

# THE SYNTHESIS OF SHARP DIAMOND-SHAPED FILTERS USING THE FREQUENCY RESPONSE MASKING APPROACH

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## ABSTRACT

The frequency response masking technique is an efficient method to realize sharp 1-D filters. This technique can synthesize sharp 1-D filters with a considerably lower complexity when compared to direct-form implementations. In this paper, we extend the frequency response masking technique to the design of 2-D diamond-shaped filters. The design procedure as well as the prototype and masking filters specifications are presented in this paper. A design example is also provided to illustrate the effectiveness of the approach.

## 1. INTRODUCTION

Linear phase FIR digital filters are frequently used in signal processing applications for their guaranteed stability and freedom from phase distortion. One disadvantage of FIR filters is that sharp filters have very high implementation complexities. Since the filter length is inversely proportional to the transition width [1], the implementation of a sharp filter would require a prohibitively long filter length. A solution to this would be to employ the frequency response masking (FRM) technique [2] to implement the filter. This technique is capable of synthesizing very sharp 1-D filters with low implementation complexities.

For the 2-D diamond-shaped (DS) filter, the results in [3] indicate that the filter support size ( $N \times N$ ) is inversely proportional to the square of the transition width. Thus, the high complexity problem is even more acute in the design of sharp DS filters. In this paper, we extend the FRM technique to synthesize sharp DS filters. We present the design procedure as well as the specifications of the prototype and masking filters. A design example is also included to illustrate the effectiveness of the technique.

## 2. DEFINITIONS AND NOTATIONS

The parameters that define the 2-D filters used in the FRM technique are as follows. The passband and stopband edges of the DS low-pass filter are denoted by  $\omega_p$  and  $\omega_s$ ;  $\omega_p$  and  $\omega_s$  are the frequency values where the respective band edges (extrapolated if necessary) meet the frequency axes (see Fig. 1(a)). The transition width of the DS filter is given by

$$\Delta_{ds} = \omega_s - \omega_p. \quad (1)$$

We define two rectangular-shaped (RS) 2-D low-pass filters which are used as masking filters. The first RS filter,

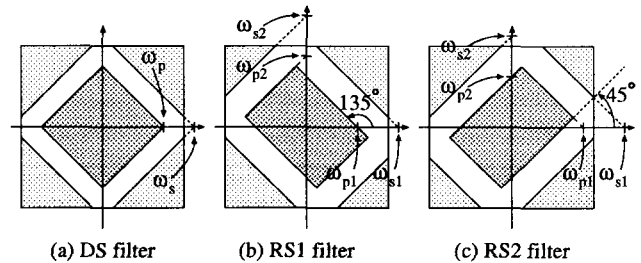


Figure 1. DS and RS filters specifications.

denoted by RS1, has the longer side of the rectangular passband at an angle of  $135^\circ$  to the horizontal frequency axis as illustrated in Fig. 1(b). The second RS filter, denoted by RS2, is obtained by rotating a RS1 filter by  $90^\circ$  and is shown in Fig. 1(c). For both the RS1 and RS2 filters, the passband and stopband edges are defined by  $\omega_{p1}$ ,  $\omega_{p2}$ ,  $\omega_{s1}$  and  $\omega_{s2}$ .  $\omega_{p1}$  and  $\omega_{s1}$  define the band edges which are at an angle of  $135^\circ$  to the horizontal frequency axis; these are respectively, the frequency values where the passband and stopband edges (extrapolated if necessary) meet the horizontal frequency axis. The band edges which are at an angle of  $45^\circ$  to the horizontal frequency axis are defined by  $\omega_{p2}$  and  $\omega_{s2}$ . These are the frequency values where the passband and stopband edges meet the vertical frequency axis. Two measures of the transition width for the RS filter,  $\Delta_1$  and  $\Delta_2$  can be defined

$$\Delta_1 = \omega_{s1} - \omega_{p1}, \quad \Delta_2 = \omega_{s2} - \omega_{p2}. \quad (2)$$

