APPROXIMATION OF COMPLEX-VALUED 2-D FREQUENCY RESPONSE SPECIFICATIONS BY SEPARABLE-DENOMINATOR DIGITAL FILTERS

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ABSTRACT

In this paper the approximation of a complex-valued specification by the frequency response of a 2-D IIR separable-denominator (SD) digital filter is considered. The approximation problem is transformed into an equivalent one, where a real-valued 2-D IIR SD digital filter with some additional characteristics has to be determined that approximates a given real-valued 2-D FIR digital filter. A theorem is presented that helps to reduce the number of parameters in the FIR-to-IIR approximation problem and a procedure to solve the problem numerically is given.

1. INTRODUCTION

In recent years a variety of techniques for the design of 2-D SD digital filters (SDDFs) has been developed. Most of these techniques are spatial domain approaches, where a given 2-D impulse response has to be approximated by the impulse response of a SDDF (e.g. [1]-[3]). Frequency response specifications have also been used for the design of SDDFs. But, except for some special cases (e.g. [4]), only the approximation of magnitude specifications has been considered. The progression of the phase was ignored.

In this paper, we are concerned with the general case of the frequency domain approximation, i.e. with the approximation of a complex-valued 2-D function $\hat{H}_0\left(e^{j\Omega_1}, e^{j\Omega_2}\right)$ by the transfer function $\hat{H}(z_1, z_2)$ of a SDDF on $z_i = e^{j\Omega_i}$ $(-\Omega_{i1} \leq \Omega_i \leq \Omega_{i1} < \pi)$ $(i = 1, 2)$. Our approximation procedure is an extension of the results presented in [5] to 2-D SD systems and is based on a transform introduced by S. Darlington [6] and a theorem due to J. L. Walsh [7, 8].

First, by applying the well known bilinear transform

$$z_i = \frac{1 + p_i \frac{T_i}{2}}{1 - p_i \frac{T_i}{2}} \quad \text{or} \quad p_i = \frac{2}{T_i} \frac{z_i - 1}{z_i + 1} \quad (i = 1, 2), \quad (1)$$

where $T_i = 2 \tan \frac{\Omega_{i1}}{2}$ $(i = 1, 2)$, the interval $-\Omega_{i1} \leq \Omega_i \leq \Omega_{i1}$ on the unit circle $z_i = e^{j\Omega_i}$ of the z_i-plane is mapped into the interval $-1 \leq \omega_i \leq 1$ on the imaginary axis $p_i = j\omega_i$ of a p_i-plane $(i = 1, 2)$ and vice versa. The specification $\hat{H}_0\left(e^{j\Omega_1}, e^{j\Omega_2}\right)$ is transformed into $H_0(j\omega_1, j\omega_2)$ by substituting $\Omega_i = 2 \arctan \frac{\omega_i T_i}{2}$ $(i = 1, 2)$ in $\hat{H}_0\left(e^{j\Omega_1}, e^{j\Omega_2}\right)$. Now we have to find a stable continuous-time SD filter $H(p_1, p_2)$ that approximates $H_0(j\omega_1, j\omega_2)$ for $p_i = j\omega_i$ $(-1 \leq \omega_i \leq 1)$ $(i = 1, 2)$.

In the next section it is shown how this approximation problem can be transformed into the problem of finding a real-valued SDDF with some additional characteristics that approximates a given real-valued 2-D FIR digital filter. Once a solution to this approximation problem is found, $H(p_1, p_2)$ can easily be calculated, and the interesting transfer function $\hat{H}(z_1, z_2)$ can be obtained by substituting p_i $(i = 1, 2)$ in $H(p_1, p_2)$ with eq. (1).

2. THE DARLINGTON TRANSFORM

In [6] the Darlington transform was used for the design of (1-D) electrical networks. Its generalization to 2-D systems is given by

$$p_i = \frac{1}{2}\left(\zeta_i - \frac{1}{\zeta_i}\right) \quad (i = 1, 2). \quad (2)$$

With eq. (2) the p_i-planes are transformed into ζ_i-planes $(i = 1, 2)$. The characteristics of this transform is discussed briefly in the following. The index "i" is omitted for simplicity. To demonstrate the one to one correspondence between the p-values and the ζ-values we introduce a two-sheet Riemann surface as depicted in Fig. 1. The essential properties of the transform are summarized in Table I. The two sheets

Table 1. Correspondences of the Darlington Transform.

