SUBBAND ADAPTIVE FILTERING WITH TIME-VARYING NONUNIFORM FILTER BANKS

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ABSTRACT

A technique is presented for subband adaptive filtering with nonuniform filter banks. The bandwidth allocations of the subband analysis and synthesis filters are adapted to the spectral characteristics of the input data in such a manner as to minimize an objective function built from the subband error powers. The nonuniform filter bank structure allows for fast convergence times for high order systems with a reduced mean square error relative to the uniform subband scheme. Results are presented for the case of a nonstationary system with time-varying spectral characteristics.

1. INTRODUCTION

The use of filter banks to decompose high order adaptive filters into several lower order parallel filters has attracted attention in the last several years. This process, depicted in Figure 1, allows fast convergence relative to the fullband filtering scheme and is computationally efficient when implemented in a parallel fashion. The chief drawback to the subband scheme is that the overall mean square error (MSE) is often several orders of magnitude higher than that achieved by the fullband system.

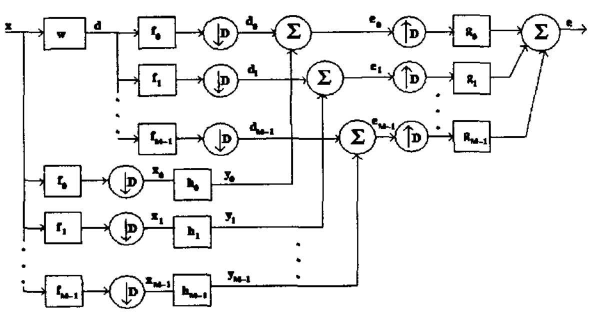

Figure 1. Subband adaptive filtering configuration for system-identification.

This work supported in part by TRW under Contract No. CU12955.55.1665B.

This excess MSE is a result of the large eigenvalue spread in the subband input correlation matrices. The decimation process creates spectral nulls in the input power spectral density. These nulls are related to small eigenvalues in the subband correlation matrices through the asymptotic equivalence of a wide sense stationary processes' correlation matrix eigenvalues and its power spectral density [2],[8]. The decimation rate is chosen to be small enough to avoid the necessity of adaptive cross filters as described in [3].

The need for robust system identification algorithms for high order time-varying systems leads naturally to the idea of nonuniform filter banks for subband processing. A disadvantage of the uniform architecture is that the spectral properties of the system are not exploited in the subband partitioning. Areas of the spectrum with small variations, i.e. easily modeled, are often split when one subband filter could model them with small error power. Similarly, complicated regions such as band edges or highly varying sections can be better modeled with multiple filters acting on smaller bandwidths. The options then are to employ a high number of analysis/synthesis filters to trap these complicated regions in tight subbands or to shape the filter bank in such a way as to isolate complicated regions and allocate large bandwidths to relatively simple regions. The former generally leads to a more resource intensive system while the latter can allow improved performance for a smaller increase in system resource expenditure. Such a system allows a trade-off between the desirable MSE properties of the fullband system and the computational complexity savings and adaptation speed of the uniform subband system.

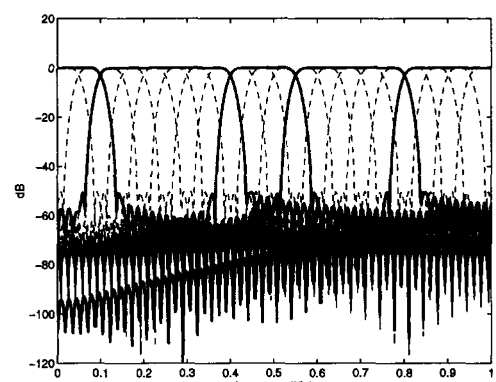

Figure 2. Example of nonuniform filter bank design for five subbands formed from 20 constituent filters with the allocation l=[2 6 3 5 4].

