A New Two-Dimensional Parallel Block Adaptive Filter with Reduced Computational Complexity

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Abstract

Two-dimensional (2-D) adaptive digital filters (ADFs) for 2-D signal processing have become a fascinating area of the adaptive signal processing. However, conventional 2-D FIR ADF's require a lot of computations. For example, the TDLMS requires $2N^2$ multiplications per pixel. We propose a new 2-D adaptive filter using the FFTs. The proposed adaptive filter carries out the fast convolution using overlap-save method, and has parallel structure. Thus, we can reduce the computational complexity to $O(log_2 N)$ per pixel.

1 Introduction

Two-dimensional (2-D) adaptive digital filters (ADFs) for system identification, image restoration and enhancement have become a fascinating area of the adaptive signal processing. Since ADFs based on 2-D finite impulse response (FIR) digital filters give us an optimum weight solution which minimizes a cost function and are always stable, they have received considerable attention.

The two-dimensional least mean square (TDLMS) method is an attractive adaptation algorithm of the 2-D FIR ADF because of its simple structure [1]. However, the 2-D FIR ADF based on the TDLMS suffers from the large computational complexity. For example, the $N \times N$ 2-D FIR ADF requires $2N^2$ multiplications per pixel for the convolution and updating the filter coefficients. Furthermore, since the TDLMS basically has a sequential structure, parallel processing techniques cannot be employed.

In order to reduce the computational load of the TDLMS, a 2-D block LMS (TDBLMS) has been proposed[2]. The TDBLMS update the filter coefficients once for every block which consists of $N \times N$ input pixel while the TDLMS updates for each input pixel. As a result, the number of multiplications per pixel can be reduced from $O(N^2)$ to $O(N)$. Furthermore, the block processing enables us to make use of the Fast Fourier Transform (FFT) for high-speed implementation of the 2-D convolution. However, when we implement the TDBLMS by hardware, the critical path becomes long because of its sequential structure. Furthermore, the stability of the algorithm depends on the appropriate selection of the stepsize parameter.

In case of 1-D ADFs, parallel ADFs utilizing the DFT frequency sampling filter (FSF) bank [3] - [4] are very attractive since we can completely avoid aliasing problem. The computational complexity of the parallel ADFs reduces to $O(log_2 N)$ per input signal. This FSF's strategy has been extended to the 2-D case in [5]. The 2-D block adaptation algorithm can be implemented with parallel structure which yields shorter critical path compared with the TDBLMS. Furthermore, the behavior of the algorithm is always stable as long as the input signal is persistently excited [3]. However, since the block processing has been done by the disjointed window, we cannot apply the overlap-save method for the linear convolution, and it requires a 2-D FIR filter of $O(N^2)$ complexity.

In this paper, we propose a 2-D block ADF with reduced computational complexity, parallel structure and stable convergence. In Section 2, we show a structure of the proposed ADF. 2-D overlap-save method for linear convolution is explained. Section 3 defines a cost function and drives an adaptation algorithm. Section 4 compares the computational complexity of the proposed method with the high-speed version of the TDBLMS, and Section 5 shows a computer simulation to verify the effectiveness of the proposed ADF.

2 Proposed 2-D Adaptive Filter

Figure 1 shows a 2-D system identification by using a proposed ADF. $x(u,v)$, $y(u,v)$ and $d(u,v)$ are an input image, an output image and a reference image, respectively. We assume that the unknown system is an $N \times N$ 2-D FIR filter.

The dotted line part in Fig.1 carries out the 2-D linear convolution by using the overlap-save method. First, at the point (i), the input image $x(u,v)$ is divided into the set of blocks, whose size is $2N \times 2N$, so as to have 50% overlap for horizontal and vertical directions as shown in Fig.2 (a) and (b). Secondly, at the point (ii), the input block is transformed into the frequency domain by the 2-D DFT to obtain $X_{st}(i,j)$ where i and j denote the block indices, and s and t are the number of the DFT bin. $X_{st}(i,j)$ is multiplied by $\hat{H}_{st}(i,j)$ which is an estimated frequency response of the unknown system $H(\omega_1, \omega_2)$; that is, $\hat{H}_{st}(i,j) \approx H(\omega_{1s}, \omega_{2t})$ where $\omega_{1s} = \frac{2\pi}{2N}s$ and $\omega_{2t} = \frac{2\pi}{2N}t$ $(s,t = 0,1,\cdots,2N-1)$. Finally, at the point (iii), $Y_{st}(i,j)$ is transformed into spatial domain by the IDFT such that we can obtain the linear convolution $y(u,v)$ as shown in Fig.2 (c). The 2-D overlap-save method results in the reduction of the computational complexity.