The band edges and transition width of the RS filter can be written in vector notation as

$$\Omega_p = [\omega_{p1} \ \omega_{p2}]^T \quad (3a)$$

$$\Omega_s = [\omega_{s1} \ \omega_{s2}]^T \quad (3b)$$

$$\Delta_{rs} = \Omega_s - \Omega_p. \quad (3c)$$

## 3. DESIGN PROCEDURE

The FRM filter design technique hinges on how the frequency spectrum can be divided into suitable complementary components. For the case of 1-D FRM, this can be done by dividing the frequency spectrum into two components [2]. There is no known simple rule that extends this procedure to the synthesis of any arbitrary-shaped 2-D filter. However, we discovered that by dividing the frequency

spectrum into four suitably chosen complementary components and using appropriate masking filters, the FRM technique can also be applied to the design of DS filters.

### 3.1. Complementary Components

Fig. 2 shows how the 2-D frequency region  $[-\pi/M, \pi/M]^2$ , where  $[a, b]^D$  denotes the set of  $D \times 1$  real vector  $X$  with components  $x_i$  in the range  $a \leq x_i < b$  and  $M$  is the up-sampling ratio, is divided to obtain the four complementary components. The term  $\omega_\alpha/M$  in Fig. 2 denotes the frequency value at the intersection of the four regions.

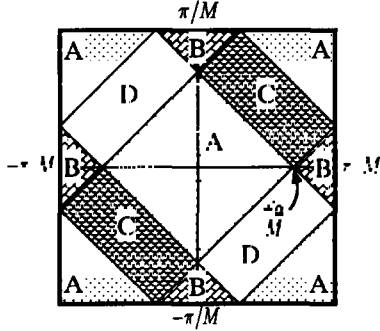


Figure 2. The four complementary components.

Let the prototype filter  $F_Y(\Omega)$ , where  $Y \in \{A, B, C, D\}$ , have the ideal frequency response such that if  $M = 1$  in Fig. 2, the gain is unity in region  $Y$  and zero elsewhere. In general  $M \neq 1$  and component  $Y$  can be obtained by applying  $F'_Y(\Omega)$  to the input, where  $F'_Y(\Omega)$  is the filter obtained by up-sampling by  $M$  times the impulse response of  $F_Y(\Omega)$  with zeros.

### 3.2. Implementation

The block diagram of the frequency response technique is illustrated in Fig. 3. In Fig. 3,  $X(\Omega)$  is the input and  $F'_A(\Omega)$ ,  $F'_B(\Omega)$ ,  $F'_C(\Omega)$  and  $F'_D(\Omega)$  are the 2-D prototype filters with the desired frequency response as described previously. The components  $A$ ,  $B$ ,  $C$  and  $D$  are given by

$$Y = F'_Y(\Omega)X(\Omega) + E/4 \quad (4)$$

where

$$E = X(\Omega)[1 - F'_A(\Omega) - F'_B(\Omega) - F'_C(\Omega) - F'_D(\Omega)]. \quad (5)$$

The term  $E/4$  has been added to ensure that the summation of the components  $A$ ,  $B$ ,  $C$  and  $D$  is identical to the input  $X(\Omega)$ . The notation  $F'_Y(\Omega)$  shall be used to denote the transfer function from  $X(\Omega)$  to component  $Y$ , i.e.

$$Y = F'_Y(\Omega)X(\Omega) \quad (6a)$$

where  $F'_Y(\Omega)$  is given by

$$F'_Y(\Omega) = \frac{1}{4} [1 - F'_A(\Omega) - F'_B(\Omega) - F'_C(\Omega) - F'_D(\Omega) + 4F'_Y(\Omega)]. \quad (6b)$$

From (6b), it is clear that the summation of  $F'_A(\Omega)$ ,  $F'_B(\Omega)$ ,  $F'_C(\Omega)$  and  $F'_D(\Omega)$  is unity, i.e.

$$F'_A(\Omega) + F'_B(\Omega) + F'_C(\Omega) + F'_D(\Omega) = 1. \quad (7)$$

Thus, these four transfer functions will be referred to jointly as "complementary" filters.