Riemann p-surface	ζ-plane
$\operatorname{Re} p < 0$ (1st sheet)	$|\zeta| < 1$ and $\operatorname{Re} \zeta > 0$
$\operatorname{Re} p < 0$ (2nd sheet)	$|\zeta| > 1$ and $\operatorname{Re} \zeta < 0$
$\operatorname{Re} p > 0$ (1st sheet)	$|\zeta| > 1$ and $\operatorname{Re} \zeta > 0$
$\operatorname{Re} p > 0$ (2nd sheet)	$|\zeta| < 1$ and $\operatorname{Re} \zeta < 0$

of the Riemann surface are connected in a crosswise manner along the imaginary axes from $p = j$ to $p = j\infty$ and from $p = -j$ to $p = -j\infty$, respectively, as indicated in Fig. 1 by edges. The interval $p = j\omega$ $(-1 \leq \omega \leq 1)$ in the first sheet of the p-surface is transferred to the right half ($\operatorname{Re} \zeta \geq 0$) of the unit circle $|\zeta| = 1$. The interval $p = j\omega$ $(-1 \leq \omega \leq 1)$ in the second sheet of the p-surface is transferred to the left half ($\operatorname{Re} \zeta \leq 0$) of the unit circle $|\zeta| = 1$.

Denoting the points on the unit circle in the ζ_i-plane by $\zeta_i = e^{j\varphi_i}$ $(-\pi < \varphi_i \leq \pi)$ we obtain from (2) with $p_i = j\omega_i$

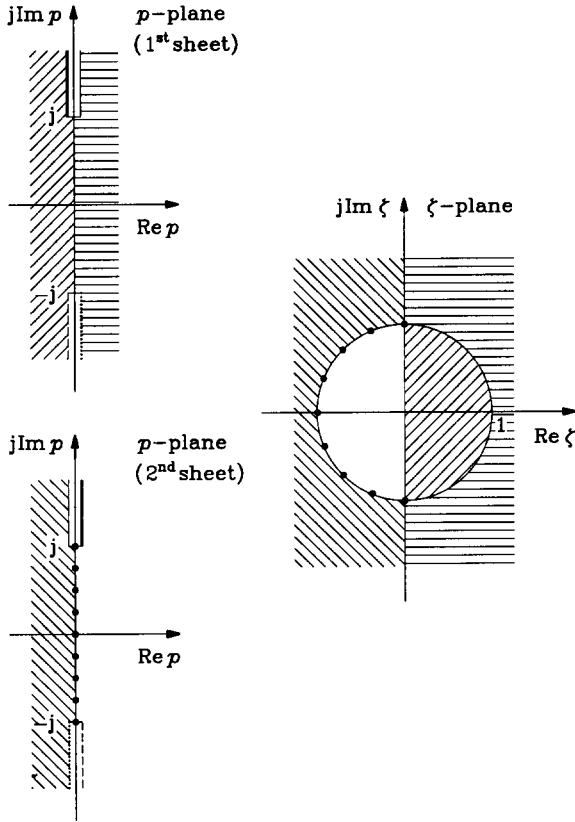$$\omega_i = \sin \varphi_i \quad (i = 1, 2) \quad (3)$$

Figure 1. Mapping of the Riemann p-surface onto the ζ-plane by the Darlington transform.

With this substitution the specification $H_0(\mathrm{j}\omega_1, \mathrm{j}\omega_2)$ is transformed into $h_0(\mathrm{e}^{\mathrm{j}\varphi_1}, \mathrm{e}^{\mathrm{j}\varphi_2}) := H_0(\mathrm{j}\sin\varphi_1, \mathrm{j}\sin\varphi_2)$. This function is periodic in φ_1 and φ_2 with period 2π. Thus, it can be expanded in a Fourier series as

$$h_0(\mathrm{e}^{\mathrm{j}\varphi_1}, \mathrm{e}^{\mathrm{j}\varphi_2}) = \sum_{\mu=-\infty}^{\infty}\sum_{\nu=-\infty}^{\infty} \gamma_{\mu\nu}\, \mathrm{e}^{-\mathrm{j}\mu\varphi_1}\mathrm{e}^{-\mathrm{j}\nu\varphi_2}, \quad (4)$$