The process of the DFT in Fig.1 can be implemented by a delay chain, decimators, and a DFT as shown in Fig.3. D stands for a sampling matrix. It is known that the DFT is a filter bank, which is called the DFT-FSF bank [3],[4]. A frequency response of the 2-D DFT-FSF in case of $N=2$ is illustrated in Fig.4. At the 2-D frequencies ω_{1s} and ω_{2t}, the frequency response in Fig.4 is equal to zero except the D.C. It means that $X_{st}(i,j)$ are degraded by aliasing except D.C. component when we use the maximal decimation. By using the accumulators, we can pinpoint the D.C. component; that is,

$$\hat{X}_{st}(i,j) = \sum_{p=1}^{i}\sum_{q=1}^{j} X_{st}(p,q). \tag{1}$$

However, since the proposed method has 50% overlap with the adjacent blocks, the outputs of the decimators are $x(Ni-p, Nj-q)$ $(p,q=0,1,\cdots,2N-1)$. This is not the maximal decimation. In this case, we must modulate $X_{st}(i,j)$ by Eq.(2) in order to shift alias free component to zero frequency. Table 1 shows the modulation frequencies at each bin in case of $N=2$ (See Appendix).

Table 1 Modulation frequencies.

s \ t	0	1	2	3
0	$(0,0)$	$(0,\pi)$	$(0,0)$	$(0,\pi)$
1	$(\pi,0)$	(π,π)	$(\pi,0)$	(π,π)
2	$(0,0)$	$(0,\pi)$	$(0,0)$	$(0,\pi)$
3	$(\pi,0)$	(π,π)	$(\pi,0)$	(π,π)

In general, the modulation can be achieved by

$$X'_{st}(i,j) = X_{st}(i,j)(-1)^{s(i+1)+t(j+1)}. \tag{2}$$

Then, the accumulators can pinpoint the DC signal of $X'_{st}(i,j)$ to obtain $\hat{X}_{st}(i,j)$ in Eq.(1).

Assume that $X(\omega_{1s},\omega_{2t})$ denotes the Fourier Transform of $x(u,v)$. Since $\hat{X}_{st}(i,j)$ includes the signal component at the discrete frequencies ω_{1s} and ω_{2t}, $\hat{X}_{st}(i,j)$ is approximately equal to $X(\omega_{1s},\omega_{2t})$.

As the same manner, we can estimate the Fourier Transform of the reference image $d(u,v)$:

$$D(\omega_{1s},\omega_{2t}) \approx \hat{D}_{st}(i,j) = \sum_{p=1}^{i}\sum_{q=1}^{j} D'_{st}(p,q). \tag{3}$$

Finally, the frequency response of the unknown system is given by

$$H(\omega_{1s},\omega_{2t}) = \frac{D(\omega_{1s},\omega_{2t})}{X(\omega_{1s},\omega_{2t})}. \tag{4}$$

We derive an adaptation algorithm which enables us to estimate $H(\omega_{1s},\omega_{2t})$ in the following section.

