ASPECTS OF SPECTRUM AND HYBRID SPECTRUM ANALYSIS FOR SENSOR SNR DETERMINATION

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ABSTRACT

Analog Sensors are sometimes described in data sheets as having "infinite" resolution. Although there are no quantization effects in their analog output, their resolution is, of course, limited by the amount of noise present. To test their dynamical performance they have to be excited with a known function, e.g. a sinusoid.
Some discussion of possible methods for sensor data evaluation after sinusoidal excitation is given in this paper. A reliable method for sensor data evaluation and Signal-to-Noise Ratio (SNR) determination is proposed. An experimental set-up for this purpose is also described.

1. INTRODUCTION

1.1. Choice of Evaluation Domain

A sensor excited with a sinusoid yields a more or less sinusoidal output signal, which can be evaluated either in the time or frequency domain to obtain figures of merit such as the SNR or harmonic distortion.
Fos such purposes, scrutiny in the frequency domain offers more easily accessible and detailed information than time domain methods. Analysis described in this paper therefore takes place mainly in the frequency domain.
While evaluation errors can occur in both evaluation domains, they are easier to ascertain in the frequency domain.

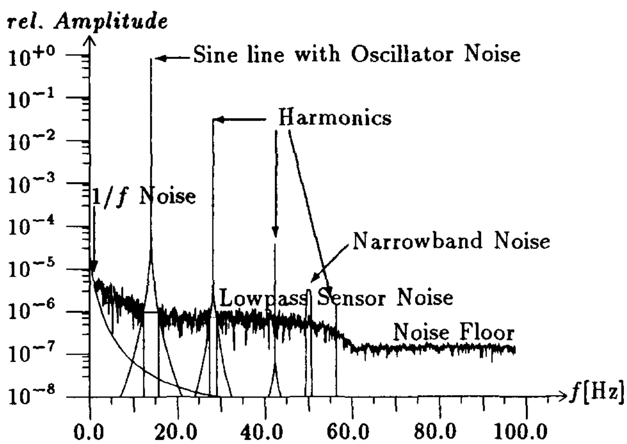

Figure 1. Schematic sensor spectrum

1.2. Components of a Sensor Signal

Fig. 1 shows a schematic spectrum containing the features typically found in a sensor spectrum after sinusoidal excitation. A real sensor spectrum will contain some or all of those features, namely sine, harmonics and noise lines as well as a noise floor and 1/f noise.
The task of evaluation now lies in extracting information from such a spectrum so that the SNR value can be reliably determined.

1.3. Evaluation of Spectra

A simple evaluation method uses a threshold above the noise floor, counting spectral values above this threshold as contributing to signal or harmonics power, values below it as contributing to the noise power (Fig. 2 (a)). Difficulties are encountered, however, if a non-ideal spectrum where lines are broadened and low-frequency and narrowband noise is present is to be evaluated (Fig. 2 (b)).

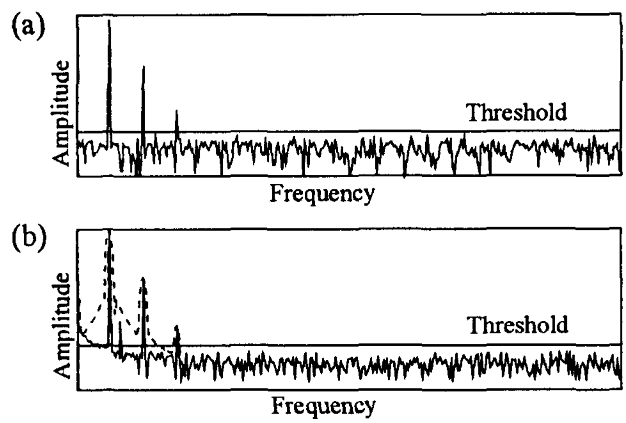

Figure 2. Threshold evaluation method

More reliable results are obtained with a line-oriented evaluation method [1]. Here the limits of every broadened line are determined by starting at the spectral value at each line crest and checking in both directions where the slope of a locally fitted straight line changes sign (Fig. 3).
The power content of each spectral value in the line is then split into a part contributing to the line power and another part forming part of the noise floor.
After this is done for all lines in the spectrum, the SNR value can be determined from the power in each of the ca-

