

TIME DEPENDENT AUTOREGRESSIVE SPECTRUM ESTIMATION OF HEART WALL VIBRATIONS

Hiroshi Kanai

Michie Sato

Noriyoshi Chubachi

Department of Electrical Engineering, Faculty of Eng.,
Tohoku University, Sendai 980-77, Japan
e-mail: hkanai@chubachi.ecei.tohoku.ac.jp

ABSTRACT

We present a new method for estimation of spectrum transition of a nonstationary signals in low signal-to-noise ratio cases. Instead of basic functions which are employed by the previously proposed time-varying AR modeling, we introduce the spectrum transition constraint in the cost function described by the partial correlation coefficients so that the method is applicable to noisy nonstationary signals of which spectrum transition patterns are complex. By applying this method to the analysis of vibration signals on the interventricular septum of the heart, noninvasively measured by the method developed in our laboratory using ultrasonics, spectrum transition pattern is clearly obtained during one beat period for a normal individual and a patient.

1. INTRODUCTION

Much work has been done on the parametric spectrum estimation using autoregressive (AR) model. A strong restriction of these methods lies in the necessary assumption that the signals may be considered to be stationary over the observation interval. *Time-varying parametric approaches of modeling* have been proposed to overcome this limitation and to take the effects of nonstationary signals into account explicitly. To estimate the parameters using a linear algorithm, the unknown time-varying parameters are approximated by linearly weighted combinations of a small number of known functions. The choice of the *basic functions* is an important part of such modeling process. A convenient way is to replace the time-varying coefficients with their second-order expansion [1], or an arbitrary order expansion [2], [3], Legendre [4], [5], Fourier [6], prolate spheroidal [7], and B-spline [8] are usually chosen for the basic functions. Since the number of unknown parameters is large, efficient equivalent representations for the modeling have been also proposed such as lattice filters [2], [7], [9].

However, if the spectrum transition pattern is

complex and/or there are large differences in the transition patterns among the individual nonstationary signals, it is difficult to estimate the transition pattern stably by choosing a set of basic functions *a priori*.

We have proposed a method for analyzing the spectrum transition of the multiframe signals of the fourth heart sounds detected during the stress test [10]. In the method, however, the analyzable signals are limited to multiple short length signals and the spectrum transition pattern between these signals are obtained. In this paper, by modifying the method we propose a new approach of modeling to *estimate the spectrum transition of a nonstationary signal by using a linear algorithm without any basic function*.

In this paper, moreover, we describe the spectrum transition constraint not by the linear predictive coefficients of the AR model but by the *partial coefficients*.

In order to noninvasively diagnose the acoustic characteristics of the heart muscle, it is necessary to measure the small vibration signals on the heart wall from the chest surface and analyze the resultant nonstationary signal during one beat period.

For the former problem of the measurement, we have developed a new method to noninvasively measure a small vibration signal on the heart wall using ultrasound [11]. For the latter problem of the analysis, we apply the developed time-varying modeling to the nonstationary small vibration signals on the interventricular septum in order to diagnose the acoustic characteristics of the heart muscle. These characteristics and the transition patterns may be applied to acoustic diagnosis of heart diseases.

2. PRINCIPLE OF SPECTRUM ESTIMATION USING PARTIAL COEFFICIENTS

Let us divide an original nonstationary signal $x(n)$ into succeeding F short signals $\{x_j(n)\}$, $n = 0, 1, \dots, N-1$, $j = 0, 1, \dots, F-1$, each is called by a *frame*, where F is the number of frames. Let us assume that each frame signal $x_j(n)$ be an AR signal

of order M , represented by the forward and backward recursions using i th order forward and backward predictive coefficients $\{a_{mi,j}\}$ and $\{b_{mi,j}\}$ of j th frame data, where $a_{m0,j} = 1$ and $b_{m,m+1,j} = 1$. The forward predictive error $x_j^+(n)$ is given by