In [1] we presented an algorithm was presented for building a nonuniform filter bank to match the unknown system's spectral characteristics. The filter bank is adaptively evolved and hence can track the spectrum of a time varying system. In this paper, the details of the algorithm will be presented and experimental results for such a nonstationary system identification will be explored as well as those of two stationary systems.

2. A NONUNIFORM FILTER BANK ALLOCATION ALGORITHM

The bandwidth allocation incorporates rate/distortion theory in the filter bank design. It has been shown [4] that under certain conditions a nonuniform filter bank can be built by selectively merging component filters from a uniform design. If the component filters were designed properly the overall nonuniform filter bank retains the near perfect reconstruction property (NPR) . This guarantees that adjacent subbands have aliasing cancellation and the subband filters have in general a high degree of stopband rejection. Figure 2 shows an example of a five subband nonuniform filter bank formed from twenty constituent filters.

In the case of a paraunitary design, we know that the reconstructed MSE is the sum of the subband error powers and we may set up a cost function as in [3]. This is given as a function of the bandwidth allocations, l, as

$$J(l) = \sum_{I=0}^{M-1} \xi_i \tag{1}$$

were the subband error power for the i^{th} band is denoted by $\xi_i = ||e_i(k)||^2$. In the case of NPR filter banks we will still use this modified cost function but with the understanding that the equality in (1) is only approximate and is dependent on the quality of the filter bank design (a technique for designing high quality NPR uniform filter banks is presented in [7]). Application of the geometric-arithmetic mean inequality yields the result that $J(l)$ is minimized when the subband MSEs are forced to be equal.

1. *Design uniform NPR filter bank with K subbands.*
2. *Choose initial allocation to build nonuniform filter bank with $M \leq K$ subbands. Denote this allocation by $l = \{l_0, l_1, ..., l_{M-1}\}$.*
3. *Perform subband adaptation on next block of data. Estimate $\underline{\xi} = \{\xi_0, \xi_1, ..., \xi_{M-1}\}$ and $\overline{\xi} = \sum_{I=0}^{M-1} \xi_i$.*
4. *Find $\{j\}$ such that $|\underline{\xi}_j - \overline{\xi}| > std(\underline{\xi})$.*
5. *If $|\{j\}| = 0$ go to step 2.*
6. *Find $i = arg\max_{i\in\{j\}, l_i > a_{min}} |\underline{\xi}_j - \overline{\xi}|$. Set $a = arg\max_{a>a_min}\{\xi_{i-1}, \xi_{i+1}\}$ and $b = arg\min_{b<a_max}\{\xi_{i-1}, \xi_{i+1}\}$.*
7. *If $\xi_i > \overline{\xi}$, if $a = \emptyset$ set $\{j\} = \{j\} - i$ and go to step 6, else $l_i = l_i - 1, l_b = l_b + 1$. If $\xi_i < \overline{\xi}$, if $b = \emptyset$ set $\{j\} = \{j\} - i$ and go to step 6, else $l_i = l_i + 1, l_a = l_a - 1$.*
8. *Go to step 3.*

Figure 3. Bandwidth Allocation algorithm.

The bandwidth partitioning algorithm is shown in figure 3. The subband MSEs, ξ_k, for an initial allocation are computed and used to determine the next partition. The bands in which the MSE falls more than a standard deviation from the mean are first determined. The indices of the subbands for which this condition is met are stored in $\{j\}$ and the distance of each $\xi_k : k \in \{j\}$ from the mean, in the order of decreasing distance. For each i,$1 \leq i \leq |\{j\}|$, if $\xi_i > \overline{\xi}$ and the current allocation for subband i is greater then the maximum allowable then a constituent filter is removed from the allocation and is added to the allocation of the adjacent subband with the smallest MSE with an allocation smaller than the maximum allowable. If $\xi_i < \overline{\xi}$ the partitioning works in the other direction. As soon as there has been a successful allocation change the search is terminated and another block of data is processed. If all of the members of $\{j\}$ are searched without meeting the conditions the partition remains unchanged.

