DESIGN OF SHARP FIR BANDSTOP FILTERS USING QUADRATURE MASKING FILTERS

Rui Yang[1] *Yong-Ching Lim*[1] *Maurice Bellanger*[2]

[1] Department of Electrical Engineering
National University of Singapore, Singapore 119260
yr@ee.nus.sg
[2] CNAM - Electronique 292, rue Saint-Martin
75141 Paris Cedex 03 - France

ABSTRACT

A novel computationally highly efficient realisation of sharp symmetrical bandstop FIR filter is proposed. The new structure is derived using the frequency-response-masking technique, where the bandedge-shaping filter is derived from half-band filter by substituting each delay of the half-band filter by M delays. The masking filters are unconventional. They are quadrature filters derived from linear combinations of the masking filters in the conventional frequency-response-masking technique. Approximate expressions for the optimal value of M and the corresponding number of multipliers are derived.

1. INTRODUCTION

The implementation of sharp cut-off filter is generally regarded as a difficult digital filter realization problem. For a given passband, stopband and peak ripple magnitude, the order of an FIR filter is approximately inversely proportional to the transition width. As a consequence, sharp FIR filters are necessarily of high orders. This results in a high computation rate, large coefficient storage, and high roundoff error. A computationally efficient FIR filter realization technique thus has attracted the attention of many authors in recent years [1]-[15]. Most of the previously proposed efficient FIR filter realization schemes are suitable for lowpass or highpass filters and, to a very limited extent [10]-[15], for bandpass/bandstop filters.

One of the computationally efficient realization for sharp filters is the frequency-response-masking technique [1]-[4]. Filters synthesized using the frequency-response-masking technique is essentially a system of FIR subfilters, whose z-transform transfer function is of the form

$$H(z) = H_a(z^M)H_{Ma}(z) + \left[z^{-\cdot} - H_a(z^M)\right] H_{Mc}(z) \quad (1)$$

In (1), $H(z)$ and $H_a(z^M)$ are the z-transform transfer functions of the overall filter system and the bandedge-shaping filter, respectively. $H_{Ma}(z)$ and $H_{Mc}(z)$ are the z-transform transfer functions of the masking filters, respectively. The structure is shown in Fig.1.

The main advantages of the frequency-response-masking technique are that it employs subfilters with very sparse coefficient vectors and that the resulting effective filter length is only slightly longer than that of the theoretical(Remez) minimum.

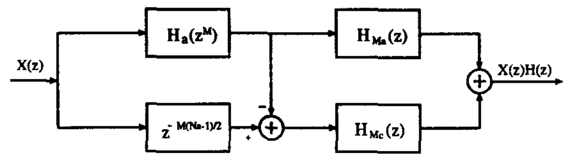

Figure 1. The structure of a filter synthesized using the frequency-response masking technique.

The frequency-response-masking technique optimal design for lowpass filter has been studied by Lim and Lian [2]. In the bandstop cases, the problem is somewhat more complicated requiring a careful selection of M so that the spectral images are well separated.

Traditionally, a bandstop filter is derived from a bandpass filter in a straightforward manner. An efficient bandpass filter design is proposed by Neuvo *et al* [10]. The implementation structure is derived from IFIR [6][7] structure. However, this implementation is limited to narrow-band bandpass filters only. Rajan *et al* [11] extended this method by using the modulated frequency-response-masking filters. This leads to a flexible approach especially efficient for very sharp bandpass filters; the structure, however, is involved and requires two bandedge-shaping filters, four masking filters; an extra adder is also required to realize the complimentary filters of the bandedge-shaping filters.

In this paper, we propose a novel variation of the frequency-response-masking approach for an efficient synthesis of symmetrical bandstop FIR filters. In this proposal, the bandedge-shaping filter is derived from a half-band filter. The masking filters are unconventional. They are quadrature filters derived from the linear combinations of the masking filters of the conventional frequency-response-masking technique [1]. (By quadrature filters we mean filters symmetrical or anti-symmetrical about the quadrature frequency $f_s/4$, where f_s is the sampling frequency.)