and it can be shown, that the $\gamma_{\mu\nu}$ are real. This is a consequence of the fact that $H_0(\mathrm{j}\omega_1, \mathrm{j}\omega_2)$ must satisfy certain symmetry conditions in order that it can be regarded as the frequency response of a real system. Substituting $\mathrm{e}^{\mathrm{j}\varphi_i} =: \zeta_i$ $(i = 1,2)$ in eq. (4), we obtain a real-valued function $h_0(\zeta_1, \zeta_2)$. It can be written as

$$h_0(\zeta_1, \zeta_2) = f(\zeta_1, \zeta_2) + f(-\frac{1}{\zeta_1}, \zeta_2) +$$
$$+ f(\zeta_1, -\frac{1}{\zeta_2}) + f(-\frac{1}{\zeta_1}, -\frac{1}{\zeta_2}), \quad (5)$$

where

$$f(\zeta_1, \zeta_2) = \sum_{\mu=0}^{\infty}\sum_{\nu=0}^{\infty} \tilde{\gamma}_{\mu\nu}\, \zeta_1^{-\mu}\, \zeta_2^{-\nu}. \quad (6)$$

This real-valued function is analytic in $|\zeta_1| > 1 \wedge |\zeta_2| > 1$ and continuous in $|\zeta_1| \geq 1 \wedge |\zeta_2| \geq 1$. In view of eq. (5) and the relationship between $h_0(\mathrm{e}^{\mathrm{j}\varphi_1}, \mathrm{e}^{\mathrm{j}\varphi_2})$ and $H_0(\mathrm{j}\omega_1, \mathrm{j}\omega_2)$

it represents the originally given complex-valued frequency response specification. If it is approximated by a real, rational, SD function

$$\Phi(\zeta_1, \zeta_2) = \frac{\displaystyle\sum_{i=0}^{N_1}\sum_{j=0}^{N_2} p_{ij}\, \zeta_1^{-i}\zeta_2^{-j}}{\displaystyle\prod_{i=1}^{N_1}(1 - \alpha_i\zeta_1^{-1}) \prod_{j=1}^{N_2}(1 - \beta_j\zeta_2^{-1})}, \quad (7)$$

then the function

$$h(\zeta_1, \zeta_2) := \Phi(\zeta_1, \zeta_2) + \Phi(-\frac{1}{\zeta_1}, \zeta_2) +$$
$$+ \Phi(\zeta_1, -\frac{1}{\zeta_2}) + \Phi(-\frac{1}{\zeta_1}, -\frac{1}{\zeta_2}). \quad (8)$$

approximates $h_0(\zeta_1, \zeta_2)$ on $\zeta_i = \mathrm{e}^{\mathrm{j}\varphi_i}$ $(-\pi < \varphi_i \leq \pi)$ $(i = 1, 2)$. In view of eq. (8) there must exist a representation of $h(\zeta_1, \zeta_2)$ as

$$h(\zeta_1, \zeta_2) = H\left(\frac{1}{2}(\zeta_1 - \frac{1}{\zeta_1}), \frac{1}{2}(\zeta_2 - \frac{1}{\zeta_2})\right). \quad (9)$$

It can easily be determined, since $h(\zeta_1, \zeta_2)$ is a rational function and with the substitution eq. (2) we finally obtain a real, rational, SD transfer function $H(p_1, p_2)$ that approximates the given specification $H_0(\mathrm{j}\omega_1, \mathrm{j}\omega_2)$ on $p_i = \mathrm{j}\omega_i$ $(-1 \leq \omega_i \leq 1)$ $(i = 1, 2)$.

The poles of $H(p_1, p_2)$ must lie in Re $p_i < 0$ $(i = 1, 2)$. These stability conditions impose constraints on the poles of $h(\zeta_1, \zeta_2)$, eq. (8). Regarding the correspondences of the Darlington transform, given in Table I, we see that the poles of $h(\zeta_1, \zeta_2)$ must lie in one of the two domains $\{\zeta_i | |\zeta_i| < 1$ and Re $\zeta_i > 0\}$ or $\{\zeta_i | |\zeta_i| > 1$ and Re $\zeta_i < 0\}$ $(i = 1, 2)$. To fulfill these conditions, the poles of $\Phi(\zeta_1, \zeta_2)$, eq. (7), must be chosen appropriately. Since the function $\Phi(\zeta_1, \zeta_2)$ should have the same region of analyticity $(|\zeta_1| > 1 \wedge |\zeta_2| > 1)$ as $f(\zeta_1, \zeta_2)$, eq. (6), its poles must satisfy $|\alpha_i| < 1 \wedge$ Re $\alpha_i > 0$ $(i = 1, 2, \ldots, N_1)$ and $|\beta_i| < 1 \wedge$ Re $\beta_i > 0$ $(i = 1, 2, \ldots, N_2)$. That is, $f(\zeta_1, \zeta_2)$ has to be approximated by a 2-D SD stable IIR digital filter transfer function, whose poles must have positive real part.