The limits on the size of each allocation are enforced in order to allow a constant decimation rate which avoids aliasing in any of the subbands. If the lower limits were removed the system can act to "reduce rank" by effectively removing one of the components of the filter bank. In all the simulations presented in this paper there were forty constituent filters acting on 5 subbands with allocation limits of 4 and 12 and a decimation rate of 3/4. The filter bank was implemented through cosine modulation as described in [6].

3. EXPERIMENTAL RESULTS

3.1. Stationary Systems

Figures 4-5 show simulation results for two different unknown systems. The nonuniform system is compared with a uniform subband filtering setup (also using cosine-modulated NPR filters) and a fullband system. In each case it can be seen that the nonuniform system exhibits lower overall error power than the uniform allocation scheme with a faster convergence rate than the fullband adaptive filter. The bandwidth allocations are plotted in figure 6. Each system and model used 512 FIR coefficients and the input in each case was unit variance white noise. The normalized least-mean squares (NLMS) algorithm [5] was used to build the subband models. The initial allocations were chosen by running the nonuniform structure with all 40 constituent filters for one data block and using the subband MSEs to heuristically form a nonuniform allocation.. In practice this process could be replaced by the incorporation of some a priori knowledge of the system structure or simply a uniform initial allocation. The allocation convergence for each system is shown in figure 6.

3.2. Time-Varying System

Figure 7 demonstrates the convergence of the nonuniform algorithm with a slowly time-varying system. The system was varied from a bandpass to a bandstop filter in a linear fashion. It is clear that the performance advantages of the nonuniform system are not affected by this choice of a nonstationary system. The adaptation speed advantage that both of the subband systems have over the fullband filter can be seen at the beginning of the first data block.

4. CONCLUSIONS

An algorithm has been presented to allocate bandwidths to a nonuniform filter bank in such a way as to reduce the overall MSE. The performance of this system has been presented for two stationary systems and for a time-varying system. It is apparent that the performance of this system

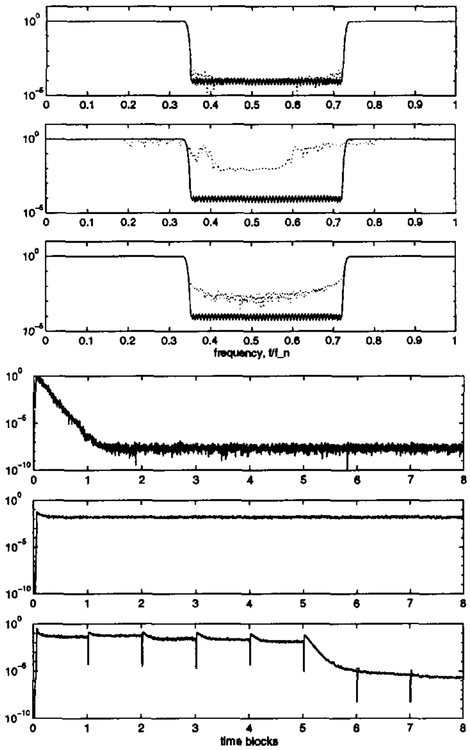

Figure 4. Filters and squared error for fullband, uniform, and nonuniform subband systems modeling a bandstop system .

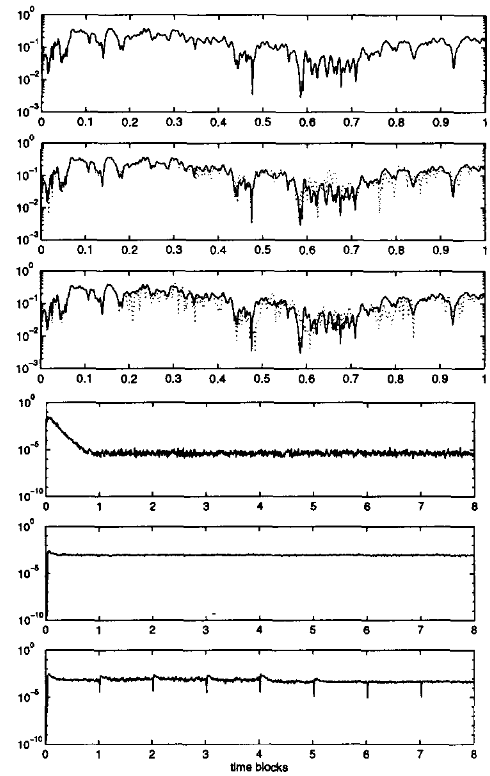

Figure 5. Filters and squared error for fullband, uniform, and nonuniform subband systems modeling a sampled room impulse response system.