2. FREQUENCY RESPONSE MASKING TECHNIQUE AND ITS VARIATION

The principle of frequency-response-masking technique is illustrated in Fig. 2 [3]. In the z-domain, the input signal $X(z)$ is filtered by a pair of complementary bandedge shaping filters $H_a(z^M)$ and $H_c(z^M)$, respectively. The sum and difference of the output of $H_a(z^M)$ and $H_c(z^M)$ are filtered by the masking filters $H_{Ma}(z)$ and $H_{Mc}(z)$, respectively. The outputs of $H_{Ma}(z)$ and $H_{Mc}(z)$ are summed to form

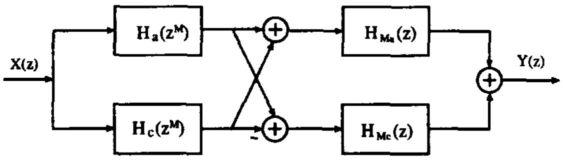

Figure 2. Principle of the frequency-response-masking technique.

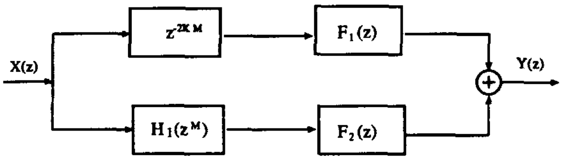

Figure 3. Alternative realization of the frequency-response-masking filter.

the final output, $Y(z)$. A variation of this system as shown in Fig. 3 was derived in [3]. $F_1(z)$ and $F_2(z)$ of Fig. 3 are defined in (2)

$$F_1(z) = \frac{1}{2}\left[H_{Ma}(z) + H_{Mc}(z)\right] \quad (2a)$$

$$F_2(z) = \frac{1}{2}\left[H_{Ma}(z) - H_{Mc}(z)\right] \quad (2b)$$

and $H_1(z^M)$ in Fig. 3 satisfies the implicit relationship of (3)

$$H_a(z^M) = \frac{1}{2}\left[z^{-2KM} - H_1(z^M)\right] \quad (3)$$

where $H_a(z^M)$ is obtained by replacing each delay of a half-band filter by M delays. The length of $H_a(z)$ is $4K+1$. This method is efficient for the design of sharp FIR lowpass filters with transition band centered at the frequency $f_c = \frac{(2p+1)f_s}{4M}$, where M and p are integers with $p < M$ and transition width less than $f_s/4M$. The sampling frequency is f_s.

The architecture of Fig. 3 is also adopted in our new approach which is eminently suitable for the synthesis of bandstop filters whose stopband center frequencies are located at $f_s/4$.

3. SYNTHESIS OF SHARP BANDSTOP FILTER

In order to simplify notations, in this paper, we shall assume that all the filters are zero phase. As a consequence, the resulting filters are non-causal. Nevertheless, causality can be easily achieved by delaying the impulse response of the filter by an appropriate number of samples. Also, we assume that the filters are all odd length filters. Define $H_{Ma}(z)$ of Fig. 2 as

$$H_{Ma}(z) = h_{Ma}(0) + \sum_{k=1}^{J} h_{Ma}(k)(z^k + z^{-k}) \quad (4)$$

The length of $H_{Ma}(z)$ is $2J+1$.

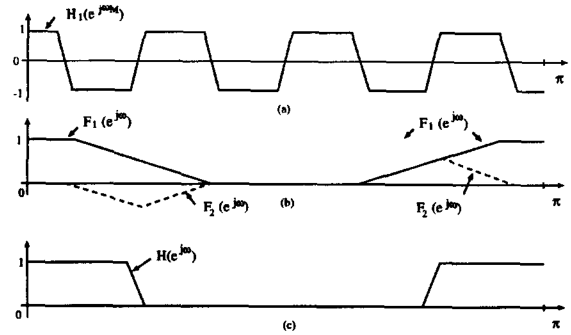

Figure 4. Frequency response for (a) the under-sampled half-band bandedge-shaping filter; (b) the quadrature masking filter $F_1(z)$, $F_2(z)$; (c) the resulting desired sharp bandstop filer.

It has been shown in [4] that if the bandedge-shaping filter is a half-band filter, it can be easily shown that the masking filters are of equal lengths. Hence,

$$H_{Mc}(z) = h_{Mc}(0) + \sum_{k=1}^{J} h_{Mc}(k)(z^k + z^{-k}) \quad (5)$$

We shall use the alternative structure shown in Fig. 3 to illustrate our new method. We choose $H_1(z^M)$ such that (3) is satisfied and instead of using the definition of (2) for $F_1(z)$ and $F_2(z)$, we redefine $F_1(z)$ and $F_2(z)$ to be quadrature filters formed from the linear combinations of $H_{Ma}(z)$ and $H_{Mc}(z)$ as depicted in (6),