Taking a finite sum of eq. (6) yields

$$f_t(\zeta_1, \zeta_2) = \sum_{\mu=0}^{L_1}\sum_{\nu=0}^{L_2} \tilde{\gamma}_{\mu\nu}\, \zeta_1^{-\mu}\zeta_2^{-\nu}, \quad (10)$$

which is a 2-D FIR transfer function close to $f(\zeta_1, \zeta_2)$, provided L_1 and L_2 are chosen sufficiently large. We now have the problem of approximating a 2-D FIR transfer function by a 2-D SD IIR transfer function. One could try to solve it e.g. by using the approach given in [3]. But there is no guarantee that the poles of the function $\Phi(\zeta_1, \zeta_2)$, eq. (7), thus obtained, all have positive real parts, as required. To avoid this difficulty, we solve the problem by a constrained minimization procedure, as described in section 4. The necessary calculations can be reduced by a considerable amount with the help of a theorem, presented in the next section.

3. THE THEOREM OF WALSH

For the approximation of 1-D functions, that are analytic in a certain region, by rational functions in the sense of least squares, J. L. Walsh has given a theorem [7], that helps to reduce the number of parameters by imposing constraints on the numerator coefficients. The generalization of this theorem to the 2-D case was made in [8] and is given below.

Theorem 1 *Among all possible functions $\Phi(\zeta_1, \zeta_2)$ of the form eq. (7), with prescibed poles α_i ($i = 1, 2, \ldots, N_1$) and β_j ($j = 1, 2, \ldots, N_2$), that are fixed and located in $|\zeta_k| < 1$ ($k = 1, 2$), that one is the best approximation in the sense of least squares to $f(\zeta_1, \zeta_2)$, analytic in $|\zeta_1| > 1 \wedge |\zeta_2| > 1$ and continuous in $|\zeta_1| \geq 1 \wedge |\zeta_2| \geq 1$, that interpolates to $f(\zeta_1, \zeta_2)$ in all points of the set*

$$S = \left\{ (\zeta_1, \zeta_2) \middle| \; \zeta_1 \in \left\{ \infty, \frac{1}{\alpha_1^*}, \ldots, \frac{1}{\alpha_{N_1}^*} \right\} \wedge \right.$$
$$\left. \zeta_2 \in \left\{ \infty, \frac{1}{\beta_1^*}, \ldots, \frac{1}{\beta_{N_2}^*} \right\} \right\}.$$

These interpolation conditions can be used to formulate a uniquely solvable set of $(N_1 + 1)(N_2 + 1)$ linear equations for the $(N_1 + 1)(N_2 + 1)$ numerator coefficients of $\Phi(\zeta_1, \zeta_2)$, as follows:

The interpolation points are denoted as

$$\zeta_{10} = \infty, \quad \zeta_{1i} = \frac{1}{\alpha_i^*} \quad (i = 1, 2, \ldots, N_1) \qquad (11)$$

and

$$\zeta_{20} = \infty, \quad \zeta_{2j} = \frac{1}{\beta_j^*} \quad (j = 1, 2, \ldots, N_2). \qquad (12)$$

We assume, that the number of distinct points in the set ζ_{kl} ($l = 0, 1, \ldots, N_k$) is r_k, where $1 \leq r_k \leq N_k + 1$ ($k = 1, 2$). Let these distinct points, after a possible renumbering of the indices, be the points ζ_{ki} ($i = 0, 1, \ldots, r_k - 1$), which means, that

$$\zeta_{ki} \neq \zeta_{kj} \quad (i \neq j; i = 0, 1, \ldots, r_k - 1; j = 0, 1, \ldots, r_k - 1) \qquad (13)$$

and

$$\zeta_{ki} \in \{\zeta_{k,0}, \zeta_{k,1}, \ldots, \zeta_{k,r_k-1}\} \quad (i = r_k, r_k+1, \ldots, N_k) \quad (14)$$

