A MODIFIED NORMALIZED LATTICE ADAPTIVE FILTER FOR FAST SAMPLING

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ABSTRACT

Most filters, adaptive or not, formulated using the delay operator, have no limit when sampling becomes fast and therefore they will have numerical problems. We will show that one reason that the normalized lattice filter has less numerical problems is because that it has a limit as the sampling period tends to zero. The transfer function in the s- domain obtained as a limit of the normalized lattice filter will, however, have only every other power in the denominator polynomial. We propose a modified normalized lattice filter that can realize any arbitrary transfer function in the discrete (z) domain and its order-recursive limit as the sampling period tends to zero can realize any arbitrary transfer function in the s-domain. Various stability properties of the new lattice are also studied.

1. INTRODUCTION

As the demand arises for faster information transmission, fast sampled processes and systems are becoming a necessity. The delay operator, i.e. $qx(t) = x(t+1)$, based methods often yield ill-conditioned processes and systems for fast sampling. This has led to a recent interest in δ-operator based algorithms [1]-[2], where $\delta = \frac{(q-1)}{\Delta}$, which converge to their corresponding continuous time counterparts for a small sampling period Δ.

In this paper, we consider the normalized lattice filter. It has many interesting properties such as that the structure is inherently limited to realizing stable discrete-time transfer functions, that the filter is more robust to finite precision errors [5] and that conditions for its time-varying stability can be established [6], [7], making it suitable for adaptive applications. Up to date, the only work on delta lattices and their limiting behaviour as the sampling rate increases is [3]. In [3], the two multipiler FIR lattice is considered and by using an alternative formulation based on the delta operator, it is shown that the filter converges to the underlying continuous time lattice structure, as the sampling period approaches zero. The limiting continuous time lattice structure has, however, continuous "stages" along the time axis and is not order recursive in form. Here "order" refers to the order of differentiation.

In this paper, we will develop a new normalized lattice structure which will have an order-recursive continuous-time limit which can realize any arbitrary all-pole transfer function in the usual s-domain. The limiting continuous time order recursive structure obtained in this paper is linked to a structure derived from [4]. We will investigate the normalized lattice IIR filter in this regard. The behaviour of the filter with a fast sampling period as well as its limit are investigated. Stability issues, both time-invariant and time varying, are further examined for the discrete time filter and the limiting continuous time structure.

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2. NORMALIZED LATTICE IIR FILTER AND ITS LIMIT

The order recursions for the normalized lattice filter are given by

$$\begin{aligned} \bar{f}_n(k) &= \cos\theta_{n+1}\bar{f}_{n+1}(k) + \sin\theta_{n+1}\bar{b}_n(k-1) \\ \bar{b}_{n+1}(k) &= \cos\theta_{n+1}\bar{b}_n(k-1) - \sin\theta_{n+1}\bar{f}_{n+1}(k), \quad (1) \end{aligned}$$

Let us now study the normalized lattice filter as the sampling period becomes infinitesimal. To accommodate time-varying situations, we now let all parameters be time-varying. From the equation for the forward prediction error in (1) with time-varying parameters $\cos\theta_{n+1}(k)$ and $\Gamma_{n+1}(k) = \sin\theta_{n+1}(k)$, we thus have

$$\bar{b}_n(k-1) = \frac{1}{\Gamma_{n+1}(k)}[\bar{f}_n(k) - \cos\theta_{n+1}(k)\bar{f}_{n+1}(k)] \quad (2)$$