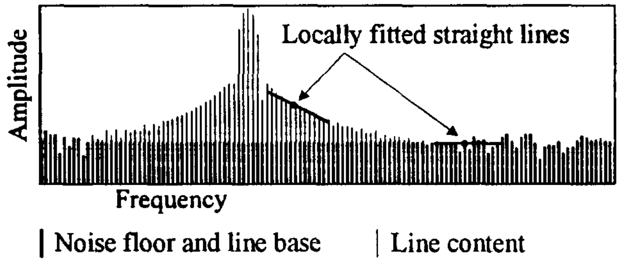

Figure 3. Line-oriented evaluation method

tegories of the preceding section according to Eq. 1.

$$\mathrm{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}} = \frac{\sum_i P_{\text{signal}_i}}{\sum_j P_{\text{noise}_j} + P_{\text{floor}}} \tag{1}$$

Each power P_X is calculated by summing all contributions to it from all spectral values h_k.

$$P_X = \sum_k h^2_{X_k}, \tag{2}$$

The power contained in the harmonics is not counted as noise power for purposes of SNR determination in this paper. If desired it can be used to determine the harmonic distortion of the sensor signal.

1.4. The Effective Number of Bits as a Figure of Merit for Analog Sensors

By analogy to ADCs, an effective number of bits (ENOB) can be calculated from the analog sensor SNR in decibels:

$$n_{\text{eff}} = \frac{\mathrm{SNR}_{dB} - 1,76}{6,02} \tag{3}$$

Such a figure of merit is useful if ADCs and sensors are to be adequately matched, e.g. in an automation environment. Effective bits, rather than decibels, are used in the remainder of this papar to express SNRs.

2. COHERENT AND REPETITIVE SAMPLING

2.1. Coherence

The examined spectra have to be free of leakage to ensure that noise lines lying close to a signal line can be detected. This includes short range leakage, so that the use of non-rectagular windows must be avoided.
This can be achieved by the use of coherent sampling, i.e. by fulfilling the coherence criterion of Eq. (4):

$$\frac{f_{\text{signal}}}{f_{\text{sampling}}} = \frac{m}{N} \tag{4}$$

This is equivalent to ensuring that an integer number m of sine periods is present in the data sample of length N.
In that case there is no leakage when the rectangular time window is used.
Such coherence can usually only be achieved if there is a common clock for all circuits used in the measurement set-up.

2.2. Occurrence of Repetitive Sampling

Repetitive sampling is a special case of coherent sampling. If a periodic function is sampled so that

$$x_n = x_{n+k} \tag{5}$$

is valid for all data points x_n of the data sample for a positive integer k this data set is repetitive (Fig. 4).

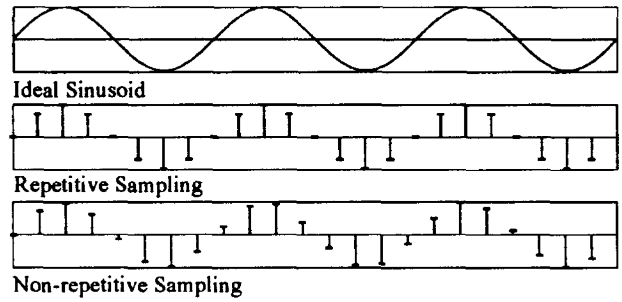

Figure 4. Repetitive and non-repetitive sampling of an ideal sinusoid

This is equivalent to a subset of the sample being repeated with a frequency $f_{\text{rep}} \leq f_{\text{sine}}$.
This leads to *quantisation lines* in the spectrum as the quantisation error, being deterministic, also occurs with the frequency f_{rep} and therefore is confined to the spectral values corresponding to f_{rep} and its multiples. If the sampling frequency simply is a multiple of the signal frequency ($f_{\text{rep}} = f_{\text{sine}}$), the additional lines coincide with the harmonics of the base frequency, causing the nonlinearities to be displayed larger than they truly are.
In Fig. 5 a spectrum of the digital-to-analog converter (DAC) signal generator of the measurement chain of Fig. 10 is shown. Most lines in the region below 10 Hz are caused by repetitive sampling ($f_{\text{rep}} = f_{\text{sine}}/9$).