$$F_1(z) = \sum_{k=0}^{\lfloor J/2 \rfloor} \Big[h_{Ma}(2k)(z^{2k} + z^{-2k}) + h_{Mc}(2k)(z^{2k} + z^{-2k}) \Big] \quad (6a)$$

$$F_2(z) = \sum_{k=1}^{\lfloor J/2 \rfloor + 1} \Big[h_{Mc}(2k-1)(z^{2k-1} + z^{-(2k-1)}) - h_{Ma}(2k-1)(z^{2k-1} + z^{-(2k-1)}) \Big] \quad (6b)$$

where $\lfloor J/2 \rfloor$ is the largest integer not larger than $J/2$. The overall filter can then be expressed as

$$H(z) = F_1(z) + H_1(z^M)F_2(z) \quad (7)$$

Let $H(e^{j\omega})$, $H_1(e^{j\omega M})$, $F_1(e^{j\omega})$, and $F_2(e^{j\omega})$ be the frequency response of the filters whose z-transform transfer functions are $H(z)$, $H_1(z^M)$, $F_1(z)$, and $F_2(z)$, respectively. The relationship between $H(e^{j\omega})$, $H_1(e^{j\omega M})$, $F_1(e^{j\omega})$, and $F_2(e^{j\omega})$ as depicted by (7) is illustrated in Fig. 4. A bandpass filter can be easily derived in a similar way or by simply taking the complementary filter of the overall bandstop filter.

It is interesting to note that both the masking filters and the bandedge-shaping filter are quadrature filters with sparse coefficient values. Thus the structure is also efficient while operating under multirate environment.

4. OPTIMUM DESIGN OF SHARP BANDSTOP FILTERS

The optimal length of a lowpass filter with transition width βf_s, where f_s is the sampling frequency, is given approximately by the expression [17],

$$N_0 \approx \frac{\Phi_1(\delta_p,\delta_s)}{\beta} + \Phi_2(\delta_p,\delta_s)\beta + 1 \tag{8}$$

When $\beta \leq 0.2$, the first term becomes dominant. Thus, (8) can be simplified to

$$N_0 \approx \frac{\Phi_1(\delta_p,\delta_s)}{\beta} \quad \text{for} \quad \beta \leq 0.2 \tag{9}$$

Applying the result of (9) the half-band filter $H_a(z)$, with transition width $M\beta$, has the filter length of

$$N_p \approx \frac{\Phi_1(\delta_1)}{M\beta} \tag{10}$$

where δ_1 is the passband and stopband ripple magnitude. The transition width of $H_{Ma}(z)$ and $H_{Mc}(z)$ are both $\frac{1}{2M}$. Hence, each of them has length N_m given by

$$N_m \approx 2M\Phi_1(\delta_p,\delta_s) \tag{11}$$

The filter lengths of $F_1(z)$ and $F_2(z)$ are each equal to N_m. In general, only 50% of the coefficients of a linear phase filter are distinct. For all three filters in Fig. 3, half of the coefficients are trivial. Thus the total number of multipliers required is

$$L \approx \frac{N_p}{4} + \frac{N_m}{2} = \frac{\Phi_1(\delta_1)}{4M\beta} + M\Phi_1(\delta_p,\delta_s) \tag{12}$$

In order to facilitate derivation, we assume that $\delta_1 = \delta_p = \delta_s = \delta_o$, where δ_o is the allowed overall peak ripple magnitude.(We have assumed that the peak ripple magnitude of the overall system is the same as those of the subfilters. This is obviously not true since the subfilters are cascaded together. However, This does not significantly affect its filter length because the filter length is more sensitive to transition width than to ripple magnitude. Furthermore, we are only interested in an approximate but useful solution.) Thus, the number of multipliers required for the overall bandstop filter in Fig. 3 is given by

$$L \approx \left(\frac{1}{4M\beta} + M\right)\Phi_1(\delta_1) \tag{13}$$

Differentiating L with respect of M and equating the derivative to zero, we have

$$M_{opt} = \frac{1}{2\sqrt{\beta}} \tag{14}$$

where M_{opt} is the value of M at minimum complexity.