are valid ($k = 1, 2$). Now, denoting the multiplicity of each point ζ_{ki} ($i = 0, 1, \ldots, r_k - 1$) by s_{ki} ($k = 1, 2$), the interpolation conditions read

$$\frac{\partial^\mu}{\partial \zeta_1^\mu} \frac{\partial^\nu}{\partial \zeta_2^\nu} \Phi(\zeta_1, \zeta_2) \bigg|_{\substack{\zeta_1 = \zeta_{1i} \\ \zeta_2 = \zeta_{2j}}} = \frac{\partial^\mu}{\partial \zeta_1^\mu} \frac{\partial^\nu}{\partial \zeta_1^\nu} f(\zeta_1, \zeta_2) \bigg|_{\substack{\zeta_1 = \zeta_{1i} \\ \zeta_2 = \zeta_{2j}}}$$
$$(\mu = 0, 1, \ldots, s_{1i} - 1; \; \nu = 0, 1, \ldots, s_{2j} - 1)$$
$$(i = 0, 1, \ldots, r_1 - 1; \; j = 0, 1, \ldots, r_2 - 1). \qquad (15)$$

Eqs. (15) describe a set of $(N_1 + 1)(N_2 + 1)$ linearly independent equations, which are linear in the $(N_1 + 1)(N_2 + 1)$ unknown numerator coefficients. The solution of this set of equations is unique. Therefore, the numerator of $\Phi(\zeta_1, \zeta_2)$ is uniquely determined by the poles.

A direct application of eqs. (15) is impractical. In the next section, it is shown, how in our case, where we take $f_t(\zeta_1, \zeta_2)$, eq. (10), as the function to be approximated, the numerator coefficients of $\Phi(\zeta_1, \zeta_2)$ can be calculated by using eqs. (15) implicitly.

4. THE FIR APPROXIMATION PROCEDURE

Let the unknown numerator of $\Phi(\zeta_1, \zeta_2)$, eq. (7), be denoted by $P(\zeta_1, \zeta_2)$ and the denominator polynomials by $Q_k(\zeta_k)$ and let the $Q_k(\zeta_k)$ be given as

$$Q_k(\zeta_k) = \sum_{\nu=0}^{N_k} q_{k\nu} \zeta_k^{-\nu} \quad (q_{k0} = 1) \quad (k = 1, 2). \qquad (16)$$

Then the unique numerator polynomial $P(\zeta_1, \zeta_2)$, that satisfies eqs. (15), can be calculated in two steps:

First we determine a polynomial

$$C(\zeta_1, \zeta_2) = \sum_{i=0}^{N_1} \sum_{j=0}^{L_2} c_{ij} \zeta_1^{-i} \zeta_2^{-j} \qquad (17)$$

such that the rational function

$$g(\zeta_1, \zeta_2) := \frac{C(\zeta_1, \zeta_2)}{Q_1(\zeta_1)} \qquad (18)$$

interpolates to $f_t(\zeta_1, \zeta_2)$, eq. (10), w.r.t. ζ_1 in the sense of the theorem. This means, that the difference function $\bar{\Delta}(\zeta_1, \zeta_2) := f_t(\zeta_1, \zeta_2) - g(\zeta_1, \zeta_2)$ vanishes in the points $\zeta_1 = \zeta_{1i}$ ($i = 0, 1, \ldots, N_1$), eq. (11), for arbitrary ζ_2. As a consequence $C(\zeta_1, \zeta_2)$ must satisfy

$$f_t(\zeta_1, \zeta_2)Q_1(\zeta_1) - C(\zeta_1, \zeta_2) = \zeta_1^{-(N_1+1)}Q_1(\zeta_1^{-1})R_1(\zeta_1, \zeta_2),$$
$$(19)$$

where

$$R_1(\zeta_1, \zeta_2) = \sum_{i=0}^{L_1-1} \sum_{j=0}^{L_2} r_{ij}^{(1)} \zeta_1^{-i} \zeta_2^{-j}. \qquad (20)$$

After some manipulation of eq. (19), a simple procedure to calculate $R_1(\zeta_1, \zeta_2)$ and $C(\zeta_1, \zeta_2)$ can be derived. It requires a digital filter operation to get $R_1(\zeta_1, \zeta_2)$ and then $C(\zeta_1, \zeta_2)$ can be obtained from eq. (19). Details are omitted for brevity.