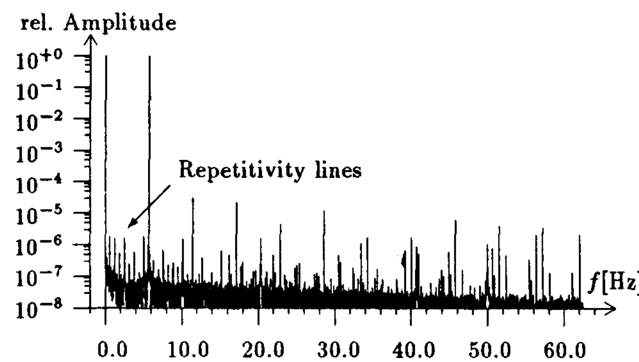

Figure 5. Generator spectrum

If repetitive sampling is to be avoided m and N in Eq. (4) must be chosen to be relatively prime.

2.3. Putting Repetitive Sampling to Use

On the other hand, if it is not necessary to determine harmonic distortion and f_{rep} is clearly below f_{sine}, i.e., if the sample starts to repeat itself only after several periods, the repetitivity can be used to confine the effects of the quantisation error into a few spectral values in the spectrum. This is particularly useful if not the quantisation noise floor itself is to be evaluated, but instead hides the effects of interest.

This is illustrated in Fig. 6, where a 10-bit ideal quantiser shows no noise floor when repetitive sampling is used owing to the above-mentioned effect. The quantisation noise is not confined entirely to those values when additional white noise is present, which reduces the effects of repetitivity (lower spectrum in Fig. 6).

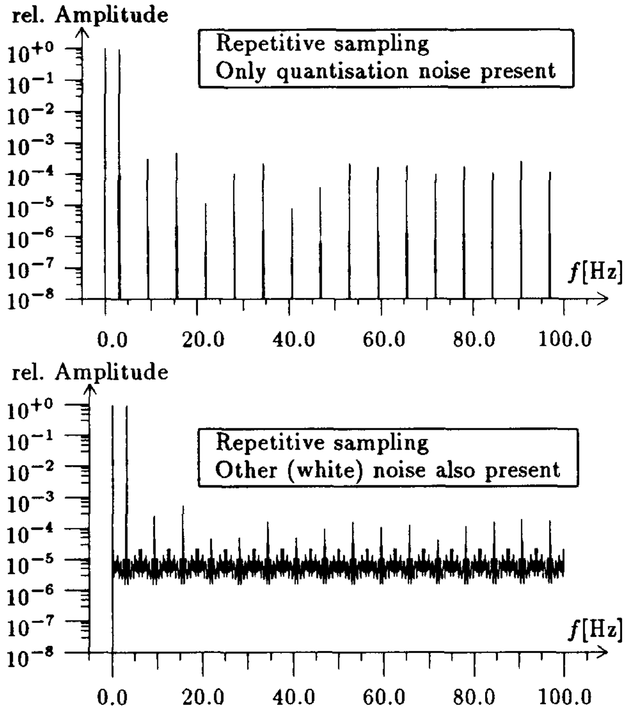

Figure 6. Repetitively sampled spectra

3. TREATMENT OF INCOHERENTLY RECORDED DATA SETS

Unfortunately it is not always possible to sample coherently. Alternatives in this case are period extraction and hybrid spectrum computation.

3.1. Period Extraction

For this method a section of the sample is chosen that contains a number of samples that is as near an integer as possible (Fig. 7).

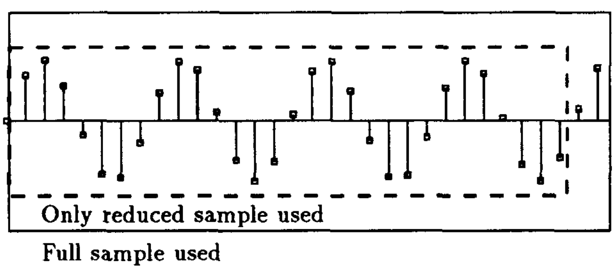

Figure 7. Period extraction

For obtaining the spectrum of the reduced sample it is usually necessary, however, to use a normal DFT rather than the more effective FFT.