Replacing $(n+1)$ by n in the equation for the backward prediction error in (1) and using (2), after some algebraic manipulations, we obtain

$$\begin{aligned} &\frac{1}{\Gamma_{n+1}(k+1)}[\bar{f}_n(k+1) - \cos\theta_{n+1}(k+1)\bar{f}_{n+1}(k+1)] \\ &= \frac{(1-\Gamma_n^2(k))}{\cos\theta_n(k)}\frac{1}{\Gamma_n(k)}[\bar{f}_{n-1}(k) - \cos\theta_n(k)\bar{f}_n(k)] \\ &-\Gamma_n(k)\bar{f}_n(k) \end{aligned}$$

which gives

$$\begin{aligned} \bar{f}_{n+1}(k+1) = &\frac{[\bar{f}_n(k+1) + \frac{\Gamma_{n+1}(k+1)\bar{f}_n(k)}{\Gamma_n(k)}]}{\Delta} \\ &\cdot\frac{\Delta}{\cos\theta_{n+1}(k+1)} - \frac{\Gamma_{n+1}(k+1)}{\Gamma_n(k)}\frac{(1-\Gamma_n^2(k))}{\Delta^2}\frac{\Delta}{\cos\theta_n(k)} \\ &\cdot\frac{\Delta}{\cos\theta_{n+1}(k+1)}\bar{f}_{n-1}(k) \quad (3) \end{aligned}$$

This is a re-arrangement of (1) in terms of only the forward prediction errors, which will be shown to have a continuous-time limit shortly.

When the sampling period becomes infinitesimal, the following limiting condition holds [8] :

$$\lim_{\Delta\to 0}\Gamma_n = (-1)^{n-1}, \ \lim_{\Delta\to 0}\frac{1-\Gamma_n^2}{\Delta^2} = \gamma_n, \ n \le N-1 \quad (4)$$

where γ_n is the ratio of the n-th order forward prediction error energy to the $(n-1)$-th order forward prediction error energy at $t = 0$ [4]. Now let us look back at equation (3). The first term on the right hand side of (3) can realize a derivative operation when the sampling period tends to zero. In fact, from (3), using the limiting conditions (4) with time-varying parameters, we obtain

$$\begin{aligned}\lim_{\Delta\to 0}\bar{f}_{n+1}(k+1) &= \lim_{\Delta\to 0}\frac{1}{\sqrt{\gamma_{n+1}(k+1)}} \cdot\frac{\bar{f}_n(k+1)-\bar{f}_n(k)}{\Delta} \\ &+ \lim_{\Delta\to 0}\gamma_n(k)\frac{1}{\sqrt{\gamma_n(k)}}\frac{1}{\sqrt{\gamma_{n+1}(k+1)}} \cdot\bar{f}_{n-1}(k), \quad n < N-2.\end{aligned}$$

Therefore,

$$\bar{f}_{n+1}(t) = \frac{1}{\sqrt{\gamma_{n+1}(t)}}\frac{d}{dt}\bar{f}_n(t)+\frac{\sqrt{\gamma_n(t)}}{\sqrt{\gamma_{n+1}(t)}}\bar{f}_{n-1}(t), \quad n < N-2. \quad (5)$$

where $\bar{f}_{n+1}(t) = \lim_{\Delta\to 0}\bar{f}_{n+1}(k)$. Equation (5) is similar to Pham-LeBreton like order recursion [4],[9].

Now for the first stage of the normalized lattice filter, $n = N$ and the limiting condition [8] is

$$\lim_{\Delta\to 0}\frac{(1-\Gamma_N^2(k))}{\Delta} = 2\xi_1(t),$$

where ξ_1 is the first coefficient of the N-th order parameter vector $\xi = [1, \ \xi_1, \ \xi_2, \ \cdots, \ \xi_N]^T$ of the corresponding continuous-time AR model [8]. Thus the limiting structure for the first stage of the normalized lattice is

$$\bar{\bar{f}}_N(t) = \frac{1}{\sqrt{2\xi_1(t)}}\frac{d}{dt}\bar{f}_{N-1}(t) + \frac{\sqrt{\gamma_{N-1}(t)}}{\sqrt{2\xi_1(t)}}\bar{f}_{N-2}(t) \quad (6)$$

where $\bar{\bar{f}}_N(k) \triangleq \frac{\bar{f}_N(k)}{\sqrt{\Delta}}$ and $\bar{\bar{f}}_N(t) = \lim_{\Delta\to 0}\bar{\bar{f}}_N(k)$. We see from above that $\bar{\bar{f}}_N(k)$, though obtained from $\bar{f}_N(k)$ by dividing by $\sqrt{\Delta}$, has a continuous-time limit given above as the sampling period goes to zero. The division by $\sqrt{\Delta}$ arises from the transition from the discrete-time domain to the continuous-time domain.

