

# OPTIMAL PHASE-LOCKED LOOP DESIGN WITH KALMAN PREDICTORS FOR SYNCHRONOUS NETWORKS

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## ABSTRACT

A systematic technique for the optimal design of phase-locked loops for synchronous networks is presented. The method is based on Kalman estimation theory under self-similar random noise processes. This approach is optimal for certain noise models and for linear phase-detectors. The results are then extended in order to maintain the minimum mean-square phase error when the reference signal of a master-slave network is lost.

## 1. INTRODUCTION

Network synchronization deals with the problem of distributing time and frequency over a network of clocks which are spread over a wide geographical area. The goal is to synchronize the time and frequency scales of all the clocks which belong to the network.

In this paper we concentrate on the master-slave technique which is certainly the most widely applied[1]. In this case all the network clocks are either directly or indirectly slaved to a network master clock.

In order to find the optimal design of the phase-locked loop (PLL) we have to take into account the properties of the noise in the reference signal and in the Voltage Controlled Oscillator (VCO). The design criterion is to minimize the phase mean-square error of the VCO.

The methods found in the literature are empirical and usually find the PLL parameters iteratively by trial and error, resulting in a sub-optimum design.

Gardner [2] shows an example where additive white noise in the reference signal and frequency flicker noise in the VCO are assumed. He proposes to find the parameters  $B_L$  (noise bandwidth) and  $\zeta$  (damping factor)

of a second-order loop by differentiating the expression of the phase error variance with respect to these parameters. As more realistic spectral densities of noise are considered this solution becomes increasingly complex.

Vanier and Têtu [3] propose to plot the VCO error spectral density for different values of  $f_n$  (natural frequency) and  $\zeta$  (damping factor), from which they attempt to find the optimal parameters by trial and error.

Wolaver [4] shows an example where additive white noise in the reference signal and phase random-walk noise in the VCO are considered. He then finds the filter parameters by placing the poles of the closed-loop transfer function at convenient locations. These selections are based on design experience rather than on well established set of rules.

In this paper, a systematic technique for the optimal design of the phase-locked loop is presented. The key idea is to use Kalman estimation theory and realistic noise models[5].

One important problem associated with master-slave networks is the possible loss of the master timing signal. In this paper we also develop a technique which minimizes the mean-square phase error when the reference signal is lost. Its performance is then compared to other techniques presented in the literature.

## 2. CLOCK ERRORS MODEL

A tractable mathematical model for the quasi-sinusoidal output signal of an oscillator is

$$v(t) = v_0 \sin[2\pi f_0 t + \phi(t)],$$

where  $\phi(t)$  is a random process denoting phase noise and  $v_0$  and  $f_0$  are the nominal signal amplitude and frequency respectively.

Let us consider the following two-state model describing the clock errors [6]:

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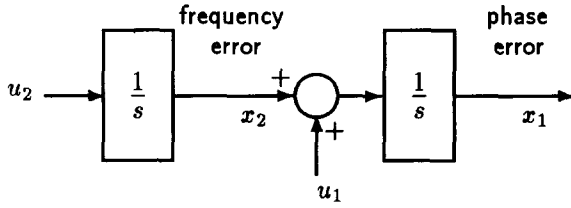


Figure 1: Two-state model describing clock errors. The independent white noise inputs  $u_1$  and  $u_2$  have spectral amplitudes  $S_f$  and  $S_g$ , respectively.