3 Adaptation Algorithm

We define the cost function for the system identification at *st-th* bin as

$$J_{st}(i,j) = \sum_{p=1}^{i}\sum_{q=1}^{j}[|\hat{D}_{st}(p,q) - \hat{H}_{st}(i,j)\hat{X}_{st}(p,q)|^2]. \tag{5}$$

By expanding Eq.(5),

$$\begin{aligned} J_{st}(i,j) &= P_{dst}(i,j) - \hat{H}_{st}(i,j)p^*_{st}(i,j) \\ &- \hat{H}^*_{st}(i,j)p_{st}(i,j) \\ &+ \hat{H}^*_{st}(i,j)\hat{H}_{st}(i,j)r_{st}(i,j) \end{aligned} \tag{6}$$

where

$$P_{dst}(i,j) = \sum_{p=1}^{i}\sum_{q=1}^{j} \hat{D}_{st}(p,q)\hat{D}^*_{st}(p,q), \tag{7}$$

$$p_{st}(i,j) = \sum_{p=1}^{i}\sum_{q=1}^{j} \hat{D}_{st}(p,q)\hat{X}^*_{st}(p,q), \tag{8}$$

$$r_{st}(i,j) = \sum_{p=1}^{i}\sum_{q=1}^{j} \hat{X}_{st}(p,q)\hat{X}^*_{st}(p,q), \tag{9}$$

and $*$ denotes a complex conjugate.

The derivative of Eq.(6) with respect to $\hat{H}_{st}(i,j)$ can be written as

$$\frac{dJ_{st}(i,j)}{d\hat{H}_{st}(i,j)} = -2p_{st}(i,j) + 2r_{st}(i,j)\hat{H}_{st}(i,j). \tag{10}$$

Let Eq.(10) be zero, then we get a least squared solution $\hat{H}_{st}(i,j)$. The proposed algorithm is shown in Table 2.

Table 2 Proposed algorithm.

for s=0 to 2N-1
for t=0 to 2N-1

$$\begin{aligned} X'_{st}(i,j) &= X_{st}(i,j)(-1)^{s(i+1)+t(j+1)} \\ D'_{st}(i,j) &= D_{st}(i,j)(-1)^{s(i+1)+t(j+1)} \\ \hat{X}_{st}(i,j) &= \sum_{p=1}^{i}\sum_{q=1}^{j} X'_{st}(p,q) \\ \hat{D}_{st}(i,j) &= \sum_{p=1}^{i}\sum_{q=1}^{j} D'_{st}(p,q) \\ r_{st}(i,j) &= \sum_{p=1}^{i}\sum_{q=1}^{j} \hat{X}_{st}(p,q)\hat{X}^*_{st}(p,q) \\ p_{st}(i,j) &= \sum_{p=1}^{i}\sum_{q=1}^{j} \hat{D}_{st}(p,q)\hat{X}^*_{st}(p,q) \\ \hat{H}_{st}(i,j) &= p_{st}(i,j)/r_{st}(i,j) \end{aligned}$$

Since the cost function is defined independently in each frequency bin, the proposed algorithm yields to a parallel structure. The one tap least square algorithm is always stable as long as $r_{st}(i,j)$ has a finite value.

4 Computational Complexity

In Table 2, the number of additions and multiplications depends on the block indices i and j. For example, the number of additions is $4ij$ per filter tap. The computational complexity becomes large when i and j are large value.

This disadvantage can be overcome by following recursive equations.

$$\begin{aligned}\hat{X}_{st}(i,j) &= \hat{X}_{st}(i-1,j-1) + X_{st}(i,j) \\ &+ \hat{X}_{1st}(i-1,j) + \hat{X}_{2st}(i,j-1) \quad (11)\end{aligned}$$

where

$$\begin{aligned}\hat{X}_{1st}(i,j-1) &= \hat{X}_{1st}(i-2,j) + X_{st}(i-1,j), \\ \hat{X}_{2st}(i,j-1) &= \hat{X}_{2st}(i,j-2) + X_{st}(i,j-1) \quad (12)\end{aligned}$$

$\hat{D}_{st}(i,j)$, $r_{st}(i,j)$ and $p_{st}(i,j)$ in Table 2 is rewritten by the same manner. The recursive equations are independent of the block indices i and j.

The number of real multiplications per pixel of the proposed ADF and the TDBLMS, which is assumed to be implemented by using the fast convolution, are compared in Table 3. Assume that the number of taps of the 2-D FIR filter is $N \times N$. A $2N \times 2N$ 2-D FFT in Fig.1 can be realized with $4N^2 \log_2 2N$ complex multiplications per block. Furthermore, we assume that four real multiplications are equivalent to one complex multiplication.