This method, easily implemented, works best for a low ratio of periods to data points in the sample. If this ratio is high, the error made relative to the length of one period is to high for this method be of use, i.e. leakage is not significantly reduced.

3.2. Hybrid Spectrum Computation

When the required coherence in the system to fulfill Eq. (4) cannot be attained and non-rectangular windowing is to be avoided, a hybrid method of spectrum calculation can be useful to obtain virtually leakage-free spectra even after non-coherent sampling [1].

For this a sine wave corresponding to the fundamental is fitted to the data set. If the sensor output is highly nonlinear, further sine-waves may have to be fitted. The Downhill Simplex method [3] is suitable for this, because it requires no derivatives to be calculated. Now the exact frequency and power content of each harmonic are known. The residuum is then transformed via DFT or FFT. The power in the lines can then be distributed between the two closest spectral values computable and added to the FFT of the residuum (Fig. 8).

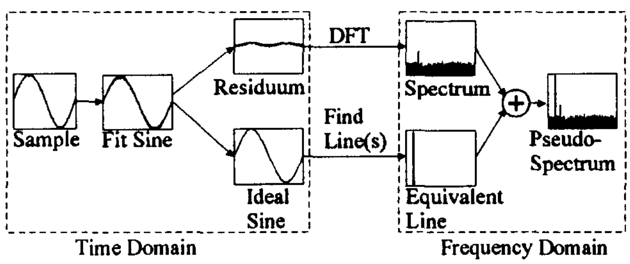

Figure 8. Procedure for hybrid spectrum computation

Although this method is far more expensive computationally than a straight FFT, minimal leakage (into only two spectral values) is obtained. Also, if the fitting procedure did not yield perfect results, this can be immediately seen in the resulting pseudo-spectrum, which in any case can be evaluated just like any other spectrum.

This is illustrated in Fig. 9 and compared to the results obtained using a simple FFT using a rectangular or Blackman window.

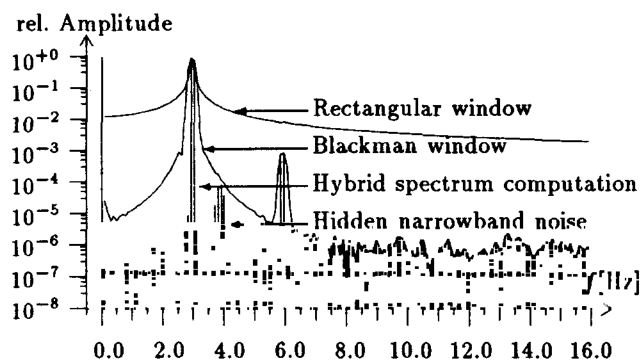

Figure 9. Comparing standard FFT and hybrid spectrum computation

The narrowband noise source, invisible to normal spectral analysis, is clearly discernible when a hybridly calculated spectrum is used. The line is broadened because, unlike the harmonics lines, it was not selected for sine fitting in the time domain.

4. MEASUREMENT SET-UP

For the evaluation of sensors a measurement chain of higher performance than the sensors under test is required. This measurement chain terminates in an ADC [6].
In this case coherent sampling is possible without excessive hardware effort.

4.1. Mechanical Set-Up

The most problematic part of the measurement chain is the distance actuator which is to provide the sinusoidal stimulation of the sensors.
A flat-membrane-loudspeaker driven by a DC HiFi Amplifier was chosen for this purpose.
Among the main advantages of using a loudspeaker are its high linearity, its flexibility when different waveforms and amplitudes are required and its electrical exitability.
The loudspeaker shows, however, a low-pass characteristic and is susceptible to ambient acoustic noise.
As an independent gage of the distance actuator's performance a laser interferometer was used which showed the resolution of the loudspeaker to be around 80 nm, clearly better than that of most tested sensors.

4.2. Signal Generation and Data Acquisition

For generation of the initial sinusoid a state-of-the-art HiFi DAC was used.
To ensure coherence of the entire measurement system the board clocks of the ADC and DAC boards as well as the sampling frequencies of both converters were derived from the same master clock.
The complete set-up is shown in Fig. 10.