$$x_{m,j}^+(n) = \sum_{i=0}^m a_{mi,j} \cdot x_j(n-i). \quad (1)$$

When the predictive order is equal to m , the power $\alpha_{m,j}$ of the predictive error for the data in the period $[M, N-1]$ of j th frame data is given by

$$\begin{aligned} \alpha_{m,j} &= \sum_{n=M}^{N-1} |x_{m,j}^+(n)|^2 \\ &= \sum_{i=0}^m \sum_{\ell=0}^m a_{mi,j} a_{m\ell,j} C_{i\ell,j}, \end{aligned} \quad (2)$$

where $C_{i\ell,j} = \sum_{n=M}^{N-1} x_j(n-i) \cdot x_j(n-\ell)$ is the covariance of data $x_j(n)$. By minimizing $\alpha_{m,j}$ with respect to the forward predictive coefficients $\{a_{mi,j}\}$, the following normal equation is given by

$$\frac{1}{2} \frac{\partial \alpha_{m,j}}{\partial a_{mi,j}} = 0 = \sum_{\ell=0}^m a_{m\ell,j} \cdot C_{i\ell,j} \quad (3)$$

$$= \sum_{n=M}^{N-1} x_{m,j}^+(n) \cdot x_j(n-\ell). \quad (4)$$

($\ell = 1, 2, \dots, m$)

Let us define the polynomials $A_{m,j}(z) = \sum_{i=0}^m a_{mi,j} \cdot z^{-i}$, and $B_{m,j}(z) = \sum_{i=1}^{m+1} b_{mi,j} \cdot z^{-i}$, where $a_{mm,j} = 1$ and $b_{m,m+1,j} = 1$. By letting us describe the z -transform of $x_j(n)$ by $X_j(z)$, and letting us employ the operator $\langle, \rangle$ as the inner product, the normal equation of Eq. (4) is given by

$$\langle A_{m,j}(z)X_j(z), z^{-\ell}X_j(z) \rangle = 0, \quad (5)$$

where $\ell = 1, 2, \dots, m$. This relation represents the orthogonality principle. Moreover, $B_{m,j}(z)$ must be orthogonal to the polynomials $B_{m-1,j}(z)$, $\dots$, $B_{0,j}(z)$ with lower order, that is,

$$\begin{aligned} &\langle B_{m,j}(z)X_j(z), B_{q,j}(z)X_j(z) \rangle \\ &= \sum_{i=1}^{m+1} \sum_{\ell=1}^{m+1} b_{mi,j} b_{q\ell,j} C_{i\ell,j} = \delta_{mq} d_{m,j}, \end{aligned} \quad (6)$$

where $q = 0, 1, \dots, m-1$, δ_{mq} is Dirac delta function, and $d_{m,j}$ is a real positive constant.

Since $A_{m,j}(z)X_j(z)$ and $B_{m,j}(z)X_j(z)$ must be orthogonal to the powers $z^{-1}X_j(z), \dots, z^{-(m-1)}X_j(z)$, $A_{m,j}(z)$ is described by [12]

$$\begin{aligned} A_{m,j}(z) &= A_{m-1,j}(z) + k_{m,j}B_{m-1,j}(z) \\ &= 1 + \sum_{i=1}^m k_{i,j}B_{i-1,j}(z), \end{aligned} \quad (7)$$

where $k_{m,j}$ is the partial coefficient for j th frame data and is determined from the above orthogonality principle. Let us define vectors $\mathbf{a}_j, \mathbf{k}_j, \mathbf{c}_j, \mathbf{x}_j$, a $M \times M$ upper triangle matrix $\mathbf{B}_j$, a $M \times M$ diagonal matrix $\mathbf{\Lambda}_j$, and a $M \times M$ covariance matrix $\mathbf{C}_j$, which is positive definite, by