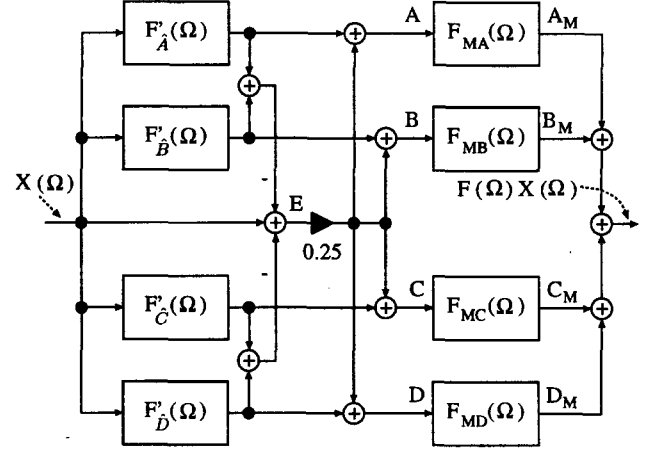


Figure 3. Block diagram of frequency response masking technique.

Each of the component  $A$ ,  $B$ ,  $C$  and  $D$  is subsequently passed through the respective masking filter ( $F_{MA}(\Omega)$ ,  $F_{MB}(\Omega)$ ,  $F_{MC}(\Omega)$  and  $F_{MD}(\Omega)$ ) to obtain  $A_M$ ,  $B_M$ ,  $C_M$  and  $D_M$ . The final output  $F(\Omega)X(\Omega)$  is obtained by the summation of  $A_M$ ,  $B_M$ ,  $C_M$  and  $D_M$ .

We shall illustrate the FRM technique for the case where  $M$  is 3. The frequency responses of  $F'_A(\Omega)$ ,  $F'_B(\Omega)$ ,  $F'_C(\Omega)$  and  $F'_D(\Omega)$  are shown in Fig. 4(a)-(d). Four masking filters,  $F_{MA}(\Omega)$ ,  $F_{MB}(\Omega)$ ,  $F_{MC}(\Omega)$  and  $F_{MD}(\Omega)$ , with the frequency responses as shown superimposed onto the complementary filters in Fig. 4(a)-(d), are applied to the filters  $F'_A(\Omega)$ ,  $F'_B(\Omega)$ ,  $F'_C(\Omega)$  and  $F'_D(\Omega)$ , respectively. By adding the outputs from the masking filters, the resulting frequency response  $F(\Omega)$ , shown in Fig. 4(e), is obtained. Notice that the corners of the passband of  $F(\Omega)$  are contributed from the passband of  $F'_A(\Omega)$ . We shall refer to this as CASE A. It is also possible that the corners of the passband of  $F(\Omega)$  are contributed by the passband of  $F'_B(\Omega)$ . Such a case will be designated as CASE B.

### 3.3. Prototype and Masking Filters Specifications

We shall now give the specifications of the prototype and masking filters. Suppose the band edges of the desired filter  $F(\Omega)$  are  $\psi_p$  and  $\psi_s$ , and the transition width is  $\Delta_\alpha = \psi_s - \psi_p$ . Let the transition widths of the prototype filters in the horizontal (or vertical) frequency dimension be  $\Delta_\alpha$  and define

$$\theta = \omega_\alpha - \frac{\Delta_\alpha}{2}, \quad \phi = \omega_\alpha + \frac{\Delta_\alpha}{2}, \quad (8)$$

it can be shown that  $\psi_p$  and  $\psi_s$  are related to  $\theta$  and  $\phi$  by