For the last stage of the normalized lattice filter, the normalized lattice recursions (1) hold with $n = 0$ and $\bar{f}_0(k) = \bar{b}_0(k)$. After some manipulations, using the limiting conditions described above, the limiting continuous time structure for the last stage is

$$\bar{f}_1(t) = \frac{1}{\sqrt{\gamma_1(t)}}\frac{d}{dt}\bar{f}_0(t) \quad (7)$$

Combining the above, the limiting variable $\bar{\bar{f}}_N(t)$ can be expressed in terms of $\bar{f}_0(t)$ is

$$\begin{aligned}\bar{\bar{f}}_N(t) &= \frac{1}{\sqrt{2\xi_1\prod_{j=1}^{N-1}\gamma_j}}[\frac{d^N}{dt^N}\bar{f}_0(t) + (\sum_{i=1}^{N-1}\gamma_i)\frac{d^{N-2}}{dt^{N-2}}\bar{f}_0(t) \\ &+ (\sum_{i=3}^{N-1}\gamma_i\sum_{j=1}^{i-2}\gamma_j)\frac{d^{N-4}}{dt^{N-4}}\bar{f}_0(t) + \cdots\cdots \\ &+ (\sum_{i=2M-1}^{N-1}\sum_{j}^{i-2}\sum_{k}^{j-2}\cdots\sum_{q=1}^{p-2}\gamma_i\gamma_j\gamma_k\cdots\gamma_p\gamma_q)\cdot \\ &\cdot\frac{d^{N-2M}}{dt^{N-2M}}\bar{f}_0(t)] \end{aligned} \quad (8)$$

where $N = 2M+1$ if N is odd and is equal to $2M$ if N is even. Refer to [9] for details.

It is seen that the polynomial $\bar{\bar{f}}_N(t)$ realized has only every other derivative of $\bar{f}_0(t)$. Thus there may be stability problems if this polynomial realizes an all-pole transfer function.

3. THE MODIFIED NORMALIZED LATTICE FILTER AND ITS LIMIT

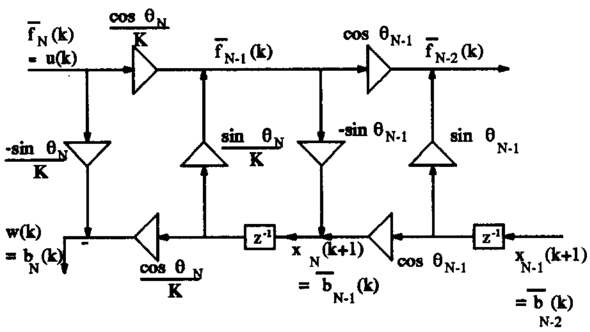

Figure 1. Modified normalized lattice filter; the parameters are time-varying.

To overcome the above problem, the approach we consider is to modify the first stage of the normalized lattice filter. Let the order recursions for the first stage of the modified normalized lattice filter be

$$\bar{f}_{N-1}(k) = \frac{\cos\theta_N(k)\bar{f}_N(k) + \sin\theta_N(k)\bar{b}_{N-1}(k-1)}{K(k)} \quad (9)$$

$$\bar{b}_N(k) = \frac{\cos\theta_N(k)\bar{b}_{N-1}(k-1) - \sin\theta_N(k)\bar{f}_N(k)}{K(k)}, \quad (10)$$

where $K(k)$ is constrained by

$$\lim_{\Delta\to 0}\frac{[K(k)-1]}{\Delta} = m(t), \quad 0 < |m(t)| < \infty,$$

$\cdots\cdots$ *Condition C1*

Equation (1) for the order recursions of the normalized lattice filter is used for stages 2 to N of this modified filter.