is superior in terms of overall reconstructed error power to that of a uniform filter bank designed in the same manner (DCT, near perfect reconstruction, etc.) It has been argued that this architecture allows greater freedom in trading off MSE and computational complexity than uniform subband filtering. This increase in flexibility comes about through the choice of the minimum and maximum allocation size in the spectrum partitioning and the choice of the decimation rate. The question of the algorithm convergence versus the initial allocation is still an open question which will be discussed in future work.

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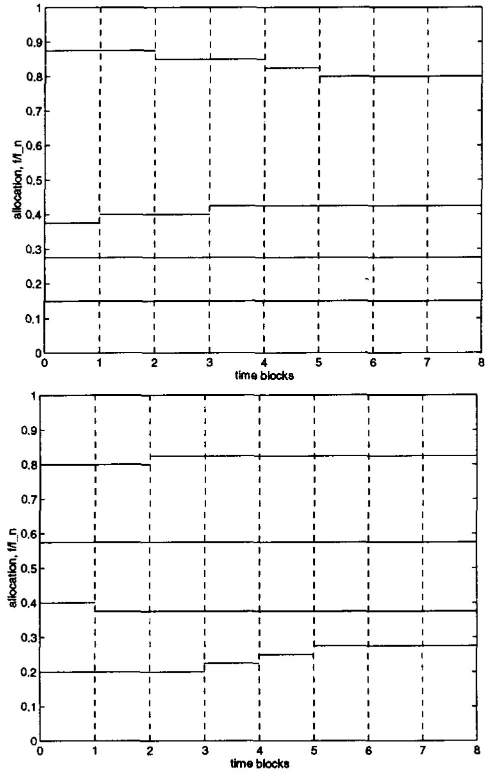

Figure 6. Allocation paths for a)bandstop system and b)echo response.

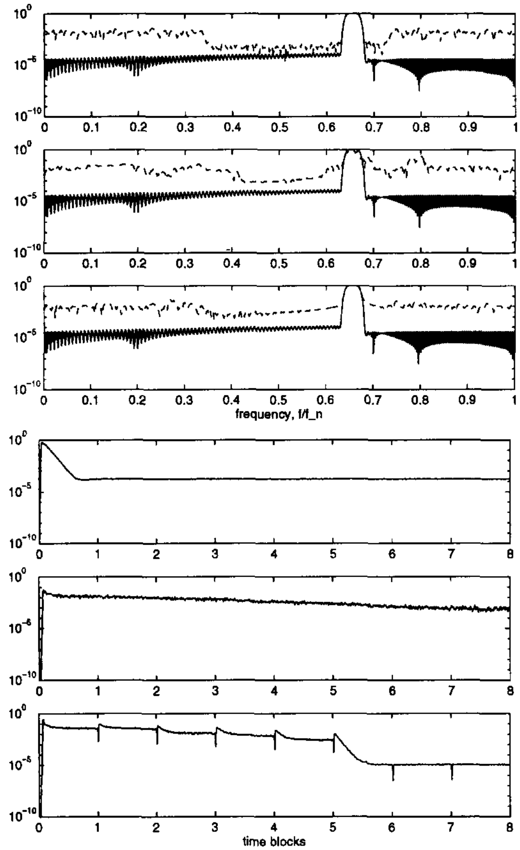

Figure 7. Final models and squared error for full-band, uniform, and nonuniform subband systems modeling a time varying system.