In the second step, $P(\zeta_1, \zeta_2)$ is obtained from $C(\zeta_1, \zeta_2)$, eq. (17), as follows. We require, that the rational function

$$v(\zeta_1, \zeta_2) := \frac{P(\zeta_1, \zeta_2)}{Q_2(\zeta_2)} \qquad (21)$$

interpolates to $C(\zeta_1, \zeta_2)$, eq. (17), w.r.t. ζ_2 in the sense of the theorem. This means, that the difference function $\bar{\Delta}(\zeta_1, \zeta_2) := C(\zeta_1, \zeta_2) - v(\zeta_1, \zeta_2)$ vanishes in the points $\zeta_2 = \zeta_{2j}$ ($j = 0, 1, \ldots, N_2$), eq. (12), for arbitrary ζ_1. As a consequence $P(\zeta_1, \zeta_2)$ must satisfy

$$C(\zeta_1, \zeta_2)Q_2(\zeta_2) - P(\zeta_1, \zeta_2) = \zeta_2^{-(N_2+1)}Q_2(\zeta_2^{-1})R_2(\zeta_1, \zeta_2),$$
$$(22)$$

where

$$R_2(\zeta_1, \zeta_2) = \sum_{i=0}^{N_1} \sum_{j=0}^{L_2-1} r_{ij}^{(2)} \zeta_1^{-i} \zeta_2^{-j}. \qquad (23)$$

From eq. (22) $R_2(\zeta_1,\zeta_2)$ and $P(\zeta_1,\zeta_2)$ can be obtained in the same way as $R_1(\zeta_1,\zeta_2)$ and $C(\zeta_1,\zeta_2)$ from eq. (19).

Now consider the difference function

$$\Delta(\zeta_1,\zeta_2) := f_t(\zeta_1,\zeta_2) - \Phi(\zeta_1,\zeta_2), \qquad (24)$$

which can be written as

$$\Delta(\zeta_1,\zeta_2) = \tilde{\Delta}(\zeta_1,\zeta_2) + \frac{\bar{\Delta}(\zeta_1,\zeta_2)}{Q_1(\zeta_1)}. \qquad (25)$$

It vanishes in the interpolation points $(\zeta_1,\zeta_2) \in S$, which indicates, that $P(\zeta_1,\zeta_2)$ has been determined such that $\Phi(\zeta_1,\zeta_2)$, eq. (7), satisfies the interpolation conditions eqs. (15).

We now take the coefficients $q_{k\nu}$ $(\nu = 1,2,\ldots,N_k)$ of the denominator polynomials $Q_k(\zeta_k)$ $(k = 1,2)$, eq. (16), as the variables of an iterative minimization procedure, where we try to minimize the l_2-norm of $\Delta(\zeta_1,\zeta_2)$. Using eqs. (19) and (22), we can show that the square of this l_2-norm can be calculated by

$$\left\| \Delta(\zeta_1,\zeta_2) \right\|^2 = \left\| R_1(\zeta_1,\zeta_2) \right\|^2 + \left\| \frac{R_2(\zeta_1,\zeta_2)}{Q_1(\zeta_1)} \right\|^2. \qquad (26)$$

In each iteration step, the polynomials $C(\zeta_1,\zeta_2)$, $R_1(\zeta_1,\zeta_2)$, $R_2(\zeta_1,\zeta_2)$ and $P(\zeta_1,\zeta_2)$ are obtained as shown above. Note that we do not need to know the roots of $Q_k(\zeta_k)$ $(k = 1,2)$ explicitly. But it must be ensured, that they lie in the unit circle and have positive real parts. This can be achieved by taking the requirements, that the $Q_k(\zeta_k)$ are Schur polynomials and that the $Q_k(-\zeta_k)$ simultaneously are Hurwitz polynomials $(k = 1,2)$, as constraints.