That is, (1) is valid for $n = N-2, N-3, \cdots, 1, 0$. Thus for the second stage, the lattice equations (1) hold with $n = N-2$. The first two stages of this modified filter is shown in Figure 1.

The modified normalized lattice filter is now analyzed in detail. For simplicity we drop the time dependence of all time-varying parameters with the understanding that they are all time-varying. From equations (9) and (10), the modified normalized lattice filter is obtained in the FIR form as

$$\begin{aligned} \bar{f}_N(k) &= \frac{K\bar{f}_{N-1}(k)}{\cos\theta_N} + \frac{\sin\theta_N}{\cos\theta_N}\bar{b}_{N-1}(k-1) \\ \bar{b}_N(k) &= \frac{\bar{b}_{N-1}(k-1)}{K\cos\theta_N} + \frac{\sin\theta_N}{\cos\theta_N}\bar{f}_{N-1}(k) \end{aligned} \tag{11}$$

Let $\bar{f}_{N-1}(k)$ and $\bar{b}_{N-1}(k)$ be expressed in terms of $\bar{f}_0(k)$ and its delayed versions as

$$\begin{aligned} \bar{f}_{N-1}(k) &= \sum_{i=0}^{N-1} \beta_i \bar{f}_0(k-i) \\ \bar{b}_{N-1}(k) &= \sum_{i=0}^{N-1} \beta_{N-1-i} \bar{f}_0(k-i) \end{aligned}$$

i.e. we have assumed that the coefficients in the expressions for $\bar{f}_{N-1}(k)$ and $\bar{b}_{N-1}(k)$ are reversed of each other. This assumption is valid since (1) is valid for $n \leq N-2$, for which such reversal relationship is classical [5]. Substituting the above two equations into (11), we obtain

$$\begin{aligned} \bar{f}_N(k) &= \frac{K\sum_{i=0}^{N-1}\beta_i\bar{f}_0(k-i)}{\cos\theta_N} \\ &+ \frac{\sin\theta_N}{\cos\theta_N}\Big(\sum_{i=0}^{N-1}\beta_{N-1-i}\bar{f}_0(k-i-1)\Big) \\ &\triangleq \sum_{i=0}^{N} d_i \bar{f}_0(k-i) \\ \bar{b}_N(k) &= \frac{\sin\theta_N\sum_{i=0}^{N-1}\beta_i\bar{f}_0(k-i)}{\cos\theta_N} \\ &+ \frac{\sum_{i=0}^{N-1}\beta_{N-1-i}\bar{f}_0(k-i-1)}{K\cos\theta_N} \triangleq \sum_{i=0}^{N}\hat{d}_i\bar{f}_0(k-i) \end{aligned}$$

From the above equations, it is clear that the coefficients in the expressions for $\bar{f}_N(k)$ and $\bar{b}_N(k)$ are not reversed of each other. In fact, the coefficients in the expressions for $\bar{f}_N(k)$ and $\bar{b}_N(k)$ are related as follows.

$$\begin{aligned} d_N &= \frac{\beta_0\sin\theta_N}{\cos\theta_N} = \hat{d}_0 \\ d_0 &= \frac{K\beta_0}{\cos\theta_N}, \quad \hat{d}_N = \frac{\beta_0}{K\cos\theta_N} \\ d_i = \frac{K\beta_i + \beta_{N-i}\sin\theta_N}{\cos\theta_N}, &\qquad \hat{d}_i = \frac{\beta_i\sin\theta_N + \frac{\beta_{N-i}}{K}}{\cos\theta_N}, \\ i &= 1, 2, \cdots, N-1 \end{aligned}$$

The reflection coefficient for the first stage is defined as

$$\Gamma_N \triangleq \frac{d_N}{d_0} = \frac{\sin\theta_N}{K} \tag{12}$$

With this definition, the limiting conditions in Section 2 still hold. It is to be noted that $\bar{f}_N(k)$ is the optimal N-th order forward prediction error but $\bar{b}_N(k)$ is not the optimal N-th order backward prediction error. It can be shown that the reflection coefficient Γ_N is still given by $\Gamma_N = \frac{E\{\bar{f}_{N-1}(k)\bar{b}_{N-1}(k-1)\}}{E\{\bar{f}^2_{N-1}(k)\}}$.