$$\begin{aligned} \mathbf{a}_j &= [a_{M1,j}, a_{M2,j}, \dots, a_{MM,j}]^T, \\ \mathbf{k}_j &= [k_{M1,j}, k_{M2,j}, \dots, k_{MM,j}]^T, \\ \mathbf{c}_j &= [c_{01,j}, c_{02,j}, \dots, c_{0M,j}]^T, \\ \mathbf{x}_j &= [x_j(M), \dots, x_j(N-1)]^T, \\ \mathbf{B}_j &= \begin{bmatrix} 1 & b_{11,j} & b_{21,j} & \dots & b_{M-1,1,j} \\ 0 & 1 & b_{22,j} & \dots & b_{M-1,2,j} \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & b_{M-1,M-2,j} \\ 0 & \dots & \dots & 0 & 1 \end{bmatrix}, \\ \mathbf{\Lambda}_j &= \begin{bmatrix} d_{1j} & 0 & \dots & 0 \\ 0 & d_{2j} & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & d_{M,j} \end{bmatrix}, \\ \mathbf{C}_j &= [C_{i\ell,j}]. \end{aligned}$$

Using these vectors and matrices, Eq. (3), Eq (7), and Eq. (6) are respectively simplified as

$$\mathbf{C}_j \cdot \mathbf{a}_j = -\mathbf{c}_j, \quad (8)$$

$$\mathbf{a}_j = \mathbf{B}_j \cdot \mathbf{k}_j, \quad (9)$$

$$\mathbf{B}_j^T \cdot \mathbf{C}_j \cdot \mathbf{B}_j = \mathbf{\Lambda}_j. \quad (10)$$

Thus, the total power $\alpha_{M,j}$ of M th-order forward predictive error in Eq. (2) is given by

$$\begin{aligned} \alpha_{M,j} &= \mathbf{a}_j^T \mathbf{C}_j \mathbf{a}_j + 2\mathbf{c}_j^T \mathbf{a}_j + \mathbf{x}_j^T \mathbf{x}_j \\ &= \mathbf{k}_j^T \mathbf{B}_j^T \mathbf{C}_j \mathbf{B}_j \mathbf{k}_j + 2\mathbf{c}_j^T \mathbf{B}_j \mathbf{k}_j + \mathbf{x}_j^T \mathbf{x}_j \\ &= \mathbf{k}_j^T \mathbf{\Lambda}_j \mathbf{k}_j + 2(\mathbf{B}_j^T \mathbf{c}_j)^T \mathbf{k}_j + \mathbf{x}_j^T \mathbf{x}_j. \end{aligned} \quad (11)$$

3. MINIMUM LIKELIHOOD ESTIMATION OF SPECTRUM TRANSITION

When each frame data of multi-frame nonstationary signal $\{\mathbf{x}_j\}$ is described by an autoregressive model, the logarithmic likelihood function ℓ , which

shows the probability of $\{\mathbf{x}_j\}$ for the unknown partial coefficients $\{\mathbf{k}_j\}$, is given from Eq. (11) by

$$\ell = - \sum_{j=0}^{F-1} \left\{ \frac{\mathbf{k}_j^T \Lambda_j \mathbf{k}_j + 2(\mathbf{B}_j^T \mathbf{c}_j)^T \mathbf{k}_j + \mathbf{x}_j^T \mathbf{x}_j}{|\mathbf{x}_j|^2} + \lambda |\mathbf{k}_{j+1} - \mathbf{k}_j|^2 \right\}, \quad (12)$$

where λ is a Lagrange multiplier. The second term in the right hand side of Eq. (13) shows the constraint for the spectrum transition between the succeeding frames. Let us determine the partial coefficients $\{\mathbf{k}_j\}$ which maximizes the logarithmic likelihood function ℓ as follows: By taking the partial derivative of ℓ with respect to $\{\mathbf{k}_j\}$ and setting the results to be zero,

$$-\frac{1}{2} \frac{\partial \ell}{\partial \mathbf{k}_j} = 0 = \frac{1}{|\mathbf{x}_j|^2} (\Lambda_j \mathbf{k}_j + \mathbf{B}_j^T \mathbf{c}_j) + \lambda (2\mathbf{k}_j - \mathbf{k}_{j-1} - \mathbf{k}_{j+1}) \quad (13)$$

