ZERO-TRACKING TIME-FREQUENCY DISTRIBUTIONS

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ABSTRACT

The zero-tracking time-frequency distribution (TFD) is introduced. The local autocorrelation function of the TFD, defined by an appropriate kernel, is used to form a polynomial whose roots correspond to the instantaneous frequencies of the multicomponent signal. Two techniques for zero-tracking based on TFD are presented. The first technique requires updating all of the polynomial signal and extraneous zeros, and is based on the formula relating to the first order approximation, the changes in the polynomial roots and coefficients. The second technique employs the zero-finding Newton's method to only obtain the zero-trajectories of interest.

1. INTRODUCTION

Recently, quadratic time-frequency distributions (TFD), including of Cohen's class [1], the affine class, and the hyperbolic class[2], have been introduced for nonstationary signal analyses. TFDs have been shown to be a powerful tool for instantaneous frequency (IF) estimation in rapidly time-varying environment [3]. These distributions do not assume a model which is signal specific, as in the case of extended Kalman filters [4], nor they are encumbered by high computational requirements, as in the case of Hidden Markov Models [5]. Further, TFDs can handle multicomponent signals, and as such, outperform existing techniques which are only applicable to a single tone scenario [6]. TFDs are obtained by taking the Fourier transform of the local autocorrelation function (LAF). The later is computed by time-averaging the bilinear data products. The averaging is performed by applying a kernel which acts on satisfying several desirable time-frequency properties including the marginal, support, instantaneous frequency, and reduced cross-terms.

In this paper, we introduce zero-tracking time-frequency distribution methods for instantaneous frequency estimation. The motivation is two fold and is analogous to that of zero-tracking adaptive filters, namely computational savings and enhanced performance. The proposed methods avoid the Fourier transform and utilize the fact that the local characteristics of the signal are captured in its LAF.

Because of the localization properties of the time-frequency distributions, the location of the spectral peak at time n represents the signal instantaneous frequency. Multiple peaks are a property of a multicomponent signal. In TFD computation, the FFT is used at each time instant. It is important to choose a long FFT block length to properly locate spectral peaks and estimate the IFs. This may require extending the data record to include more data samples, or by zero padding. The cost of applying a high resolution FFT on a data sample by a data sample basis along with a search routine can be avoided by directly extracting the information from the local autocorrelation function without Fourier transformation.

By constructing a polynomial whose roots are located at the TFD peak positions, one can apply zero tracking algorithms to provide the zero trajectories, i.e., the signals instantaneous frequencies. Two methods for zero updating are introduced. The first follows the same approach adopted in zero-tracking adaptive filters [7], but with application to the local autocorrelation function, instead of the filter coefficients. In this case, the zero trajectories are provided using the formula relating the changes in the LAF coefficients to the polynomial zeros. The second method is introduced to mitigate the problems inherent in the first method, namely the need to update all zeros including those which are extraneous, i.e., do not correspond to the signal IFs. In the second method, new polynomial zero positions are obtained by applying an iterative technique similar to that of *Newton's method* [8] for root finding. It is shown that this method works well in low signal-to-noise ratio and under both evolutionary and abrupt changes in frequency.

2. POLYNOMIAL DERIVATION

The basic idea of zero-tracking applied to the underlying problem is to find a polynomial in which all the signal instantaneous frequencies (IFs) are among its roots. We then proceed to identify the signal roots and follow their trajectories.

Let $R_l(n)$ denote the local autocorrelation function at time n and lag l. The TFD is given by

$$Y_n(e^{j\omega}) = \sum_{l=-L/2}^{L/2} R_l(n)e^{-j2\omega l} \qquad (1)$$

where $L/2$ is the maximum lag of interest. $Y(e^{j\omega})$ is real and it peaks at the instantaneous frequencies of the multicomponent signal, represented by $\omega_1(n), \omega_2(n),$ ..., $\omega_M(n)$. Accordingly,

$$\left.\frac{dY(e^{j\omega})}{d\omega}\right|_{\omega_i(n), 1 \le i \le M} = -j2\sum_{l=-L/2}^{L/2} lR_l(n)e^{-j2\omega l} = 0 \quad (2)$$

Substituting $e^{j2\omega} = z$, (2) can be rewritten as

$$\sum_{l=-L/2}^{L/2} lR_l(n)z^{-l} = 0 \qquad (3)$$

By multiplying the above equation by $z^{L/2}$, we obtain

$$\sum_{l=-L/2}^{L/2} lR_l(n)z^{-l+L/2} = 0 \qquad (4)$$

which is the L-th order polynomial:

$$f_n(x) = a_0(n) + a_1(n)z + a_2(n)z^2 + ... + a_L(n)z^L = 0 \quad (5)$$

$$a_l = (-l + L/2)R_{-l+L/2}(n), \quad l = 0, 1..., L \qquad (6)$$

The instantaneous frequencies are among the above polynomial roots in the form of $e^{j2\omega_i}, 1 \le i \le M$, and therefore can be obtained from the polynomials' zero trajectories.

It is important to note that since the polynomial $f_n(x)$ is based on the LAF, time-frequency kernels which yield reduced cross-terms should be used for proper IF estimation.