Due to the modification and the above change in the relationship between Γ_N and $\sin\theta_N$, a question naturally arises as whether the modified normalized lattice filter can realize an arbitrary all-pole (or all-zero) transfer function. We now give below, in algorithmic form, how to realize an arbitrary all-pole transfer function in the discrete-time using this new lattice structure. Let $D_N(z^{-1}) \triangleq \sum_{i=0}^{N} d_i z^{-i}$ and $\hat{D}_N(z^{-1}) \triangleq \sum_{i=0}^{N} \hat{d}_i z^{-i}$.

Algorithm:
Given transfer function $= \frac{1}{A(z^{-1})}$, where $A(z^{-1}) = \sum_{i=0}^{N} a_i z^{-i}$ is arbitrary with $a_0 \neq 0$.
Initialization: Calculate $D_N(z^{-1})$ by substituting

$$d_i = a_i$$

Calculation of Reflection Coefficient: The reflection coefficient Γ_N is then computed as

$$\Gamma_N = \frac{\sin\theta_N}{K} = \frac{a_N}{a_0}, \ \cos\theta_N = \sqrt{1-\sin^2\theta_N}$$

Computation of $\hat{D}_N(z^{-1})$: First compute

$$\hat{d}_0 = d_N = a_N, \ \ \hat{d}_N = \frac{d_0}{K^2} = \frac{a_0}{K^2}$$

Next, solve the following set of equations for β_i

$$\frac{K\beta_i + \beta_{N-i}\sin\theta_N}{\cos\theta_N} = d_i = a_i, \ i = 1, 2, \cdots, N-1 \tag{13}$$

We can choose K as $K = 1 + m\Delta$. Then, the β_i's calculated are used to compute

$$\hat{d}_i = \frac{\beta_i\sin\theta_N + \frac{\beta_{N-i}}{K}}{\cos\theta_N}$$

Then form $\hat{D}_N(z^{-1}) = \sum_{i=0}^{n} \hat{d}_i z^{-i}$.

Once $D_{N-1}(z^{-1})$ and $\hat{D}_{N-1}(z^{-1})$ have been computed, the algorithm follows the same procedure as for the regular normalized lattice filter as in [6]. With the definition of Γ_N in (12), $D_{N-1}(z^{-1})$ and $\hat{D}_{N-1}(z^{-1})$ will have a degree reduction of one from $D_N(z^{-1})$ and $\hat{D}_N(z^{-1})$. Also, the coefficients in the polynomials $D_i(z^{-1})$ and $\hat{D}_i(z^{-1})$, for $i = 1, 2, \cdots, N-1$, will be reversed of each other. Therefore $\frac{1}{A(z^{-1})}$ is realized by this modified normalized lattice filter. It can be shown that equation (13) will have an unique solution if

$$K^2 > 1, \ |\sin\theta_N| < 1.$$

We now proceed to manipulate equations (9), (10) and the lattice equations for the second stage to obtain a limiting structure for the first stage of the normalized lattice filter, since all other stages will still have the same continuous-time limit as the conventional normalized lattice filter analyzed in Section 2. Similar to the approach in Section 2, we perform manipulations, similar to (1) to (3), on (9) and (1). After some calculations, this gives

