \phi(M+1) \end{aligned} \quad (22)$$

$$\varphi(M+1) = \phi(M+1) + (M+1).\phi_l \quad (23)$$

which corresponds to (18) for $m = M+1$. So, the recursive method can be used to recover the phase $\varphi(m)$ for $m = M+1 \dots K/2$, with K the last sample index of the sampled signal $s(t)$. Such a recursion will be made with different values of l leading to different estimations of $\varphi(m)$. We will average the exponential factors $\exp(j\varphi(m))$ instead of $\varphi(m)$ because the $\varphi(m)$ are only determined up to modulo 2π .

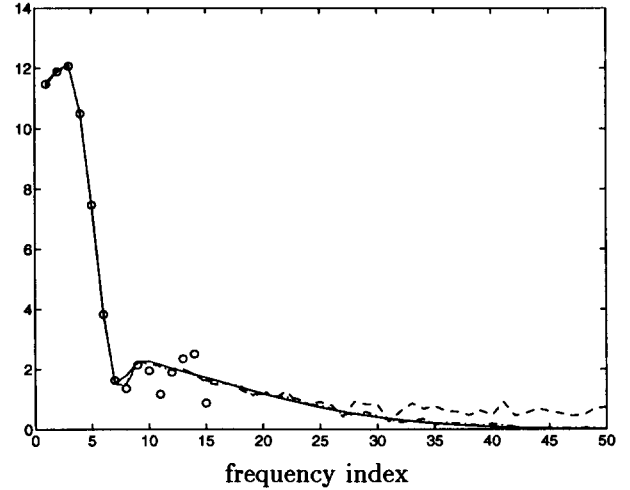


Figure 1: *Magnitude reconstruction.* True magnitude: —, using the bispectra recursive algorithm: ---, using the incomplete phase knowledge: o o o, using the spectral technique: - . - .

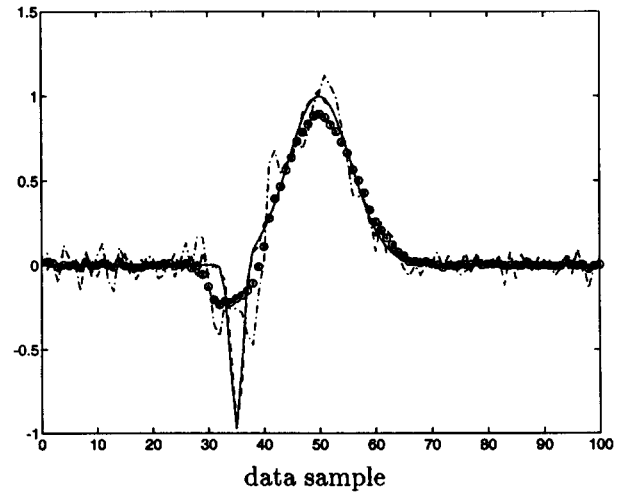


Figure 2: *Signal reconstruction.* true signal: —, using the proposed bispectral technique: ---, averaged signal (2): o o o, using the bispectral technique with the averaged signal phase: - . - .

5. SIMULATION AND APPLICATION

In order to compare the different approaches, we propose a simulation involving the signal shown in figure (2) (true signal). The standard deviation of the random delay is 20 samples, the noise is white and gaussian so that the SNR is 0dB, the number of realizations is equal to 600. We have shown in fig.(1) the reconstructed magnitude using the incomplete phase knowledge ($M = 15$), the bispectra recursive algorithm including the least-squares estimation, the spectral technique. The spectral technique is based on the following property :

$$E |X(\omega)|^2 = |S(\omega)|^2 + N(\omega) \quad (24)$$

with $N(\omega)$ the power spectrum of the noise.

As in the practical case, a noise reference is available (just before the stimuli occurrence) leading to the estimated power spectrum $\hat{N}(\omega)$. So an estimation of $|S(\omega)|$ is easily obtained using $\hat{N}(\omega)$ and (24).

We can see in fig.(1) that the proposed method exhibits a higher variance than other ones. In regards to these results we will not make use of the proposed method for the magnitude reconstruction in the following application.

In fig. (2), the comparison of the different results illustrates the interest of the phase recovery using the bispectrum.

The previous methods have been applied to the somatosensory evoked potential which is a response to a 20 mA electrical stimuli. The signal has been recorded 40ms after the stimuli with a sampling frequency equal to 3200 Hertz, the number of records N is 600 and the SNR is approximately equal to 0dB. We have used the bispectra recursive algorithm including the least-squares estimation and the spectral approach (24). Considering the constraints introduced in section 2., we fixed $M = 13$. In fig. (3), we can notice that the recovered signals respect the time shift in regard to the averaged one. As expected, the recovered signals contain more high frequencies since the averaged signal is low-pass filtered but the signal from the spectral approach is less noisy.

6. CONCLUSION

The averaged signal contains some useful information. We have shown that this information could be a part of the phase of the recurrent signal $s(t)$. In order to avoid the drawbacks of the signal averaging techniques, we have used the bispectrum averaging approach. The difficulties of such an approach is the amplitude and phase recovery. We have proposed an amplitude recovery algorithm including the knowledge of the information previously introduced. Unfortunately, it doesn't lead to a performance improvement in comparison with

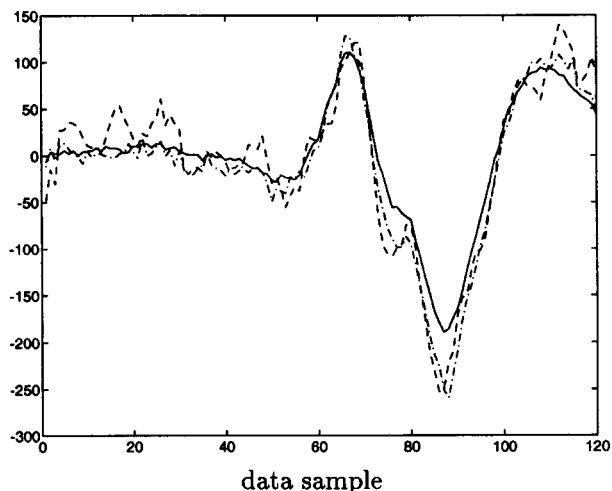


Figure 3: Signal reconstruction. averaged signal : —, using the proposed bispectral technique : - - -, using the spectral technique combined with the proposed phase reconstruction method:

classical algorithms. Nevertheless, it doesn't mean that the performance improvement based on this knowledge is an impossible task. For the phase recovery issue, this knowledge can be used with benefits. We have shown that we can make use of the recursive algorithm to recover the unknown part of the signal phase. To illustrate the performance of such an approach, an application example in the biomedical field is given. It is clear that it leads to a details enhancement while preserving the latency estimation of the somatosensory evoked potential.

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