A NEW APPROACH TO THE PHASE ERROR AND THD IMPROVEMENT IN LINEAR PHASE IIR FILTERS

Bojan Djokić[1] *Miroslav D. Lutovac*[2] *Miodrag Popović*[3]

[1] MOBTEL BK-PTT, Bulevar umetnosti 16 A, 11070 Belgrade, Yugoslavia
[2] IRITEL Institute, Batajnički put 23, 11080 Belgrade, Yugoslavia
[3] Faculty of Electrical Engineering, Univ. of Belgrade, Bul. revolucije 73, 11001 Belgrade, Yugoslavia

ABSTRACT

This paper deals with the real-time implementation of the IIR filters having linear phase. Powell and Chau have devised an efficient method for the design and realization of real-time linear phase IIR filters using suitable modifications of the well-known time reversing technique. In their method, the input signal is divided into L-sample segments, time-reversed, and twice filtered using two IIR filter blocks whose transfer functions are the same. The performance has been further improved by Willson and Orchard, where double zeros on the unit circle have been separated, giving better amplitude response in the stop-band and slightly smaller amplitude and phase distortions. In this paper, new improvements are described based on the reordering of polynomials in the numerator of the filter transfer function, or on the appropriate selection of the transfer function. The truncation noise, phase and amplitude errors of new realizations show considerable improvement over the previous solutions.

1. INTRODUCTION

Recently, a notable interest has been shown in real-time implementation of IIR filters having linear phase. Powell and Chau [1] have devised an efficient method for the design and realization of the real-time linear phase IIR filters using suitable modification of the well-known time reversing technique [2]. In their method, shown in Fig. 1, the input signal is divided into L-sample segments, time-reversed, and twice filtered using two IIR filter blocks whose transfer functions are the same, e.g. $H_1(z) = H_2(z) = H(z)$. The transfer function $H(z)$ is usually of elliptic type, giving the best selectivity. It has been shown that the proposed procedure is much faster than previous methods, having at the same time low distortions of amplitude and phase characteristics.

The performance of this technique has been further improved by Willson and Orchard in [3], where double zeros on the unit circle have been separated, giving better amplitude response in the stop-band and slightly smaller amplitude and phase distortions. In their implementation, the transfer functions $H_1(z)$ and $H_2(z)$ are different, but have the same denominator.

In this paper, two new improvements to the Powell-Chau and Willson-Orchard methods are described. One of these solutions is based on the reordering of polynomials in the numerators of the filter transfer functions. Another solution is based on the appropriate synthesis of the whole transfer function.

2. REORDERING OF NUMERATOR POLYNOMIALS

Let the transfer functions of the two IIR filter blocks in Fig. 1 be:

$$H_1(z) = \frac{A(z)}{D(z)} \quad \text{and} \quad H_2(z) = \frac{B(z)}{D(z)} \qquad (1)$$

where the polynomials $A(z)$ and $B(z)$, whose zeros lie on the unit circle, may be equal as in [1], or different as in [3]. In general, they have no influence to the phase characteristic. But, the phase error which is caused by the segmentation of input sequence into length-L segments, may be influenced by the numerator polynomials, as can be concluded from the results in [1] and [3]. Also, it may be noticed that the effects of the segmentation of the input signal are concentrated into the time-reversed part of the block diagram where infinite impulse responses of the blocks are interrupted after L samples.

Following the results obtained by Willson and Orchard in [3], it is interesting to examine whether some other rearrangements of polynomials $A(z)$ and $B(z)$ into transfer functions $H_1(z)$ and $H_2(z)$ may be better in respect to truncation noise, phase and amplitude errors.

First, truncation noise (noise caused by truncation to L-sample segments) of several realizations will be examined. As in [1], the total harmonic distortion (THD) of the output signal is measured in response to a single frequency input $x(n) = \sin(\omega_0 n)$, $0 \le n \le N-1$. The input signal frequency ω_0 is chosen so that an integer number of periods is contained in the N input samples, e.g. $\omega_0 = 2\pi l/N$. In this case, we have used the values: $l = 37$, $N = 4096$, and section length $L = 200$. The THD of the input signal alone is approximately -150 dB. Some representative results are shown in Fig. 2, where effects of placing the polynomial $B(z)$ before or after time-reversed section $H_1(z)$, or included in $H_1(z)$, are shown. In all examples, the polynomials $A(z)$, $B(z)$, and $D(z)$ are synthesized to satisfy the requirements from Example 6 in [1]:

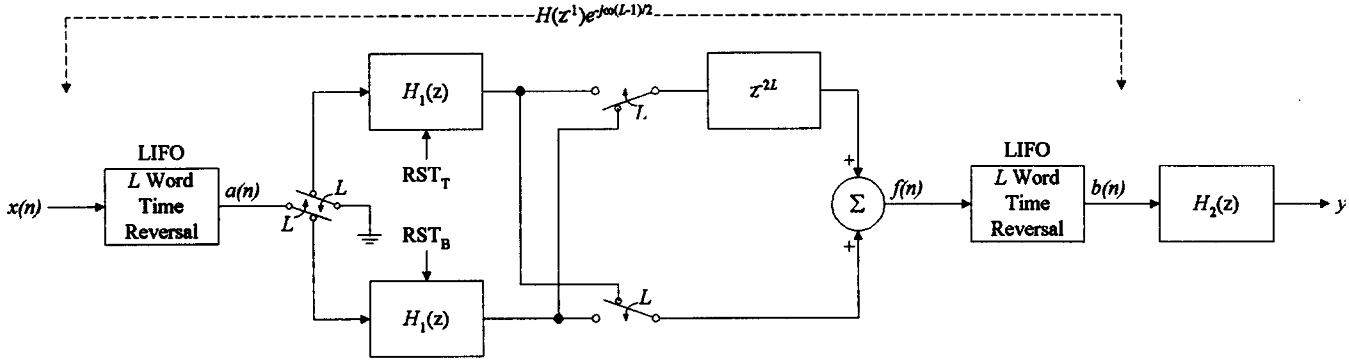

Figure 1. Block diagram of the Powell-Chau implementation of linear phase IIR filter.

$$F_p = 0.3,\ F_s = 0.325,\ \delta_p = 0.01\text{ dB},\ \delta_s = 70\text{ dB} \quad (2)$$

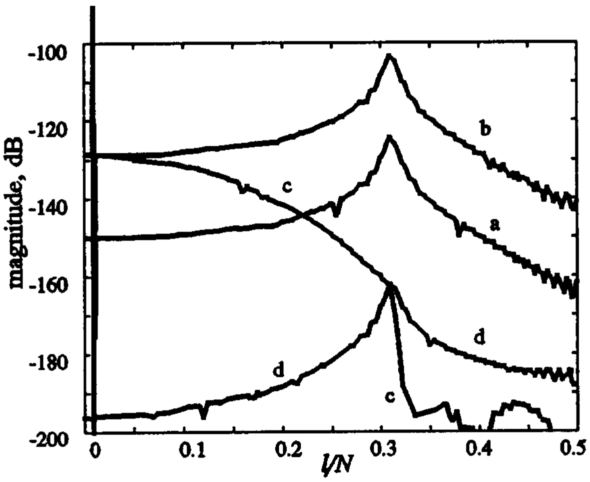

Figure 2. Envelopes of single frequency responses: l = 37, N = 4096, L = 200. Time-reversed sections are designated by z^{-1}, and direct section by z.

a) $[A(z^{-1})/D(z^{-1})]$

b) $B(z)\ [A(z^{-1})/D(z^{-1})]$

c) $[A(z^{-1})/D(z^{-1})]\ B(z)$

d) $[A(z^{-1})B(z^{-1})]/D(z^{-1})$

In Fig 2, only the envelopes of spectra of all examined realizations are plotted. First, the curve **a** represents the spectrum of the time-reversed section $H_1(z) = A(z)/D(z)$ (without direct section $H_2(z) = B(z)/D(z)$) of the original solution [1]. If FIR part of the direct filter $B(z)$ is implemented before time-reversed section $H_1(z)$, the input signal in $H_1(z)$ is increased and consequently the truncation noise at the output of $H_1(z)$ is increased at all frequencies (curve **b**). On the contrary, if $B(z)$ is realized after time-reversed section $H_1(z)$, the truncation noise is filtered by $B(z)$, and the noise at higher frequencies is more attenuated (curve **c**).

Finally, when $B(z)$ is included into the time-reversed section $H_1(z)$, the noise is considerable decreased at lower frequencies (curve **d**). As can be seen in Fig. 2, this realization produces the lowest truncation noise in the pass-band and slightly larger noise in the stop-band than the solution represented by curve **c**. It should be noticed that the noise of the last realization in the stop-band is still below the noise at the pass-band edge frequency. Hence, from the truncation noise point of view, the following choice of time-reversed and direct transfer functions is optimal:

$$H_1(z) = \frac{A(z)B(z)}{D(z)} \quad \text{and} \quad H_2(z) = \frac{1}{D(z)} \quad (3)$$

where the numerator polynomials $A(z)$ and $B(z)$, and denominator polynomial $D(z)$, are the same as in [1] or [3].