The white noise spectral amplitudes  $S_f$  and  $S_g$  are determined from the typical Allan variance parameters for an oscillator. For a typical crystal oscillator we will consider the following Allan variance parameters[6]:

$$h_0 = 9.43 \times 10^{-20} \text{ seg} \quad ; \quad h_{-1} = 1.8 \times 10^{-19} \\ h_{-2} = 3.8 \times 10^{-21} \text{ seg}^{-1}$$

With a discretization step size  $\Delta t$  equal to the clock nominal period  $T_0$ :

$$\Delta t = T_0 = \frac{2\pi}{\omega_0} = 1.25 \times 10^{-4} \text{ seg} \quad (1)$$

we have that the state transition matrix  $\mathbf{F}_k$  and the process noise covariance  $\mathbf{Q}_k$  are given by

$$\mathbf{F} = \begin{bmatrix} 1 & 1.25 \times 10^{-4} \text{ seg} \\ 0 & 1 \end{bmatrix} \quad (2)$$

$$\mathbf{Q} = \begin{bmatrix} 6.9394 \times 10^{-14} \text{ rad}^2 & 6.8997 \times 10^{-18} \frac{\text{rad}^2}{\text{seg}} \\ 6.8997 \times 10^{-18} \frac{\text{rad}^2}{\text{seg}} & 1.1039 \times 10^{-13} \frac{\text{rad}^2}{\text{seg}^2} \end{bmatrix} \quad (3)$$

### 3. PLL WITH KALMAN PREDICTOR

Let us consider a master-slave network and a node from the first level. The node receives the reference signal from the master clock

$$A \sin[\omega_0 t] + N(t) \quad (4)$$

where  $N(t)$  is the additive white Gaussian noise in the channel. Also, the node has its own local clock whose stability can be described by the two-state model mentioned earlier. We assume that the local clock is implemented in analog form as a voltage controlled oscillator (VCO).

The VCO output signal is given by

$$\sin[\omega_0 t + C_0 \int_0^t u(\tau) d\tau - \phi(t)]$$

where  $u(t)$  is the VCO control input signal in Volts,  $C_0$  is the VCO constant in rad/(volts.seg),  $\omega_0 = 2\pi/T_0$  is the oscillator nominal frequency and  $\phi(t)$  is the phase error process of the oscillator. The block diagram of the PLL to be considered is given as follows.

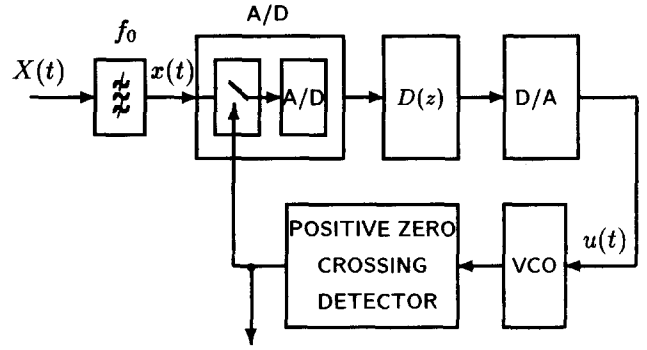


Figure 2: Block diagram of the Phase-Locked Loop.

The corresponding mathematical model is given in Figure 3, where  $A$  is the amplitude of the received signal,  $C_1$  [1/volts] is the A/D converter constant and  $C_2$  [volts] is the D/A converter constant.

It can be shown that in order to minimize the mean square phase error of the local clock  $e_\phi(k)$ , the blocks characterized by  $C_1$ ,  $D(z)$ ,  $C_2$  and  $C(z)$  must behave as a Kalman predictor[5].

As a consequence, by using Kalman's estimation theory concepts and the two-state model for the clock errors, we can find that the optimal loop filter  $D(z)$  in the steady state is

$$D(z) = G_1 + \frac{G_2}{1 - z^{-1}}$$

where

$$G_1 = \frac{K^1}{T_0 C_0 C_1 C_2} \quad ; \quad G_2 = \frac{K^2 \Delta t}{T_0 C_0 C_1 C_2}$$

and where  $K^1$  and  $K^2$  are the Kalman gains in the steady state.