$$\psi_p = \frac{2m\pi + \theta}{M} \quad (9a)$$

$$\psi_s = \frac{2m\pi + \phi}{M} \quad (9b)$$

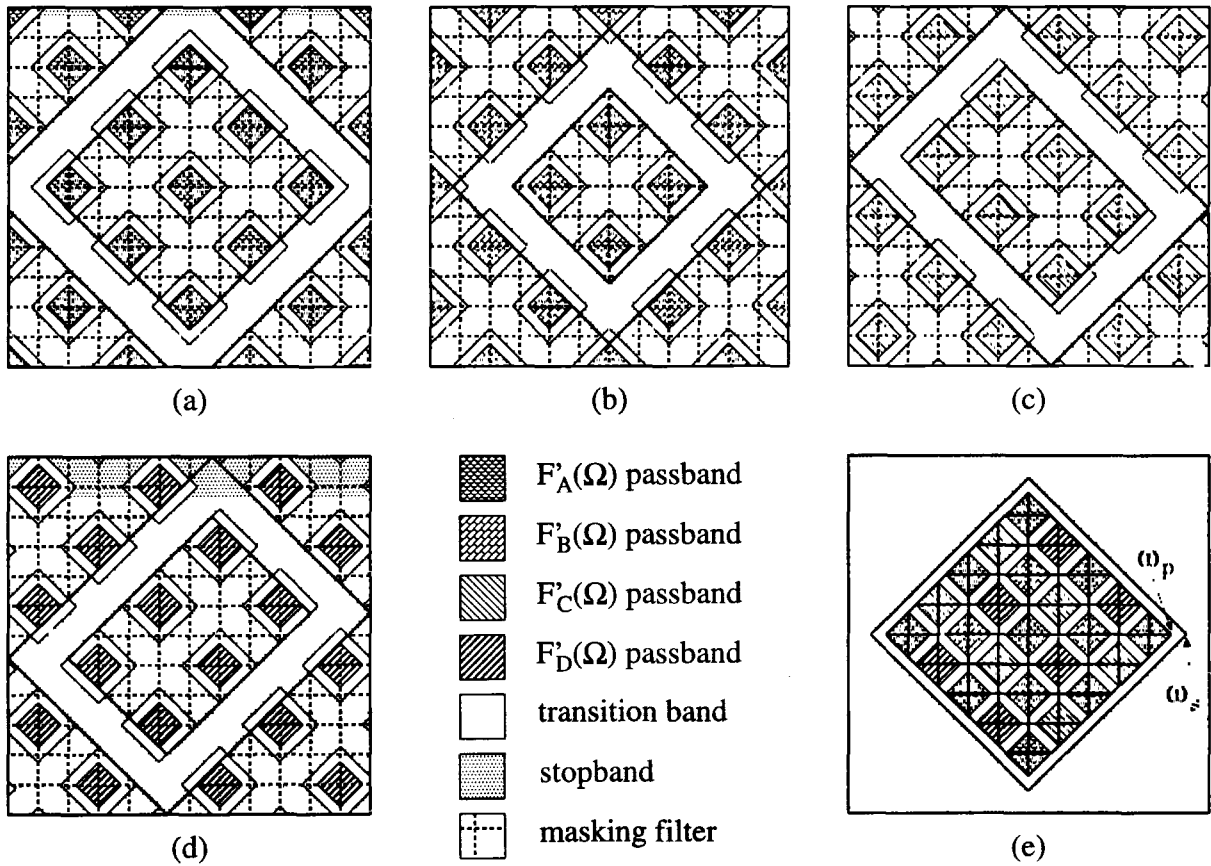


Figure 4. Complementary components and filter masks for CASE A.

for CASE A, and

$$\psi_p = \frac{2m\pi - \phi}{M} \quad (10a)$$

$$\psi_s = \frac{2m\pi - \theta}{M} \quad (10b)$$

for CASE B, where  $m$  is an integer less than  $M$ .