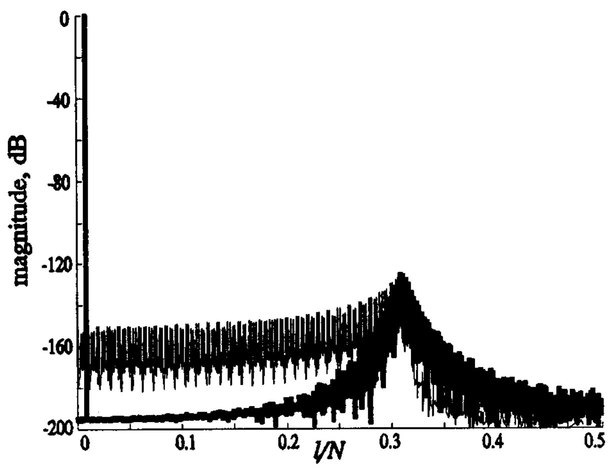

Figure 3. Spectra of single frequency responses: l = 37, N = 4096, L = 200. New realization - thick line, old realization - thin line.

Due to low-pass character of the direct transfer function $H_2(z)$, the truncation noise is further attenuated in the stop-band. The results from previous example are repeated in Fig. 3 for original realization [1] and for the new realization using (3). The results of the Powell-Chau technique are shown by the thin line while the results for new realization are shown by the thick line. It can be easily seen that the harmonic distortion obtained by using the new realization is smaller for 20-40 dB in most part of the pass-band. This means that the finite section length produces smaller harmonic distortions when both numerator polynomials are concentrated in time-reversed section as in (3).

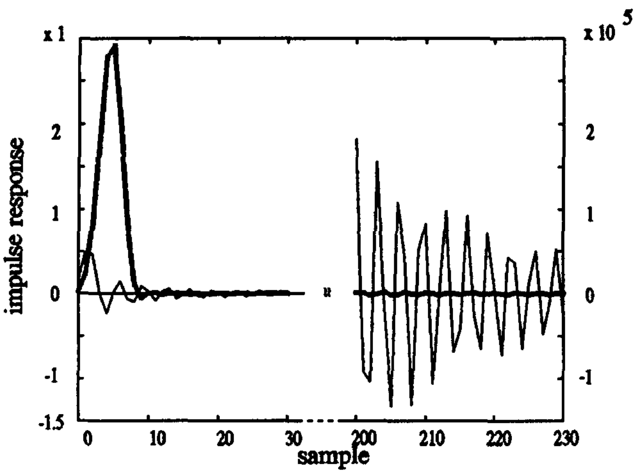

Figure 4. Impulse responses of:
$H(z) = A(z)/D(z)$ - thin line
$H(z) = A(z)B(z)/D(z)$ - thick line.

This choice of the transfer functions $H_1(z)$ and $H_2(z)$ also has positive influence to the rate of decrease of the impulse response, as shown in Fig. 4. As can be seen, after few samples of impulse response which are larger than in original realization [1], the remaining samples are significantly smaller. This is a consequence of increased number of zeros in the time-reversed transfer function $H_1(z)$.

Having reduced truncation noise in the pass-band, we can also expect that the phase response will be improved in the pass-band. In the following experiment the phase errors, produced by this method and method from [1], are compared. The same filter, which satisfies the specifications (2), was excited by a unit pulse sequence having 4096 samples. The results are presented in Fig. 5. As can be seen, the phase error in the pass-band is very small, having the maximum of about $2 \cdot 10^{-6}$ rad. Compared to the previous realization, the new realization produces 2-3 times smaller maximum phase error in the pass-band. This experiment was performed again using the impulse response of a high-order all pass filter as input signal with very similar results.

The comparisons with Willson-Orchard method show similar results in favor of the new method using the same example given in [3].

In order to achieve the lowest possible number of multipliers, $H_1(z)$ is realized as cascade connection of the FIR filter $B(z)$ and the IIR filter $A(z)/D(z)$. The IIR filter $A(z)/D(z)$ is realized as in [1]. The IIR filter $1/D(z)$ can be realized as cascade connection of the second order sections without direct branches, or even with the same realizations as in [1] by proper

