AN ARCHITECTURE FOR REALIZATION OF THE CROSS-TERMS FREE POLYNOMIAL WIGNER-VILLE DISTRIBUTION

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Abstract— **A method for the Polynomial Wigner-Ville distributions realization, in the case of multicomponent signals, is presented. It is based on the author's recently proposed S-method. Using this method one may, theoretically, get the sum of the Polynomial Wigner-Ville distributions of each component separately. Architecture for the Polynomial Wigner-Ville distributions realization, starting from the short time Fourier transform, is given. Method is illustrated on a numerical example.**

I. INTRODUCTION

Out of the general Cohen class of quadratic shift covariant distributions, the Wigner distribution (WD) is the only one (with signal independent kernel) which produces the ideal concentration along instantaneous frequency for the linear frequency modulated signals [4], [6]. In order to improve the concentration, when the instantaneous frequency is a polynomial function of time, the Polynomial Wigner-Ville distributions (PWVD) are proposed by Boashash et all., [1], [2]. Since, these distributions belong to the class of higher order time-varying spectra, they suffer from very emphatic cross-term effects, what makes their application to the multicomponent signals very unsuitable. In this paper we will show that the recently proposed S-method [6], [7], [10], [11], may be efficiently used for the reduction (removal) of the cross-terms in the PWVD of multicomponent signals. Theoretically, we get a sum of the PWVD of each component separately, what is exactly that we achieved in the example presented in the paper.

II. REVIEW OF THE S-METHOD

The S-method has been derived from the analysis of the WD in the case of multicomponent signals. In this case the WD produces very emphatic cross-terms which may even completely mask the auto-terms and make this distribution useless for analysis. This was the reason why many other quadratic distributions had been introduced (Choi-Williams, Zao-Atlas-Marks, Born-Jordan, Butterworth...). Common for all of these distributions is to reduce cross-terms and other interferences, at the same time satisfying as many desired properties as possible. But, the cross-term reduction inherently leads to the auto-terms degradation, [14]. In order to preserve the auto-terms quality, as high as in the Wigner distribution, and at the same time to reduce cross-terms, the S-method has been introduced [13]:

$$SM(t,\omega)=\int_{-\infty}^{\infty} P(\theta)STFT(t,\omega+\theta)STFT^*(t,\omega-\theta)\frac{d\theta}{\pi}. \quad (1)$$

Using this method one may obtain the auto-terms quality as in the Wigner distribution, eliminating (reducing) cross-terms in a numerically very efficient way (more efficient than the Wigner distribution realization). The S-method realization is based on a direct application of the short time Fourier transform (STFT). Frequency domain $P(\theta)$ controls the cross-terms reduction. The S-method belongs to the general class of smoothed pseudo Wigner distributions (SPWD) - Cohen class of distributions. Its kernels in (τ,θ) and (τ,t) domain are given by

$$c(\tau,\theta)=P(\theta/2)*_\theta A_{ww}(\tau,\theta)/\pi c$$
and
$$\varphi(\tau,t)=2p(2t)w(t+\tau/2)w(t-\tau/2)$$

respectively [14]. *Let us emphasize that the S-method is a member of the general SPWD class, and not just an alternative way of writing distributions from this class. While each of the above mentioned distributions (Choi-Williams, Zao-Atlas-Marks, Born-Jordan, Butterworth...) belongs to the SPWD, none of them can be written in form (1).*

Discrete form of the S-method, for a rectangular window $P(\theta)$, is given by:

$$\begin{aligned} SM(n,k) &= SPEC(n,k)+ \\ &+2\sum_{i=1}^{L_d} Re\{STFT(n,k+i)STFT^*(n,k-i)\} \end{aligned} \quad (2)$$

Note that for $L_d=0$ we get the spectrogram, while $L_d=N/2$ produces the pseudo WD. Number of terms L_d may be also signal dependent. The signal dependent form of S-method may be also considered in the following way: (a) signal components are separated on the basis of its STFT; (b) the WD of each component is calculated separately, within its frequency range, using (2). One such realization is presented in [9].