3. ZERO-TRACKING ALGORITHMS

Two techniques for zero-tracking based on the LAF of the TFD are considered below.

A. Orfanidis-Vail's Method

This method is based on using the formula, relating to the first-order approximation, the changes in the polynomial coefficients and its roots. Equation (5) can be written in the factorized form as

$$a_0(n) + a_1(n)z^{-1} + a_2(n)z^{-2} + ... + a_L(n)z^{-L}$$
$$= a_0(n)(1 - z_1(n)z^{-1})(1 - z_2(n)z^{-1})...(1 - z_L(n)z^{-1}) \qquad (7)$$

where $z_i(n)$ is ith root. The roots are updated by

$$z_i(n+1) = z_i(n) + \Delta z_i(n), \quad 1 \le i \le L \qquad (8)$$

where

$$\Delta z_i(n) = \sum_{m=0}^{L} \frac{\partial}{\partial a_m(n)} z_i(n)\Delta a_m(n) \qquad (9)$$

The partial derivatives in the above equation are given by

$$\frac{\partial}{\partial a_m(n)} z_i(n) = \frac{z_i(n)^{L-m}}{a_0(n)\prod_{j \ne i}(z_i(n) - z_j(n))} \qquad (10)$$

This algorithm can therefore be summarized as: 1) At time n, the LAF is available, and so are the coefficients $a_m(n)$. 2) Compute the zero updates using (9), and update the zeros using (8).

The problem with the above zero-tracking method is that all the polynomial roots need to be updated, as evident from equation (10). This entails heavy computations and becomes inefficient, especially for small values of M as well as high spectral resolution requirements. As shown below, zero-tracking based on Newton's method can allow only selected zeros corresponding to the IFs to be updated.

B. Zero-Tracking Based On Newton's Method

A well-known numerical root finding method, *Newton's Method*, is very suitable for the underlying problem. Given the form of the polynomial (5), the *i*th root can be found by

$$z_i^{(k)} = z_i^{(k-1)} - \frac{f(z_i^{(k)})}{f'(z_i^{(k)})} \qquad (11)$$

where k is the iteration number. If the initial guess of the root, $z_i^{(0)}$, is in the neighborhood of the exact root, then the Newton's method is quadratically convergent [8].

It is assumed that the IF corresponding to the *i*th root is changing over adjacent time samples such that the root at the *n*th time sample is in the neighborhood of the new root at time n+1. Newton's Method can then be applied by using the last root value as the initial guess in (11). Due to the quadratical convergence, the new root value, and subsequently the corresponding IF, will be quickly reached after very few iterations.

A situation may arise, however, in which the incorrect zero is tracked. This may occur if we start with an extraneous zero and follow its trajectory, or by starting with the correct zero and losing its trajectory to another

one, which is extraneous. In order to circumvent or mitigate this problem, we propose the following four guidance procedures.

• The Zero-Tracking Correction Technique

It is needed to find the exact roots corresponding to the IFs at the early stage of tracking and whenever tracking of the correct roots is lost due to fast changing IF or other problems. In this case, we use TFD and peak picking to identify the signal zeros.

• Power Monitor

To find out whether the tracking follows the wrong trajectory, we simply monitor the power of the signal at the IF $\omega_i(n)$.

$$P(\omega_i(n)) = \sum_{l=-L/2}^{L/2} R_l(n)e^{-j2\omega_i(n)l} \qquad (12)$$

Whenever the power of a tracked IF is lower than a threshold, it is declared that the algorithm has lost the correct zero. The zero-tracking correction technique must then be applied.

• Loss of Resolution

Due to the finite extent of the LAF, or equivalently, the finite order of the polynomial in (5), two closely spaced IFs will be presented by one peak in the TFD, and subsequently, one zero in the corresponding polynomial. In this case, the two zero trajectories will merge into one. In time-varying environment, loss of resolution may, however, lasts for small periods of time. For example, in the case of two crossing chirps, only the vicinity of the intersection point may represent a resolution problem. Once the two chirps move farther apart, TFD should show two peaks corresponding to the autoterms, and subsequently the tracking algorithm should again yield two trajectories. When two zeros merge to one, we keep record of the merged peak value. Whenever the peak maximum value drops in half, the algorithm should go to the correction technique to find the exact roots.

• Convergence Monitor

A simple way to test the divergence of the *Newton's Method* is given in [8]. The inequality, $\left|f(x^k)\right| > \left|f(x^{k-1})\right|$, means that the root finding method has lost convergence. If this occurs, we need to apply the correction technique.

C. The Basic Algorithm

The complete version of the proposed zero-tracking algorithm is summarized as follows

1) At time n, perform the zero-tracking correction technique. Find the IFs and the corresponding roots $z_{n,i}$.

2) Calculate the new polynomial coefficients $a_0, a_1,...,a_L$ at time n+1 using equation (4).