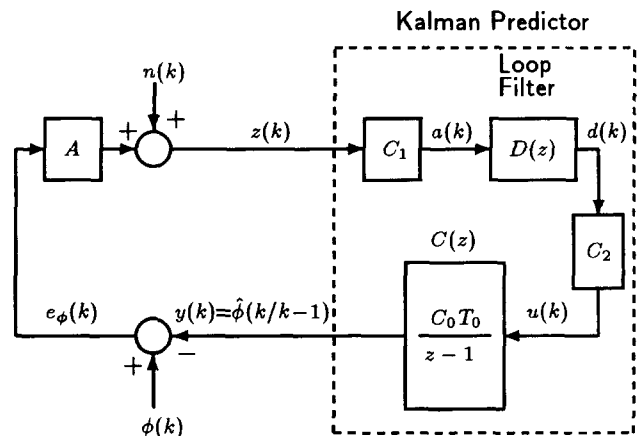


Figure 3: Kalman predictor "hidden" in the loop.

In Figure 4 we show simulation results for the predictor using the values of  $\Delta t$ ,  $T_0$ ,  $\mathbf{F}$  and  $\mathbf{Q}$  from eqs. (1),

(2) and (3), with  $A = 1$  Volt/rad and where  $R = 10^{-10}$  (Volt)<sup>2</sup> is the observation noise variance. One can observe the good performance of the predictor from this graph.