Thus, the optimum value of L, denoted by L_{opt} is given by

$$L_{opt} \approx \frac{\Phi_1(\delta_1)}{\sqrt{\beta}} \tag{15}$$

In [16], Mintse *et al* estimate a practical design rule for bandpass digital filters. The order of a bandpass filter, denoted by N_b is given by

$$N_b \approx C_\infty(\delta_p,\delta_s)/\Delta F_m + g(\delta_p,\delta_s)\Delta F_m + 1 \tag{16}$$

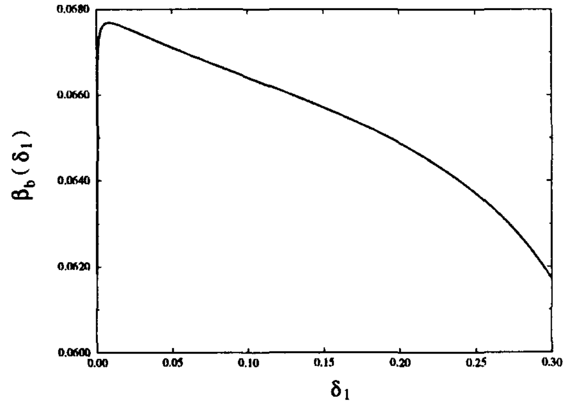

Figure 5. plot of $\beta_b(\delta_1)$ vs. δ_1.

where δ_p and δ_s are the passband and stopband ripples, respectively; and ΔF_m satisfies

$$\Delta F_m = min\{\Delta F_l, \Delta F_r\} \tag{17}$$

where ΔF_l and ΔF_r are the left and right transition widths of a bandpass filter, respectively. When $\Delta F_m \leq 0.15$ the first term is the dominant term. Thus, we have

$$N_b \approx C_\infty(\delta_p,\delta_s)/\Delta F_m \quad \text{for} \quad \Delta \mathrm{F_m} \leq 0.15 \tag{18}$$

For a bandstop filter with $\Delta F_m = \beta$, the length N_d is therefore given by

$$N_d \approx \frac{C_\infty(\delta_1)}{\beta} \tag{19}$$

with the assumption that both passband and stopband ripples are δ_1. Taking advantage of the symmetry of the filter and the fact that half of the coefficients are trivial, the number of multipliers required is

$$L_d \approx \frac{C_\infty(\delta_1)}{4\beta} \tag{20}$$

The fractional saving η is given by

$$\eta = \frac{L_d - L}{L_d} \approx \frac{C_\infty(\delta_1) - (1/M + 4M\beta)\Phi_1(\delta_1)}{C_\infty(\delta_1)} \tag{21}$$

Substituting M_{opt} into (21),

$$\eta = \frac{L_d - L}{L_d} \approx \frac{C_\infty(\delta_1) - 4\sqrt{\beta}\Phi_1(\delta_1)}{C_\infty(\delta_1)} \tag{22}$$

The new method is effective if $\eta > 0$, that is when $\beta < \beta_b(\delta_1)$, where

$$\beta_b(\delta_1) = \left[\frac{C_\infty(\delta_1)}{4\Phi_1(\delta_1)}\right]^2. \tag{23}$$

Note that $\beta_b(\delta_1)$ is a function of the ripple magnitude δ_1. The values of $\beta_b(\delta_1)$ for different values of δ_1 ranging from 0.00001 to 0.3 are shown in Fig. 5. It can be seen from Fig. 5 that for a wide range of ripple magnitude(0.00001 to 0.3), our new technique is efficient for β less than 0.0617. The saving increases when the transition width decreases. Fig. 6 shows η versus β plots for $\delta_1 = 0.01$ and 0.0001. It is evident from Fig. 6 that the fractional saving, η, is influenced strongly by the value of the transition width, β, but is not sensitive to the value of overall peak ripple magnitude.

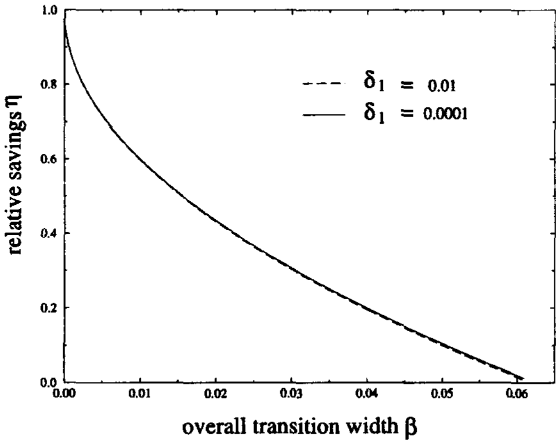

Figure 6. relative savings in multipliers η vs. desired transition width β.