5. AN EXAMPLE

With the described procedure a SDDF $\hat{H}(z_1,z_2)$ with $N_1 = N_2 = 15$ was designed, whose frequency response approximates the specification $\hat{H}_0\left(e^{j\Omega_1}, e^{j\Omega_2}\right) = e^{-(\Omega_1\Omega_2)^2} + j\Omega_1^2\left(\sin\Omega_2\right)^3$ in the frequency domain $(-0.4\pi \leq \Omega_1 \leq 0.4\pi;\ -0.7\pi \leq \Omega_2 \leq 0.7\pi)$. Fig. 2 shows the real and imaginary parts of the specification $\hat{H}_0\left(e^{j\Omega_1}, e^{j\Omega_2}\right)$ and of the frequency response of $\hat{H}(z_1,z_2)$. The approximation error

$$e := \left[\frac{1}{\Omega_{11}\Omega_{21}} \int\limits_{0}^{\Omega_{11}} \int\limits_{0}^{\Omega_{21}} \left| \hat{\Delta}\left(e^{j\Omega_1}, e^{j\Omega_2}\right) \right|^2 d\Omega_2\, d\Omega_1 \right]^{\frac{1}{2}},$$

where $\hat{\Delta}\left(e^{j\Omega_1}, e^{j\Omega_2}\right) := \hat{H}_0\left(e^{j\Omega_1}, e^{j\Omega_2}\right) - \hat{H}\left(e^{j\Omega_1}, e^{j\Omega_2}\right)$, $\Omega_{11} = 0.4\pi$ and $\Omega_{21} = 0.7\pi$, was $e = 0.0465$.

6. CONCLUSION

A procedure for the design of SDDFs was described. The approximation of a given complex-valued frequency response specification was achieved with the help of the bilinear transform, the Darlington transform and a constrained minimization procedure. The problem was reformulated as an FIR-to-IIR approximation problem, whose solution was considerably simplified by a theorem of Walsh. An example was presented to show the applicability of our approach.
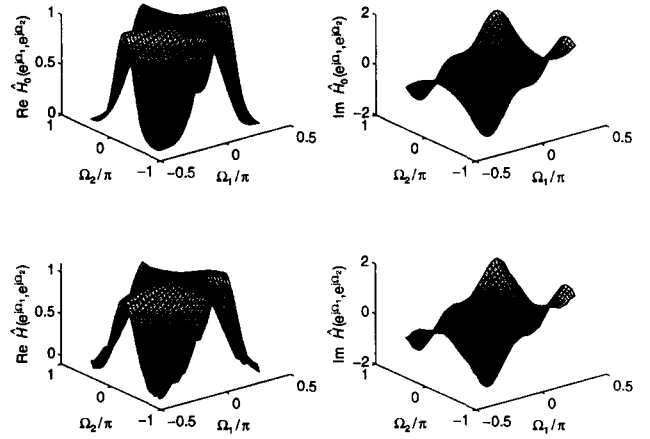

Figure 2. Real- and imaginary parts of the given specification and of the frequency response of the designed SDDF.

REFERENCES

[1] B. Lashgari, L. M. Silverman, J.-F. Abramatic, "Approximation of 2-D Separable in Denominator Filters", *IEEE Trans. Circuits Syst.*, vol. CAS-30, pp. 107-121, Feb. 1983.

[2] T. Hinamoto, "Design of 2-D Separable-Denominator Recursive Digital Filters", *IEEE Trans. Circuits Syst.*, vol. CAS-31, pp. 925-932, Nov. 1984.

[3] T. Lin, M. Kawamata and T. Higuchi, "Design of 2-D Separable-Denominator Digital Filters Based on the Reduced-Dimensional Decomposition", *IEEE Trans. Circuits Syst.*, vol. CAS-34, pp. 934-941, Aug. 1987.

[4] M. Ahmadi, M. T. Boraie, V. Ramchandran and C. S. Gargour, "Design of 2-D Recursive Digital Filters with Constant Group Delay Characteristics Using Separable Denominator Transfer Function and a New Stability Test", *IEEE Trans. Acoust. Speech, Signal Processing*, vol. ASSP-33, pp. 1316-1318, Oct. 1985.

[5] R. Unbehauen, "On the Approximation of Complex-Valued Frequency Response Specifications", *Frequenz*, vol. 44, pp. 92-95, March 1990.

[6] S. Darlington, "Network Synthesis Using Tchebyscheff Polynomial Series," *Bell Syst. Techn. Journal*, vol. 31, pp. 613-665, July 1952.

[7] J. L. Walsh, *Interpolation and Approximation by Rational Functions in the Complex Domain*, American Math. Society, Providence, Rhode Island, 1965.

[8] X. Nie, *Beiträge zur Synthese zweidimensionaler Digitalfilter* ("Contributions to the Synthesis of two-dimensional digital filters", in German), Ph.D. thesis, Lehrstuhl für Allgemeine und Theoretische Elektrotechnik, University Erlangen-Nürnberg, 1992.