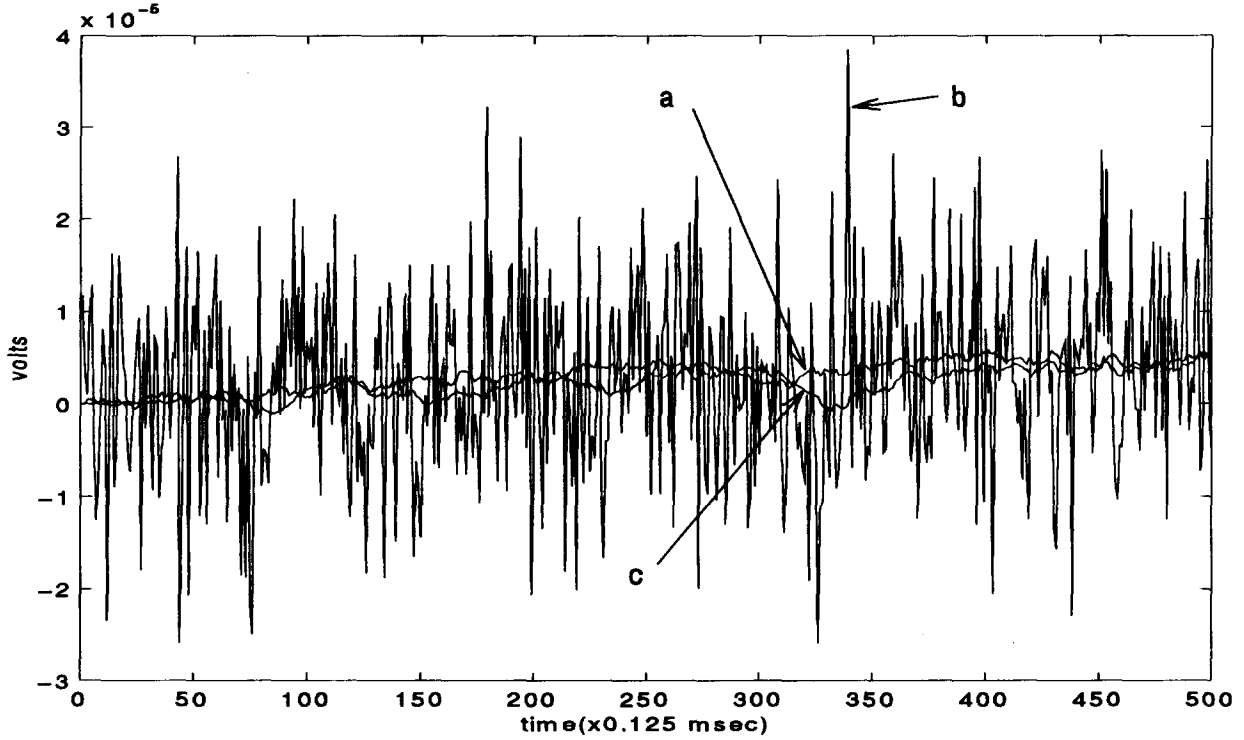


Figure 4: Phase error process:  $A\phi(k)$  (a), Observation:  $A(\phi(k) - \hat{\phi}(k/k-1)) + n(k)$  (b), Prediction:  $A\hat{\phi}(k/k-1)$  (c).

One problem associated with master-slave networks is the possible loss of the master timing signal. This problem can be partially corrected by designating alternate master clocks or running the slave clock independently, until the master signal can be restored. In most cases, the latter approach requires a very accurate, and therefore expensive, back-up clock at each node. In this paper we develop an alternative technique that consists in compensating the clock error with an “ $n + 1$  steps” optimal predictor by using the two-state clock errors model.

Let us consider the loss of the master timing signal starting from  $k = i$ . For the  $n + 1$  steps optimal prediction of the phase error  $\hat{\phi}(i + n/i - 1)$ ,  $n = 1, 2, \dots$ , the phase error variance is given by

$$E[(x_{i+n}^1 - \hat{x}_{i+n/i-1}^1)^2] = E[(x_i^1 - \hat{x}_{i/i-1}^1)^2] + (n\Delta t)^2 E[(x_i^2 - \hat{x}_{i/i-1}^2)^2] + 2(n\Delta t)E[(x_i^1 - \hat{x}_{i/i-1}^1)(x_i^2 - \hat{x}_{i/i-1}^2)] +$$

$$+(n\Delta t)S_f + \frac{1}{3}S_g(n\Delta t)^3,$$

where we have used the components of the covariance matrix for the prediction errors.

For the other two techniques (free run and holdover) the phase error variances are given by

$$E[(x_{i+n}^1 - \hat{x}_{free}^1)^2] = E[(x_i^1 - \hat{x}_{i/i-1}^1)^2] + (n\Delta t)^2 E[(x_i^2)^2] + 2(n\Delta t)E[(x_i^1 - \hat{x}_{i/i-1}^1)(x_i^2 - \hat{x}_{i/i-1}^2)] + (n\Delta t)S_f + \frac{1}{3}S_g(n\Delta t)^3$$

and

$$E[(x_{i+n}^1 - \hat{x}_{holdover}^1)^2] = E[(x_i^1 - \hat{x}_{i/i-1}^1)^2] + (n\Delta t)^2 E[(x_i^2 - \hat{x}_{i/i-1}^2)^2] + 2(n\Delta t)E[(x_i^1 - \hat{x}_{i/i-1}^1)(x_i^2 - \hat{x}_{i/i-1}^2)] + (n\Delta t)S_f + \frac{1}{3}S_g(n\Delta t)^3 + n^2[(K_{i-1}^1)^2 A^2 E[(x_{i-1}^1 - \hat{x}_{i-1/i-2}^1)^2] + (K_{i-1}^1)^2 E[n_{i-1}^2]]$$

respectively.