5. AN EXAMPLE

We shall choose the design of a bandstop filter with stopband centered at $f_s/4$ as an example to illustrate our method. The stopband width of the example is 0.19 and its peak ripple magnitude in both passband and stopband is 0.01. An equi-ripple direct form realisation requires a filter of length $N = 197$; taking into consideration the symmetry of the coefficients and the trivial coefficient values leads to 50 multipliers per output sample.

For our improved method, the bandedge-shaping filter is derived from prototype half-band filter of length 51; this corresponds to 13 non-trivial distinct coefficients. The masking filter length is 25. Taking the symmetry of the coefficients and the trivial coefficients into consideration, the two masking filter F_1, F_2 requires a total of 13 multipliers. Hence, the total number of multipliers required by the entire system of filters is 26; this corresponds to a saving of almost 50%!

6. CONCLUSION

In this paper, we have presented a new efficient technique based upon frequency-response-masking technique for the synthesis of sharp bandstop filters. Quadrature filters are employed as the bandedge-shaping filters as well as the masking filters.

REFERENCES

[1] Y.C. Lim, "Frequency-response masking approach for the synthesis of sharp linear phase digital filter," *IEEE Trans. Circuits Syst.*, vol. CAS-33, pp.357-364, April 1986.

[2] Y.C. Lim and Y. Lian, "The optimum design of one- and two-dimensional FIR filters using the frequency response masking technique," *IEEE Trans. Circuits Syst-sII.*, vol. CAS-40, part II, pp.88-95, February 1993.

[3] M.G. Bellanger "Improved design of long FIR filters using the frequency masking technique," *IEEE Int. Conf. Acoust. Speech. Signal Processing*, in 1996, pp.1272-1275.

[4] T. Saramäki, Y.C. Lim, and R. Yang, "The synthesis of half-band filters using the frequency-response-masking technique," *IEEE Trans. Circuits Syst. - II*, vol.CAS-42, pp.58-60, Jan. 1995.

[5] J.W. Adams and A.N. Willson, Jr., "A new approach to FIR digital filters with fewer multipliers and reduced sensitivity," *IEEE Trans. Circuits Syst.*, vol.CAS-30, pp.277-283, May. 1983.

[6] Z. Jing and A.T. Fam, "A new strucure for narrow transition band lowpass digital filter design," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol.ASSP-32, pp.362-370, Apr. 1984.

[7] Y. Neuvo, C.-Y. Dong, and S.K. Mitra, "Interpolated finite impulse response digital filters," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol.ASSP-32, pp.563-570, June. 1984.

[8] T. Saramäki, "Subfilter approach for designing efficient FIR filters," *IEEE Int. Symposium on Circuits & Syst.*, in 1988. pp.2903-2915.

[9] S. Chu and C.S. Burrus, "Efficient recursive realizations of FIR filters: Part 2," *Circuits Syst. Signal Processing.*, vol.3, pp.21-57, 1984.

[10] Y. Neuvo, G. Rajan and S.K. Mitra, "Design of narrow-band FIR bandpass digital filters with reduced arithmetic complexity," *IEEE Trans. Circuits Syst.*, vol.CAS-34, pp.409-419, Apr. 1987.

[11] G. Rajan, Y. Neuvo, and S.K. Mitra, "On the Design of Sharp Cutoff wide-band FIR filters with reduced arithmetic complexity," *IEEE Trans. Circuits Syst.*, vol.CAS-35, pp.1447-1454, Nov. 1988.

[12] M.T. McCallig, "Design of digital FIR filters with complex conjugate pulse response," *IEEE Trans. Circuits Syst.*, vol.CAS-23, pp.1103-1105, Dec. 1978.

[13] A.H.M. ABed and G.D. Cain, "The host windowing technique for FIR filter design," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol.ASSP-32, pp.683-694, Aug. 1984.

[14] F. Mintzer and B. Liu, "The design of optimal multirate bandpass and bandstop filters," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol.ASSP-26, pp.534-543, Dec. 1978.

[15] M.C. Bellanger, J.L. Daguet, and G.P. Lepagnol, "Interpolation, extrapolation, and reduction of computation speed in digital filters," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol.ASSP-22, pp.231-235, Aug. 1974.

[16] F. Mintzer and B. Liu "Practical design rules for optimum FIR bandpass digital filters," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol.ASSP-27, pp.204-206, Apr. 1979.

[17] O. Herrmann, L.R. Rabiner, and D.S.K. Chan, "Practical design rules for optimum finite impulse ronse lowpass digital filters," *Bell Syst. Tech. Journal*, vol.52, pp.769-799, July-August 1973.