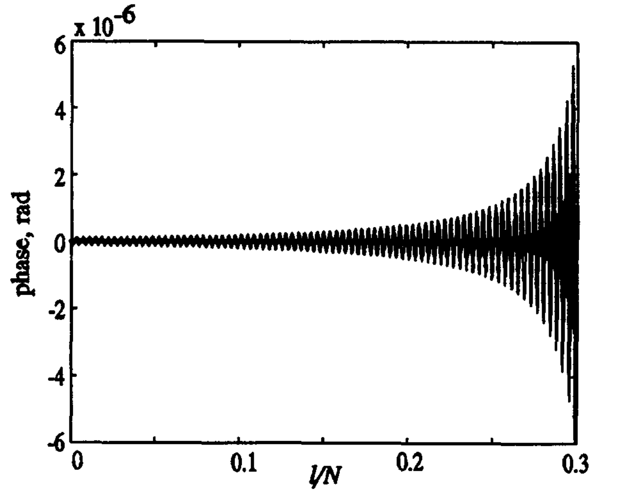

Figure 5. Pass-band phase responses: $N = 4096$, $L = 200$. New realization - thick line, old realization [1] - thin line.

selection of output node. The number of multipliers for $A(z)/D(z)$ and $1/D(z)$ is the same as in [1] for $A(z)/D(z)$ and $B(z)/D(z)$, respectively. An additional multiplier is required for every second order section of FIR filter $B(z)$. For analyzed example the number of multipliers is increased from 21 in [1] to 27.

Since $B(z)/D(z)$ is now realized as two different filters, $B(z)$ and $1/D(z)$, the quantization noise due to multiplication is increased (in the previous example less than 6 dB). In this paper we assume that the quantization noise can be reduced by the increased wordlength and is lower than the noise caused by truncation to L-sample segments.

3. SELECTION OF TRANSFER FUNCTION

The rate of decrease of the impulse response depends upon the damping of the most lightly damped natural frequency, which is the zero of the denominator of the transfer function. Therefore, a damping factor could be improved by appropriate selection of transfer function $H(z)$. In this paper, it is shown that a Chebyshev rational function [4] based on analog prototype of elliptic filters with minimal Q-factors [5] results in improvement of the phase error. Instead of the 7th-order elliptic transfer function given in [1]

$A(z) = B(z) = 0.18 + 0.825z^{-1} + 1.935z^{-2} + 2.865z^{-3} + 2.865z^{-4} + 1.935z^{-5} + 0.825z^{-6} + 0.18z^{-7}$,

$D(z) = 1 + 1.86z^{-1} + 3.09z^{-2} + 2.643z^{-3} + 1.95z^{-4} + 0.784z^{-5} + 0.258z^{-6} + 0.0253z^{-7}$,

a new 15th-order transfer function is derived as cascade connection of two fifth-order minimum Q-factor analog prototype elliptic filters [5] and one fifth order filter with complex zeros [4]:

$$H_1(z) = \frac{A(z)}{D(z)} = \frac{A_5(z)}{D_5(z)}\,\frac{A_5(z)}{D_5(z)}\,\frac{C_5(z)}{D_5(z)} \qquad (4)$$

where
$A_5(z) = 0.3146\ (1+z^{-5}) + 1.0463\ (z^{-1}+z^{-4}) + 1.7720(z^{-2}+z^{-3})$,
$D_5(z) = 1+ 1.4777z^{-1} + 1.9866z^{-2} + 1.1614z^{-3} + 0.5606z^{-4} + 0.0794z^{-5}$,
$C_5(z) = 1.2338\ (1 + 1.1545z^{-1} + 1.6815z^{-2} + 0.7753z^{-3} + 0.441z^{-4} + 0.0262z^{-5})$, and $A(z) = B(z) = 0.12209 + 0.9531z^{-1} + 3.86896z^{-2} + 10.5577z^{-3} + 21.3977z^{-4} + 33.865z^{-5} + 43.0148z^{-6} + 44.4977z^{-7} + 37.6979z^{-8} + 26.103z^{-9} + 14.6211z^{-10} + 6.48699z^{-11} +2.19321z^{-12}+0.52432z^{-13}+0.07514z^{-14}+0.0032z^{-15}$,
$D(z) = 1 + 4.43317z^{-1} + 12.5109z^{-2} + 24.3252z^{-3} + 36.8334z^{-4} + 44.1562z^{-5} + 43.4015z^{-6} + 34.9764z^{-7} + 23.342z^{-8} + 12.7438z^{-9} + 5.65339z^{-10} + 1.97439z^{-11} + 0.52379z^{-12} + 0.09679z^{-13} + 0.0106z^{-14} + 0.0005z^{-15}$.

The magnitude response is shown in Fig. 6.