By solving the simultaneous equation of (13), the partial coefficients $\{\mathbf{k}_j\}$ of all frame data are estimated under the constraint for the spectrum transition between the frame data. Let us denote $(\Lambda_j/|\mathbf{x}_j|^2 + 2\lambda I)$ by a diagonal matrix $\mathbf{D}_j$ and let us define $FM \times FM$ matrix $\mathbf{G}$ and FM -dimension vectors $\mathbf{g}$ and $\mathbf{k}$ as follows:

$$\mathbf{G} = \begin{bmatrix} \mathbf{D}_0 & -\lambda \mathbf{I} & 0 & \cdots & 0 & -\lambda \mathbf{I} \\ -\lambda \mathbf{I} & \mathbf{D}_1 & -\lambda \mathbf{I} & 0 & \cdots & 0 \\ 0 & -\lambda \mathbf{I} & \mathbf{D}_2 & \ddots & \ddots & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & \vdots & \ddots & \ddots & \ddots & -\lambda \mathbf{I} \\ -\lambda \mathbf{I} & 0 & \cdots & 0 & -\lambda \mathbf{I} & \mathbf{D}_{F-1} \end{bmatrix},$$

$$\mathbf{g} = \left[\left(\frac{\mathbf{B}_0^T \mathbf{c}_0}{|\mathbf{x}_0|^2} \right)^T, \dots, \left(\frac{\mathbf{B}_{F-1}^T \mathbf{c}_{F-1}}{|\mathbf{x}_{F-1}|^2} \right)^T \right]^T,$$

$$\mathbf{k} = [\mathbf{k}_0^T, \mathbf{k}_1^T, \dots, \mathbf{k}_{F-1}^T]^T,$$

where $-\lambda \mathbf{I}$ in the top right and bottom left of the matrix $\mathbf{G}$ is introduced by assuming the frame data are same between 0-th frame and F th frame, that is, $\mathbf{x}_0(n) = \mathbf{x}_F(n)$. Substituting these matrix and vectors for Eq. (13), optimum estimation of the spectrum transition of a nonstationary signal under the constraint for the spectrum transition between frame data is achieved by solving the following linear simultaneous equations:

$$\mathbf{G}\mathbf{k} = -\mathbf{g}, \quad (14)$$

Therefore, the partial coefficients of the multi-frame data are estimated by

$$\hat{\mathbf{k}} = -\mathbf{G}^{-1}\mathbf{g}, \quad (15)$$

where $\mathbf{G}^{-1}$ is the inverse matrix of $\mathbf{G}$. Using $\hat{\mathbf{k}} = [\hat{\mathbf{k}}_0^T, \hat{\mathbf{k}}_1^T, \dots, \hat{\mathbf{k}}_{F-1}^T]^T$, the estimates of the vectors $\mathbf{a}_j$ of linear predictive coefficients in the j th frame data is obtained by

$$\hat{\mathbf{a}}_j = \mathbf{B}_j \hat{\mathbf{k}}_j. \quad (16)$$

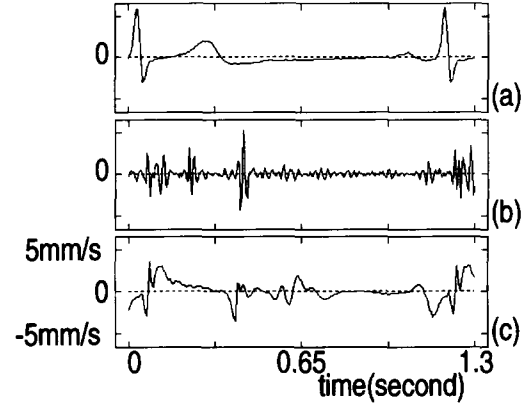


Figure 1. (a) an electrocardiogram, (b) heart sounds, and (c) small vibration signal $x(n)$ with high frequency components on the interventricular septum of the heart of a normal subject measured by the newly developed method in our lab.