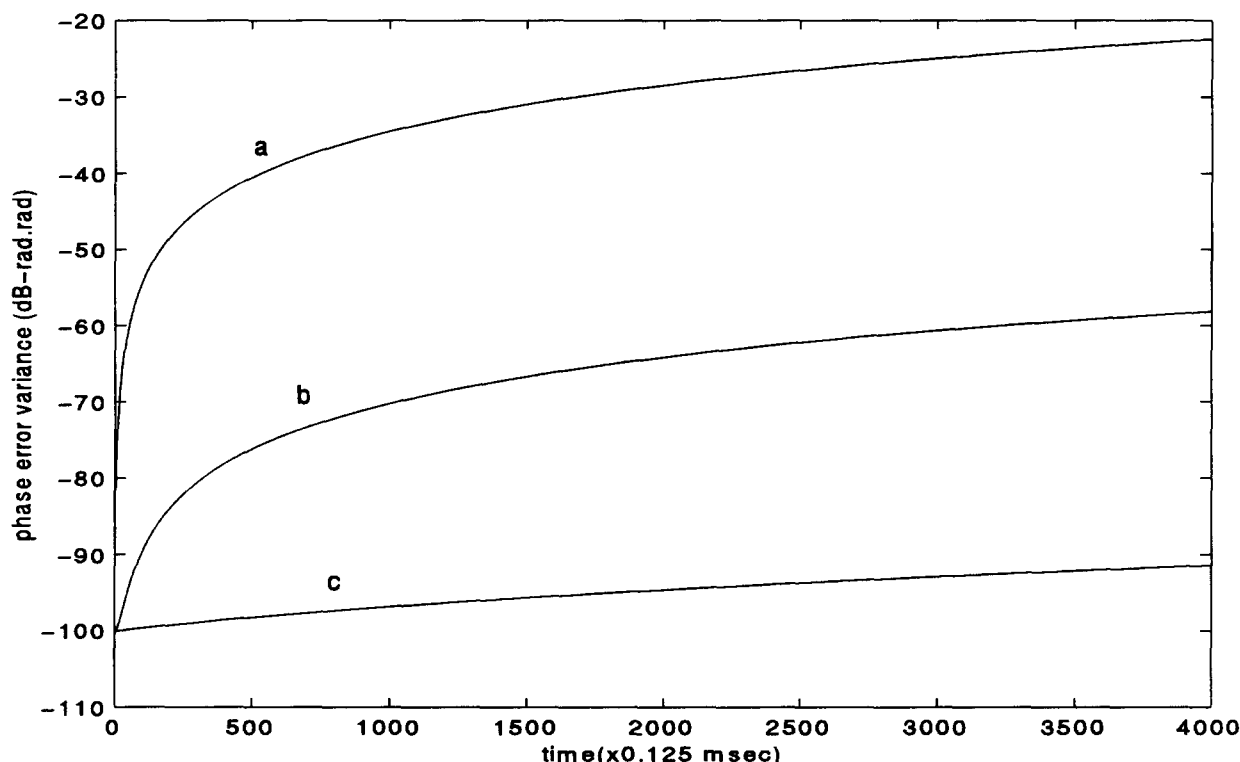


Figure 5: Reference lost after 10 months: Free run (a), Holdover (b),  $n + 1$  steps optimal prediction (c)

In Figure 5 we show the results for the different approaches: (a) Free run, (b) Holdover and (c)  $n+1$  steps optimal prediction. It is clear that by using this last technique the phase error variance grows very slowly with time compared to the other two methods.

#### 4. ACKNOWLEDGMENT

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## References

- [1] Lindsey, W. C.; F. Ghazvinian; W. C. Hagmann and K. Dessouky, "Network Synchronization", Proc. IEEE, 73, 10, 151-174 (1985).
- [2] Gardner, F. M., *Phaselock Techniques*, Wiley-Interscience, New York (1979).
- [3] Vanier, J. and M. Têtu, "Phase-Locked Loops Used with Masers: Atomic Frequency Standards", IEEE Trans. Commun., 30, 10, 2355-2361 (1982).
- [4] Wolaver, D. H., *Phase-Locked Loop Circuit Design*, Prentice-Hall Inc., New Jersey (1991).
- [5] Gustavo A. Hirchoren, "Otimização de Sincronismo em Redes Digitais Síncronas com Preditor de Kalman", M.Sc. Dissertation, DECOM-FEEC-UNICAMP, September, 1996 (in Portuguese).
- [6] Brown, R. G. and P. Y. C. Hwang, *Introduction to random signals and applied Kalman filtering*, John Wiley & Sons, Inc (1992).