In a synthesis problem,  $\psi_p$  and  $\psi_s$  are known and  $m$ ,  $M$ ,  $\theta$ ,  $\phi$  and the band edges of the masking filters must be determined. We shall express  $m$ ,  $\theta$  and  $\phi$  in terms of  $\psi_p$ ,  $\psi_s$  and  $M$ . Since

$$0 < \theta < \phi < \pi, \quad (11)$$

to ensure that (9) yield a solution for CASE A, we have

$$m = \lfloor \psi_p M / (2\pi) \rfloor \quad (12a)$$

$$\theta = \psi_p M - 2m\pi \quad (12b)$$

$$\phi = \psi_s M - 2m\pi \quad (12c)$$

where  $\lfloor x \rfloor$  denotes the largest integer less than  $x$ . For CASE B, we have

$$m = \lceil \psi_s M / (2\pi) \rceil \quad (13a)$$

$$\theta = 2m\pi - \psi_s M \quad (13b)$$

$$\phi = 2m\pi - \psi_p M \quad (13c)$$

where  $\lceil x \rceil$  denotes the smallest integer larger than  $x$ . For any given set of  $\psi_p$ ,  $\psi_s$  and  $M$ , either CASE A ((9) and

(12)) or CASE B ((10) and (13)) (but not both) will yield a set of  $\theta$  and  $\phi$  that satisfies (11). Irrespective of CASE A or CASE B,  $\Delta_{ds}$  is given by

$$\Delta_{ds} = \psi_s - \psi_p = \frac{\Delta_\alpha}{M}. \quad (14)$$

The transition width of the prototype filters is therefore  $M$  times the transition width of the synthesized filter. The value of  $\omega_\alpha$  may be obtained from (8) by substituting for the values of  $\Delta_\alpha$ ,  $\theta$  and  $\phi$ . The specifications of the masking filters depend on whether  $F(\Omega)$  belongs to CASE A or CASE B; these are given in Table 1 and Table 2. As in the 1-D case, "don't care" bands may be introduced into the specifications of the masking filters to reduce the complexity. The value of  $M$  can be obtained by estimating the filter complexity for each  $M$  and selecting the value which corresponds to the lowest complexity.

#### 4. DESIGN EXAMPLE

We shall illustrate the FRM technique by synthesizing a DS filter with  $\omega_p = 0.5\pi$  and  $\omega_s = 0.52\pi$ . We choose  $M = 7$  and check with (9)–(13). CASE B applies with  $m = 2$ ,  $\theta = 0.36\pi$ ,  $\phi = 0.5\pi$ ,  $\Delta_\alpha = 0.14\pi$  and  $\omega_\alpha = 0.43\pi$ . Using these values, the band edges of the masking filters can be obtained from Table 2. Prototype filters with filter support sizes of  $31 \times 31$  each and masking filters with filter support sizes ranging from  $23 \times 23$  to  $29 \times 29$  were used