4. EXPERIMENTAL RESULTS

We applied this method to the analysis of small vibration signals $x(n)$, measured in our laboratory using the noninvasive measurement method [11] on the interventricular septum in the heart wall of a normal male subject of 26 years old in Fig. 1(c) for the noninvasive acoustical diagnosis of myocardial dysfunction.

One beat signal $x(n)$ in the first beat period in Fig. 1(c) is divided into succeeding 30 short frame signals $x(n; j)$, each of which has 30 points in length, by multiplying the Hamming window with a length of 30 points. That is, each frame signal is about 150 ms in length since the signal is A/D converted at a sampling period of 5 ms. Adjacent short signals overlap each other by their three-quarter-length. Since each duration time of the first heart sound (I) and the second heart sound (II) in Fig. 1(b) is about 150 ms in length, let us assume that each frame signal $x(n; j)$ is stationary over each frame.

Since the SNR is not so high and the duration time of each frame signal is very short, there are large fluctuations and many phantom peaks appear in spectra of Figs. 2(a) and 2(b) estimated by independently applying the discrete Fourier transform (DFT) and the maximum entropy method (MEM) to each frame signal ($N=30$, $F=30$).

On the other hand, by applying the proposed method ($M = 8$) to the same multi-frame signals, the resultant spectrum transition patterns

are shown in Fig. 3(a) for the same signal as Fig. 2. In these experiments the value of λ of Eq. (12) is 0.2. In the resultant spectra in Fig. 3(a), the frequency transition from the systole to the diastole is clearly obtained.

For a male young patient with cardiomyopathy, by applying the proposed method ($M = 8$) to the multiframe signals, the resultant spectrum transition patterns are shown in Fig. 3(b). In these experiments the value of λ is 0.2.

By comparing these results in Figs. 3(a) and 3(b), for the normal subjects there are large changes in dominant frequency and power of the vibration from the diastole to the systole, while for the patient these changes are smaller than those for the normal. These qualitative phenomena will be quantitatively confirmed using many samples in near future.

5. CONCLUSIONS

We present a new method to estimate spectrum transition of a nonstationary signal in low SNR cases using a linear algorithm without any basic function. By applying the proposed method to the heart wall vibrations, we found there are clear spectrum transition patterns.

The electrocardiogram or the heart sounds contain only low frequency components and each of them does not continue within one beat period. However, small vibration signals accurately measured by our method contain the information enough to diagnose all four stages in one cardiac cycle. Thus, a new scientific field of noninvasive acoustic diagnosis of the heart dysfunction will be developed soon by the measurement of the heart wall vibrations and their analysis as proposed in this paper.

REFERENCES

- [1] T. S. Rao, "The Fitting of Non-Stationary Time-Series Models with Time-Dependent Parameters," *J. of the Royal Statist. Soc. Series B*, Vol. 32, pp.312-322, 1970.
- [2] L. A. Liporace, "Linear Estimation of Non-Stationary Signals," *J. Acoust. Soc. Amer.*, Vol. 58, pp.1288-1295, 1975.
- [3] K. C. Sharman and B. Friedlander, "Time-Varying Autoregressive Modeling of a Class of Nonstationary Signals," in *Proc. IEEE ICASSP*, 1984, pp. 22.2.1-22.2.4. 70
- [4] F. Kozin, "Estimation and Modeling of Nonstationary Time-series," in *Proc. Symp. Appl. Comput. Meth. Eng.*, Vol. 1, Los Angeles, CA, 1977, pp. 603-612.
- [5] G. Alehgrin and M. Barlaud, and J. Menez, "Unbiased Parameter Estimation of Nonstationary Signals and Noise," *IEEE Trans.* Vol. ASSP-34, pp. 1319-1322, 1986.
- [6] M. Hall, A. V. Oppenheim, A. Willsky, "Time-Varying Parametric Modeling of Speech," in *Proc. IEEE Decision and Control Conf.*, New Orleans, LA, pp. 1085-1091, 1977.
- [7] Y. Grenier, "Time-Dependent ARMA Modeling of Nonstationary Signals," *IEEE Trans.* Vol. ASSP-31, pp. 899-911, 1983.
- [8] M. J. Flaherty, "Application of Polynomial Splines to a Time-Varying Autoregressive Model of speech," in *Proc. IEEE ICASSP*, 1988, pp. 2220-2223.