S-METHOD APPLICATIONS: (I) Multidimensional form of the S-method is presented in [16]; (II) Application to the time-scale distributions is described in [6], [5]; (III) Implementation of the cross-terms free higher order L-Wigner distributions, using this method, is given in [6], [7], [10], [15]; (IV) Very highly concentrated distributions, that may satisfy the marginal conditions [20], are implemented using the S-method in [21], [22]; (V) Cross-terms free form of the Wigner bispectrum and other Wigner higher order spectra are considered in [18], [19]. In the next section an application on the cross-terms free (reduced) realization of the PWVD is presented. Further details about the S-method itself may be also found in [6], [7], [10], [11], [14], [16], [17], [21].

III. PWVD DEFINITION AND METHOD DERIVATION

Polynomial Wigner-Ville distributions are derived from the condition that the distribution of a frequency modulated signal $x(t) = A\exp(\phi(t))$, having polynomial phase function $\phi(t) = \sum_{i=0}^{p} a_i t^i$, is equal to the ideally concentrated one $W_x(t,\omega) = 2\pi\delta(\omega - \phi'(\omega))$. It has been shown [1], [2] that such a distribution may be obtained as a Fourier transform of the polynomial kernel $K_x(t,\tau) = \prod_{k=-q/2}^{q/2} x^{b_k}(t + c_k\tau)$, with respect to τ:

$$W_x(t,\omega) = \int_{-\infty}^{\infty} \prod_{k=-q/2}^{q/2} x^{b_k}(t + c_k\tau) e^{-j\omega\tau} d\tau \tag{3}$$

where $q \geq p$ is an even number. Coefficients b_k and c_k should be determined, for a given p and q so that the ideal distribution is achieved ($b_0 \equiv 0$).

In practical and numerical applications, only the PWVD of order $q = 4$ has been used, so we will, without loss of generality, demonstrate the realization procedure on this distribution, since the same technique may be applied to any order distribution. The PWVD with $q = 4$ is defined by, [1]:

$$W_x(t,\omega) = \int_{-\infty}^{\infty} x^2(t + 0.675\tau) x^{*2}(t - 0.675\tau) \\ x^*(t + 0.85\tau) x(t - 0.85\tau) e^{-j\omega\tau} d\tau. \tag{4}$$

Rewrite distribution (4) in a frequency scaled form

$$W_x(t,\omega) = \frac{1}{2.7} \int_{-\infty}^{\infty} x^2(t + \tfrac{\tau}{4}) x^{*2}(t - \tfrac{\tau}{4}) \\ x^*(t + A\tfrac{\tau}{2}) x(t - A\tfrac{\tau}{2}) e^{-j\frac{\omega}{2.7}\tau} d\tau \tag{5}$$

where $A = 0.85/1.35$. The multilinear kernel of the PWVD creates a multiplicity of cross-terms. For example, if we have only a two-component signal, the number of cross terms in (4) is 13. This illustrates the unapplicability of the original definition for the processing of multicomponent signals. In order to present a procedure for the efficient PWVD realization in the case of multicomponent signals, note that (3) may be expressed as a convolution of the L-Wigner distribution (with $L = 2$), [8], [10], and a scaled Wigner distribution:

$$W_x(t,\omega') = \frac{1}{5.4\pi} LWD_2(t,\omega') *_{\omega'} WD_A(t,\omega') \tag{6}$$

where:

$$LWD_2(t,\omega) = \int_{-\infty}^{\infty} x^2(t + \frac{\tau}{4}) x^{*2}(t - \frac{\tau}{4}) e^{-j\omega\tau} d\tau,$$

$$WD_A(t,\omega) = \int_{-\infty}^{\infty} x^*(t + A\frac{\tau}{2}) x(t - A\frac{\tau}{2}) e^{-j\omega\tau} d\tau.$$