$$\begin{aligned}\frac{\bar{f}_N(k+1)}{\sin\theta_N(k+1)} &= \frac{\bar{f}_{N-1}(k+1)}{\sin\theta_N(k+1)\cos\theta_N(k+1)} \\ &+ \frac{(K(k+1)-1)}{\sin\theta_N(k+1)\cos\theta_N(k+1)}\bar{f}_{N-1}(k+1) \\ &+ \frac{\sin\theta_{N-1}(k)}{\cos\theta_N(k+1)}\bar{f}_{N-1}(k) \\ &+ \frac{(1-\Gamma^2_{N-1}(k))}{\sin\theta_{N-1}(k)\cos\theta_N(k+1)}\bar{f}_{N-1}(k) \\ &- \frac{\cos\theta_{N-1}(k)}{\sin\theta_{N-1}(k)\cos\theta_N(k+1)}\bar{f}_{N-2}(k) \quad (14)\end{aligned}$$

With the definition of the reflection coefficient in (12), it can be shown [9] that the following limiting conditions hold

$$\begin{aligned}\lim_{\Delta\to 0}\sin\theta_N(k) &= (-1)^{(N-1)} \\ \lim_{\Delta\to 0}\frac{1-\sin^2\theta_N(k)}{\Delta} &= 2(\xi_1(t)-m(t))\end{aligned}$$

where $m(t)$ is defined in Condition $C1$. Then the limit of (14) is

$$\begin{aligned}\bar{\bar{f}}_N(t) &= \frac{1}{\sqrt{2\xi_1(t)-2m(t)}}\frac{d}{dt}\bar{f}_{N-1}(t) \\ &+ \frac{\sqrt{\gamma_{N-1}(t)}}{\sqrt{2\xi_1(t)-2m(t)}}\bar{f}_{N-2}(t) \\ &+ \frac{m(t)}{\sqrt{2\xi_1(t)-2m(t)}}\bar{f}_{N-1}(t) \quad (15)\end{aligned}$$

Compared with (6), (15) has an additional term which is the key in realizing an arbitrary all-pole transfer function. A similar recursion for this stage has been seen in [4]. If one forms the polynomial $\bar{\bar{f}}_N(t)$ similar to (8), it will be seen that the polynomial has every derivative of $\bar{f}_0(t)$, unlike (8). An important issue is how one goes about choosing $K(k)$. This has been discussed in [9].

4. STABILITY ISSUES

Time-varying and time-invariant stabilities of the modified normalized lattice filter and the limiting continuous-time structure have been investigated in detail in [9]. Due to limitations of space, the results are only presented here. The modified normalized lattice filter is represented in a state-space representation and the following result holds. *Theorem:*

The time-varying modified normalized lattice filter is BIBO stable in the discrete-time provided that the following holds :

$$\begin{aligned}K^2(k) &> 1 \text{ for all } k, \\ |\sin\theta_n(k)| &< 1 \text{ for all } k \text{ and } n = 1,2,\cdots,N \quad (16)\end{aligned}$$

Next, the stability of the limiting continuous-time structure is investigated. Consider the conditions

$$\begin{aligned}0 &< \bar{U} \le m(t) \le \bar{V} < \infty,\ \ 0 < U \le \bar{\xi}_1(t) \le V < \infty, \\ 0 &< \bar{\bar{U}}_i \le \gamma_i(t) \le \bar{\bar{V}}_i,\ \ i = 1,2,\cdots,N-1,\ \ \text{for all } t. \quad (17)\end{aligned}$$

where all U's and V's are constants. For time-invariant parameters, (17) are exactly the conditions for BIBO stability of the limiting continuous-time structure. For the time-varying case, it can be shown that (17) guarantees Lyapunov stability of the limiting continuous-time structure which means that the zero input recursion of the time-varying limiting continuous-time structure obtained from the modified normalized lattice filter could never go unstable. It can be shown that condition (17) gives time-varying BIBO stability of the second-order limiting continuous-time structure. It is seen that conditions (16) and (17) are equivalent in the light of the limiting conditions (4).

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