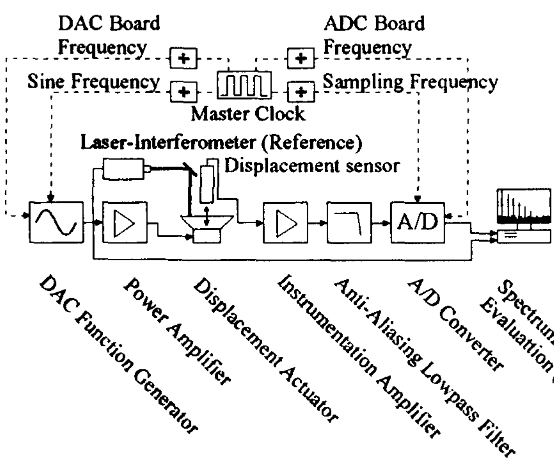

Figure 10. Measurement set-up

5. EXPERIMENTAL RESULTS

By adapting the outlined measurement method to different sensors the SNR (or, equivalently, the ENOB) was obtained for a number of distance, velocity and acceleration sensors including eddy current, differential transformer, strain gage and capacitive sensors and compared with the results of interferometric measurements.
An eddy current sensor spectrum is shown in Fig. 11. Ignoring nonlinearities, for this sensor a performance of around 12 effective bits was obtained.
This ENOB remained valid in a frequency range between 0.5 and 80 Hz and for mechanical amplitudes between 5 μm and 5 mm, the operating range in which the sensors were tested [5].
Other commercial sensors showed SNRs of between 8 and 13 effective bits [4].

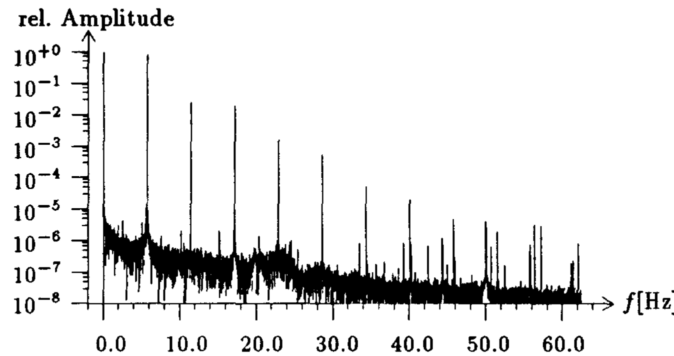

Figure 11. Spectrum of eddy current sensor

As can be seen in Fig. 11, the low frequency component of the sensor noise determines its SNR, which was also true for the other commercial sensors tested.
A strain gage bender bar set-up yielded a higher performance corresponding to an ENOB of about 18. Here also, the low frequency component of the noise determined the SNR.

REFERENCES

[1] A. Breitenbach: A Closer Look at Analog-to-Digital Converter Spectra, *Proceedings of the IMEKO TC-4 International Workshop on ADC Modelling at Smolenice Castle*, 1996, pp. 50-55

[2] A. Breitenbach, J. Tammemägi: The Effective Number of Bits of Analog Sensors
Proceedings of the Baltic Electronics Conference '96, Tallinn, pp. 63-66

[3] W. Press, S. Teukolsky, W. Vetterling, B. Flannery: *Numerical Recipies in C, 2nd Edition*, Cambridge University Press, 1992, pp. 408-412

[4] G. Lebelt, A. Breitenbach: Einsatz des Analog-Digital-Umsetzers PCM1760 zur Bestimmung des Signal/Rauschverhältnisses analoger Sensoren
In: *Die Meß-und Automatisierungstechnik; Bd 1: Meßtechnik und Meßsignalverarbeitung*, pp. 27-31
expert-Verlag, Renningen-Malmsheim, 1996

[5] A. Breitenbach, E. Schrüfer: Bestimmung der Zahl der effektiven Bit bei analogen Sensoren
X. Meßtechnisches Symposium, Tagungsband, pp. 111-118, München 1996.

[6] A. Breitenbach: An Experimental Set-Up for Determining the Signal-to-Noise Ratio of Sensors, *Baltic Electronics, Vol. 2 Issue 3*, Dec. 1996