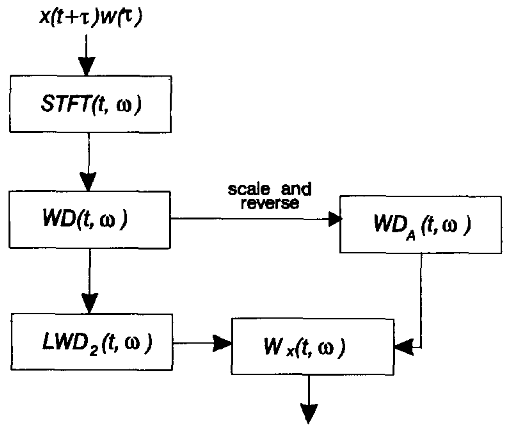

Fig. 1. Block shema for the PWVD realization.

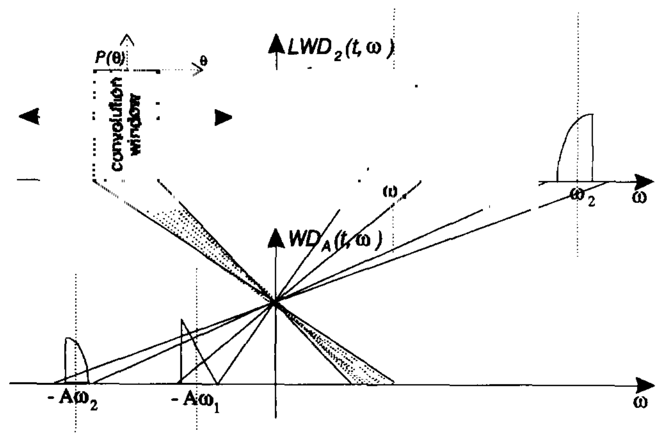

Fig. 2. Illustration of the windowed convolution in the PWVD of a two-component signal.

A block schema for the PWVD realization, starting from the signal, via STFT, Wigner distribution, and L-Wigner distribution is shown in Fig.1. Since we have already described the method for the cross-terms free (cross-terms reduced) realization of the Wigner distribution and L-Wigner distribution ([6], [7], [10], [11] and Sec.II), the only remaining step is to realize the convolution in (6) so that it does not introduce any additional cross term, as well as produces the auto-terms at their natural positions. In order to examine the convolution in (6), let us consider a two-component signal at an instant t, whose WD and LWD are shown in Fig.2. It is obvious that if an auto-term exists at and around the instantaneous frequency ω_1, it will be placed in $LWD_2(t,\omega')$ at and around the same frequency ω_1. This auto-term is located in $WD_A(t,\omega')$ at and around $-A\omega_1$. Thus, in order to calculate $W_x(t,\omega')$ at a given frequency ω' we should calculate convolution (6) using only an interval around ω' in $LWD_2(t,\omega')$ and using an interval around $-A\omega'$ in $WD_A(t,\omega')$, Fig.2. Theoretically, this interval should be greater or equal to the auto-term width and less than the distance between auto-terms. But, in practical realizations we usually a priori assume its value. Note, if window $P(\theta)$ is too wide, the cross-terms will start appearing, while too narrow window will degrade auto-terms with respect to their original PWVD form (recently we derived a technique for variable and self adaptive window $P(\theta)$ width, that may be applied to this case in a straightforward manner, [9], [21], [22]). Of course, the position of the convolution value (obtained through a window $P(\theta)$) is kept at the position of ω' in $LWD_2(t,\omega')$, since this is a true position of the auto-term in the non-scaled frequency axis ω. This way, the auto-terms of the PWVD, at their natural positions are obtained.