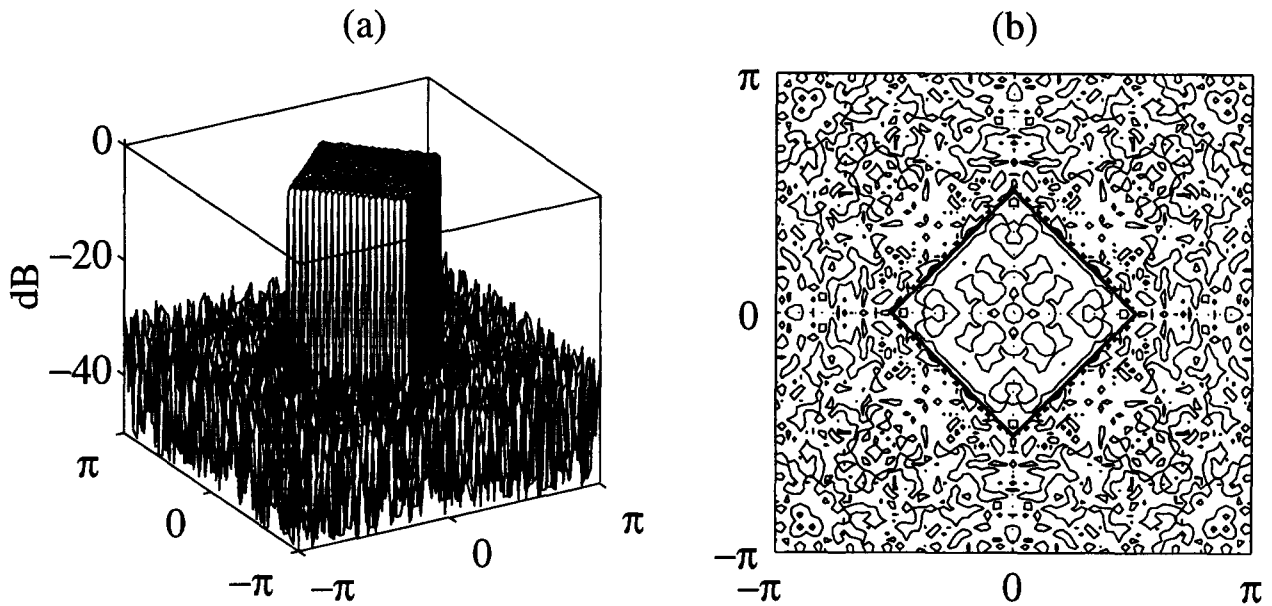


Figure 5. The frequency magnitude response of the synthesized DS filter, (a) perspective plot, (b) contour plot.

Table 1. Masking filters specifications for CASE A

Filter	Type	$\omega_p, \Omega_p$	$\omega_s, \Omega_s$
$F_{MA}(\Omega)$	DS	$\frac{2m\pi+\theta}{M}$	$\frac{2(m+1)\pi-\phi}{M}$
$F_{MB}(\Omega)$	DS	$\frac{2m\pi-\theta}{M}$	$\frac{2m\pi+\phi}{M}$
$F_{MC}(\Omega)$	RS1	$\frac{2m\pi-\theta}{M}$ $\frac{2m\pi+\theta}{M}$	$\frac{2m\pi+\phi}{M}$ $\frac{2(m+1)\pi-\phi}{M}$
$F_{MD}(\Omega)$	RS2	$\frac{2m\pi+\theta}{M}$ $\frac{2m\pi-\theta}{M}$	$\frac{2(m+1)\pi-\phi}{M}$ $\frac{2m\pi+\phi}{M}$

Table 2. Masking filters specifications for CASE B

Filter	Type	$\omega_p, \Omega_p$	$\omega_s, \Omega_s$
$F_{MA}(\Omega)$	DS	$\frac{2(m-1)\pi+\phi}{M}$	$\frac{2m\pi-\theta}{M}$
$F_{MB}(\Omega)$	DS	$\frac{2m\pi-\phi}{M}$	$\frac{2m\pi+\theta}{M}$
$F_{MC}(\Omega)$	RS2	$\frac{2m\pi-\phi}{M}$ $\frac{2(m-1)\pi+\phi}{M}$	$\frac{2m\pi+\theta}{M}$ $\frac{2m\pi-\theta}{M}$
$F_{MD}(\Omega)$	RS1	$\frac{2(m-1)\pi+\phi}{M}$ $\frac{2m\pi-\phi}{M}$	$\frac{2m\pi-\theta}{M}$ $\frac{2m\pi+\theta}{M}$

in the synthesis. The perspective and contour plots of the frequency magnitude response of the synthesized filter are shown in Fig. 5(a) and (b), respectively. The passband and stopband ripple magnitudes are both approximately -23 dB. Using the relationships presented in [3], a direct-form DS filter with the same specifications would require a filter support size of at least 129 by 129. By counting only the number of multipliers required, there is a complexity advantage of two. Higher orders of complexity reduction is possible for the design of filters with sharper transition bands.

## 5. CONCLUSION

The frequency response masking technique is an efficient way to synthesize sharp 1-D filters. In this paper, we extend the frequency response masking technique to the design of 2-D DS filters. The design procedure, the prototype and masking filters specifications as well as a design example, are presented in this paper. With this technique, sharp DS filters can be synthesized with considerably lower complexities compared to direct-form implementations.

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