- [9] E. Karlsson and M. H. Hayes, "ARMA Modeling of Time Varying Systems with Lattice Filters," in *Proc. IEEE ICASSP*, 1986, pp. 2335-2338.
- [10] H. Kanai, N. Chubachi, K. Kido, Y. Koiwa, T. Takagi, J. Kikuchi, and T. Takishima: "A New Approach to Time Dependent AR Modeling of Signals and Its Application to Analysis of the Fourth Heart Sound," *IEEE Trans. on SP*. Vol. 40, pp. 1198-1205, 1992.
- [11] H. Kanai, M. Sato, Y. Koiwa, and N. Chubachi: "Transcutaneous Measurement and Spectrum Analysis of Heart Wall Vibrations," *IEEE Trans. on UFFC*. Vol. 44, pp. 791-810, 1996.
- [12] J. D. Markel, A. H. Gray, Jr.: *Linear Prediction of Speech*. Springer-Verlag, 1978.

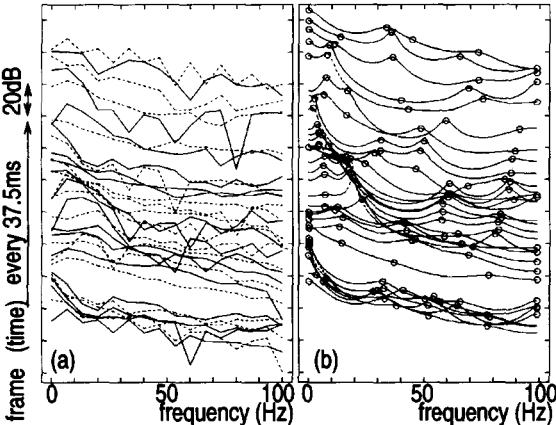


Figure 2. For the vibration signal $x(t)$ in Fig. 1(c) of a normal subject, (a) the spectra of signals $x(n; j)$ obtained by the DFT with the Hamming window of 30 point in length, (b) the spectrum transition estimated by the maximum entropy method. The even and odd frames are shown in solid lines and dotted lines, respectively. Each estimated pole frequency is indicated by circle.

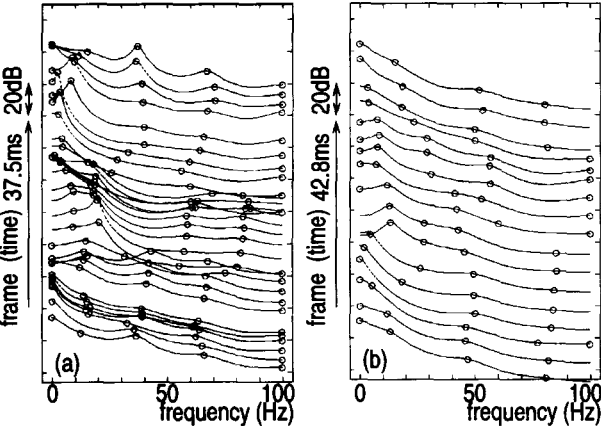


Figure 3. The spectrum transition patterns estimated by the proposed method in this paper. (a) For the vibration signal of a normal subject and (b) for a patient with cardiomyopathy. Each estimated pole frequency is indicated by circle.