In the discrete implementation of the above procedure, the only problem that remains is the evaluation of $WD_A(t,\omega')$ on the discrete set of points on frequency axis, $\omega' = -k\Delta\omega'$. Since $WD_A(t,\omega')$ is nothing but a scaled and reversed version of $WD(t,\omega')$, its values at $-k\Delta\omega'$ are the values of $WD(t,\omega')$ at $k\Delta\omega'/A$. But, these points do not correspond to any sample along frequency axis. Thus, the interpolation has to be done (one way of doing it is in an appropriate zero padding of the signal, as indicated in [1]). A discrete form of convolution (6), including window $P(\theta)$ and the above considerations, is:

$$W_x(n,k) = \sum_{i=-P}^{P} P(i)LWD_2(n,k+i)WD(n,k+[i/A]) \quad (7)$$

where P is the width of $P(\theta)$ in the discrete domain, while $[i/A]$ is the nearest integer to i/A. The terms in summation in (7), when $k+i$ or $k+[i/A]$ is outside the basic period, are considered as being zero, in order to avoid possible aliasing. Architecture for the hardware (or software) implementation of the cross-term free (reduced) PWVD may be easily done according to (7) and using the systems for the cross-terms free realizations of the WD and LWD that are presented in [7], [9], [10], [11], [13]. It is shown in Fig.3.

IV. NUMERICAL EXAMPLE

Consider a multicomponent signal, with two real FM signals,

$$x(t) = \cos(50t^3/3 + 75t) + \cos(30t\,|t| + 265t) \quad (8)$$

within the interval $-1 \le t \le 1$. Signal is sampled at $2/N$, with $N = 256$. In the realization of the PWVD, an equivalent Hanning window $w(\tau)$ is used, i.e., signal at the input is multiplied by $w^{1/6}(\tau)$. As in [1] the length of $w(\tau)$ is assumed over the entire considered time interval ($T = 2$). Although, a narrower window would produce more concentrated distribution, we used this width in order to emphasize the artifacts and their reduction by the Polynomial distributions. Rectangular windows $P(\theta)$ are used in all convolutions: in the WD of the width $W_P = 35\pi$ (in the analog domain), while in the LWD and PWVD (since the auto term widths are significantly reduced as compared to the ones in the STFT) the

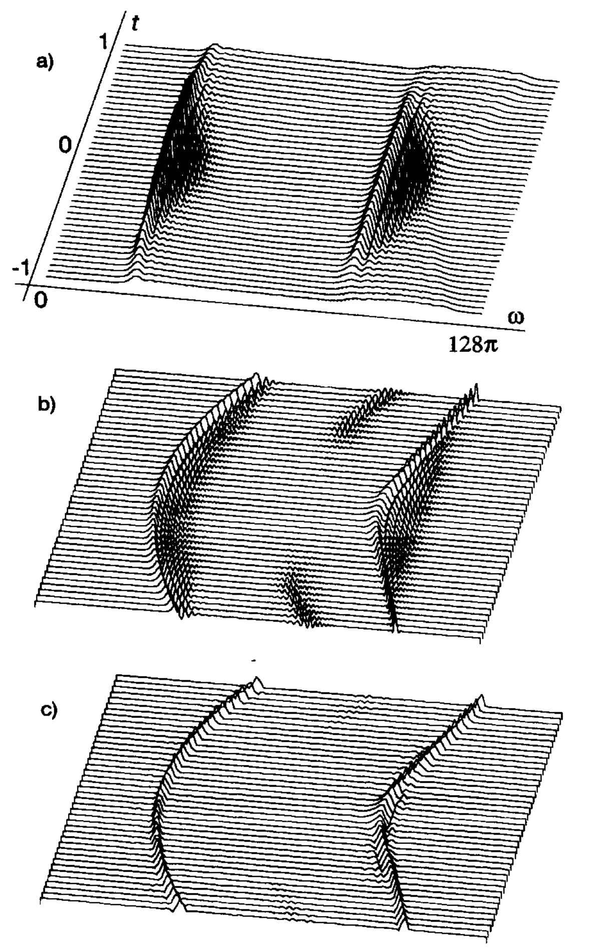

Fig. 4. a) Spectrogram, b) S-method, c) Polynomial Wigner-Ville distribution realizaed using the S-method.

window $P(\theta)$ width was $W_P = 17.5\pi$. For the reasons described above, an interpolation with factor 4 is used, i.e., signal is zero padded up to $4N$. Note that the signal sampling is done according to the Nyquist rate, i.e., twice less than it should be in the "ordinary" Wigner distribution, while the length of sequence for the FFT calculations, including zero padding, is only twice longer. Also, since the cross-term effects appearing between positive and negative frequencies will be eliminated (reduced) in the same way as the other cross-terms, there is no need for the analytic signal calculation. The results are presented in Fig.4. The cross-term free PWVD, highly concentrated at the instantaneous frequency (not only in the polynomial phase component, but in the non-polynomial FM component, as well) is shown in Fig.4c.