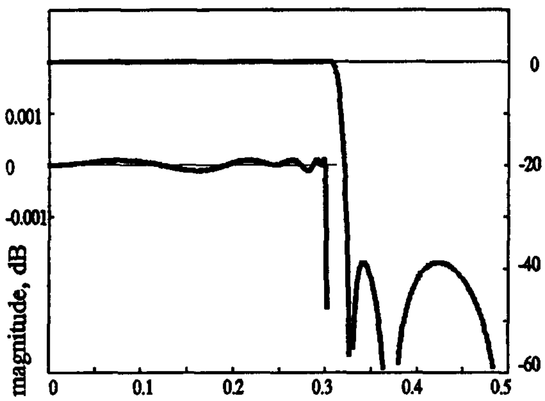

Figure 6. Magnitude response of $H_1(z) = H_2(z) = [A_5(z)A_5(z)C_5(z)] / [D_5(z)D_5(z)D_5(z)]$

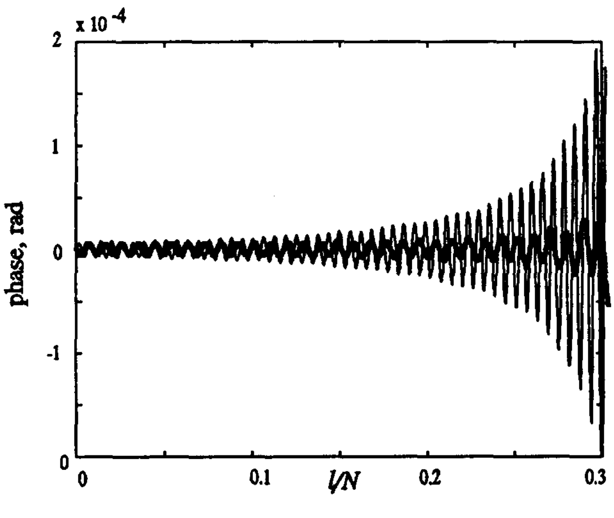

Figure 7. Pass-band phase responses: $N = 2048, L = 150$. New realization - thick line, old realization [1] - thin line.

Compared to the previous realization [1], the new realization produces smaller maximum phase error in the pass-band as shown in Fig. 7. It should be noticed that the truncation noise is approximately the same as in [1]. The better phase performances are obtained for shorter length L.

In spite of the increased filter order from 7 in [1] to 15, the complexity of the new realization is not necessarily increased. The design method presented in [6] enable multiplierless realization, that is not possible in conventional design. A half of the multipliers can be designed with minimal number of shift and add operations and without coefficient quantization. The second half of multipliers can be optimized for minimal quantization error. The complexity of a new digital filter, when implemented as a custom integrated circuit, a reprogrammable logic device or a low-cost micro-controller without an in-built multiplier, is reduced and the phase response is improved.

4. CONCLUSION

In this paper, two new improvements to the realization of the linear phase IIR filters are described. One method is based on the rearrangement of the numerator polynomials of the two IIR filter functions which are used in the real-time realizations in [1] and [3]. The new realizations have better total harmonic distortion when sine input is used, and smaller phase error due to finite section length. They produce shorter sample delay for the same phase error, or lower phase error and THD improvement for the same sample delay. Considerable improvements in phase response and lower truncation noise can approve increased number of multipliers and possible increased wordlength.

The other method is based on the analog prototype transfer functions with reduced Q factors. The faster rate of decrease of the impulse response is obtained by enlarging the real part of transfer function pole, i.e. by decreasing the critical Q-factor of the analog prototype filter. The improvement of the phase response is better for shorter L-segments.

REFERENCES

[1] S.R. Powell, and P.M. Chau, "A technique for realizing linear phase IIR filters," IEEE Trans. SP, vol. SP-39, no. 11, pp. 2425-2435, 1991.

[2] J.J. Kormylo, and V.K. Jain, "Two-pass recursive digital filter with zero phase shift," IEEE Trans. ASSP, vol. ASSP-30, pp. 384-387, 1974.

[3] A.N. Willson, and H.J. Orchard , "An improvement to the Powell and Chau linear phase IIR filters," IEEE Trans. SP, vol. SP-42, no. 10, pp. 2842-2848, 1994.

[4] D. Rabrenović, and M. Lutovac, "A Chebyshev rational function with low Q factors", Int. J. Circuit Theory Appl., Vol. 19, pp. 229-240, 1991.

[5] D. Rabrenović, and M. Lutovac, "Elliptic filters with minimal Q-factors", Electronics Letters, pp.206-207, vol. 30, No. 3, 1994.

[6] Lj.D. Milić and M.D. Lutovac, "Design of multiplierless elliptic IIR filters", ICASSP'97, Paper No. 1583, Munich, Germany, 1997.