3) Calculate $f(z_{n,i})$ using (5), and $f'(z_{n,i})$ by

$$f'(z_{n,i}) = a_1 + 2a_2z_{n,i} + 3a_3z_{n,i}^2 + ... + La_Lz_{n,i}^{L-1} \qquad (13)$$

4) $z_{n+1,i} = z_{n,i} - \dfrac{f(z_{n,i})}{f'(z_{n,i})}$

5) Calculate $f(z_{n+1,i})$, and put $z_{n,i} = z_{n+1,i}$

6) If $\left|f(z_{n+1,i})\right| > \left|f(z_{n,i})\right|$ or $P(z_{n,i}) < threshold$, apply the correction technique in step 1).

9) If $z_{n,i}$ is equal to any other IF root, keep record of the maximum value of the merged peak until it drops in half, then apply the correction technique in step 1).

10) If $\left|f(z_{n+1})\right| <$ accuracy threshold, go to the next time sample, and apply step 2), otherwise go to the step 3).

The calculation of zero-tracking in term of number of multiplication is approximately *4L*, where *L* is the number of roots. The time efficiency is linear.

4. SIMULATIONS

First, consider the case of single tone frequency hopping signal with SNR=20dB. We use Choi-Williams kernel applied to the LAF with $\sigma=1$ and L=14. The result is shown in Fig.1. The zero-tracking correction technique was used once for each hop.

Second, we use a single tone signal with a fast sinusoidal changing IF and high noise level, SNR=5dB. Again, Choi-Williams kernel with $\sigma=1$ and 14 roots is applied. The IF tracking is shown in Fig.2. In this example, the correction technique was used only once at the beginning of tracking.

Next, two component IFs are tested. The signal consists of two crossing chirps with SNR=20dB. To provide good spectral resolution, we use a polynomial of 30 roots and Choi-Williams kernel with $\sigma=10$. The two chirps are nicely tracked in Fig.3. The correction technique was only applied twice, at the initial phase and at the split point past the chirp crossing region.

CONCLUSION

Two zero-tracking algorithms have been introduced for instantaneous frequency estimation. Both algorithms operate on the local autocorrelation function of the time-frequency distribution and construct a polynomial whose zeros correspond to the IFs of the multicomponent signal. The zero trajectories of this polynomial are either provided by using the formula relating, to the first order approximation, the changes in the polynomial coefficients and roots, or by applying *Newton's method* for zero-finding.

Unlike TFDs, providing the IFs via the zero-trajectories of a polynomial whose coefficients are generated

from the LAF does not require the application of either FFT or peak finding techniques every time sample.

Computer simulations illustrating the performance of the zero-tracking algorithm using Newton's method under evolutionary and abrupt frequency changes are presented. All simulations demonstrate the fast tracking and convergence properties of the proposed algorithm when applied to LAFs of reduced interference distributions.
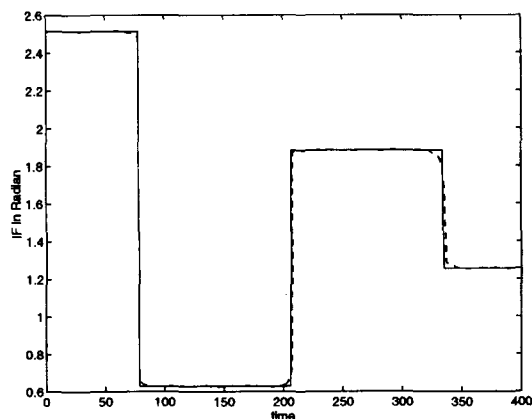

Fig.1 Frequency Hopping.
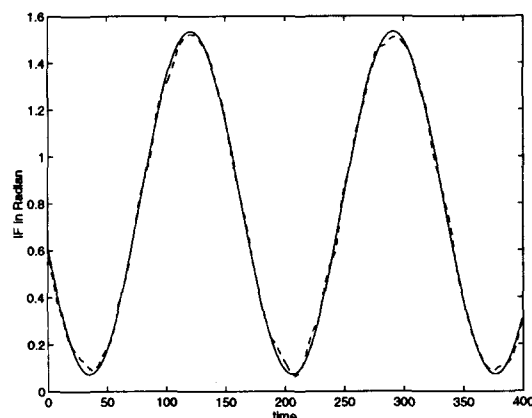'-' is the exact IF, ' --' is the IF estimate.

Fig.2 Sinusoidal IF.
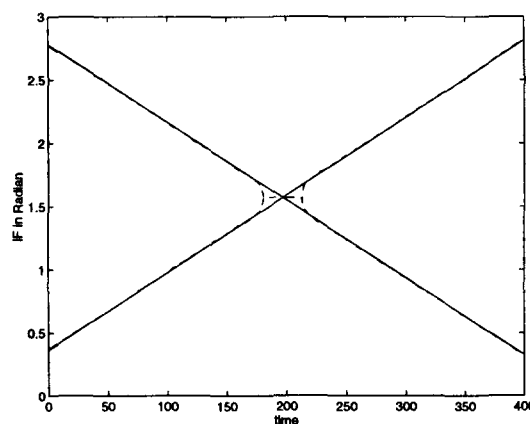'-' is the exact IF, ' --' is the IF estimate

Fig.3 Two Crossing Chirps.
'-' is the exact IF, ' --' is the IF estimate.

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