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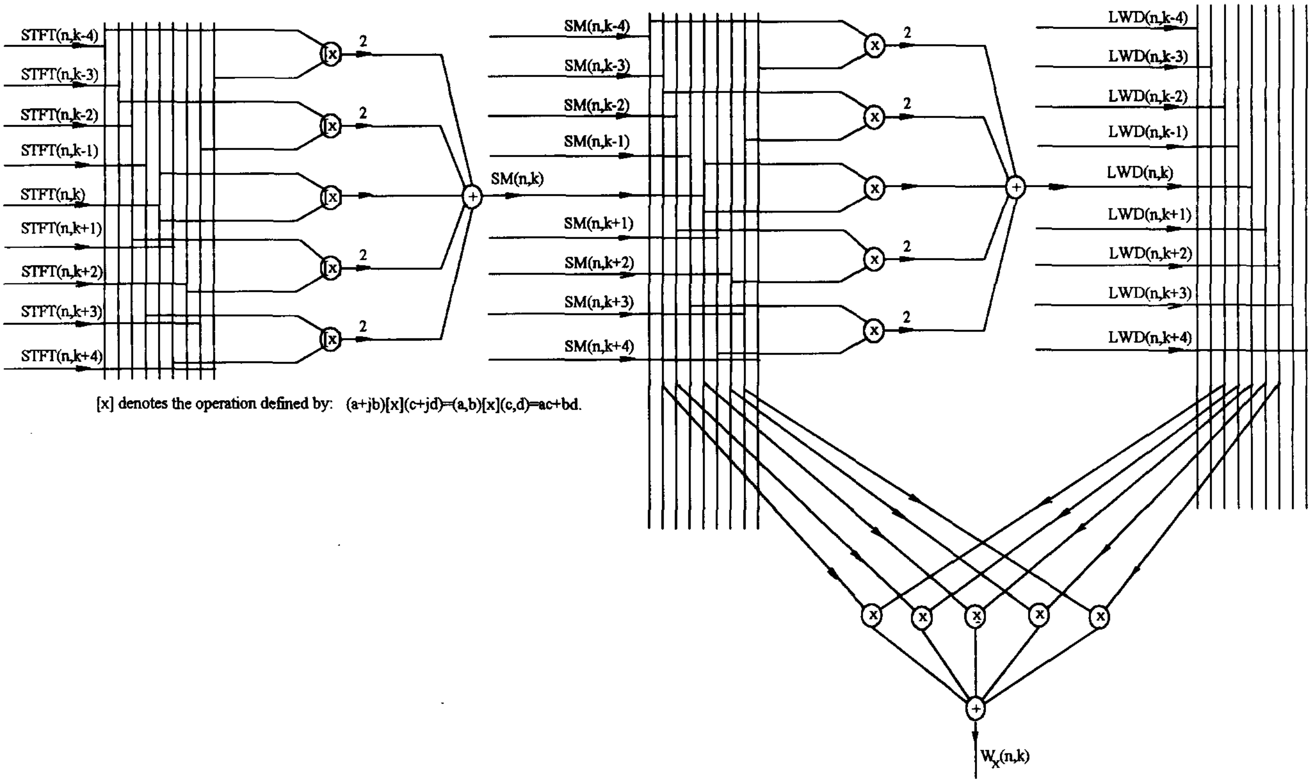

Fig. 3. Architecture for the cross-terms free polynomial Wigner-Ville distributions realization.

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