Table 3 Comparison of multiplications per pixel.

	Proposed structure	TDBLMS
N	$16(3\log_2 N + 10)$ [2]	$16(5\log_2 N + 7)$
16	352[2]	432
32	400[2]	512
64	448[2]	592

[·] denotes the number of real divisions.

As shown in Table 3, the computational complexity of the proposed parallel ADF is reduced compared with the high-speed version of the TDBLMS.

5 Simulation Result

The 2-D system identification in Fig.1 is done by the computer simulation in order to show the applicability of the proposed algorithm.

[Conditions]
Unknown system :

$$\begin{aligned}H(z_1,z_2) &= 0.914 + 0.926z_2^{-1} + 0.338z_2^{-2} + 0.494z_1^{-1} \\ &+ 0.677z_1^{-1}z_2^{-1} - 0.843z_1^{-1}z_2^{-2} - 0.894z_1^{-2} \\ &+ 0.517z_1^{-2}z_2^{-1} - 0.777z_1^{-2}z_2^{-2}\end{aligned}$$

FFT size (is equal to the block size.) : 8×8,
Input image $x(u,v)$:White Gaussian with the variance of 1 and the mean value of 0.
(The image size is 128×128.)

We define the normalized mean square error (MSE):

$$MSE = 10log_{10}[\frac{E[(d(u,v) - y(u,v))^2]}{E[d(u,v)^2]}] \quad [dB] \quad (13)$$

where $E[\cdot]$ denotes an average operation. The MSE curve is shown in Fig.5. Fig.5 shows that the precise system identification and linear convolution is carried out by using the proposed ADF.

6 Conclusion

A 2-D parallel ADF with reduced computational complexity has been proposed. It has been shown that the computational complexity is reduced compared with the TDBLMS. In addition, the proposed algorithm is stable as long as the input signal is persistently excited. The result of a computer simulation has shown that the proposed 2-D parallel ADF has good performance for the 2-D adaptive filtering.

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Appendix

The 2-D DFT of the input image $x(u,v)$ is

$$x'_{st}(u,v) = \sum_{p=0}^{K}\sum_{q=0}^{K} x(u+p-K, v+q-K)e^{-j\frac{2\pi}{2N}sp}e^{-j\frac{2\pi}{2N}tq} \quad (14)$$

where $K = 2N - 1$. By taking the z-transform of Eq.(14), the transfer function of the DFT in st-th bin, $G_{st}(z_1,z_2)$, is given by

$$G_{st}(z_1,z_2) = \sum_{p=0}^{K}\sum_{q=0}^{K} z_1^{p-K}z_2^{q-K}e^{-j\frac{2\pi}{2N}sp}e^{-j\frac{2\pi}{2N}tq}. \quad (15)$$

Thus, the frequency response of $G_{st}(z_1, z_2)$ is

$$G_{st}(\omega_1,\omega_2) = \frac{sin(\omega_1 - \frac{2\pi}{2N}s)N}{sin\frac{1}{2}(\omega_1 - \frac{2\pi}{2N}s)} \cdot \frac{sin(\omega_2 - \frac{2\pi}{2N}t)N}{sin\frac{1}{2}(\omega_2 - \frac{2\pi}{2N}t)}\theta(\omega_1,\omega_2) \quad (16)$$

where

$$\theta(\omega_1,\omega_2) = e^{-j\omega_1\frac{2N-1}{2}}e^{-j\omega_2\frac{2N-1}{2}}.$$

Since $X_{st}(i,j)$ in Fig.1 is a decimated version of $x'_{st}(u,v)$, the amplitude spectrum of $X_{st}(i,j)$ is

$$\begin{aligned}|X^f_{st}(\omega_1,\omega_2)| &= |\frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} G_{st}(\omega'_{1k},\omega'_{2l}) \\ &\quad \cdot X(\omega'_{1k},\omega'_{2l})| \\ &= |\frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1}\frac{sin(\omega_1 - 2\pi k - \pi s)}{sin\frac{1}{2}(\frac{\omega_1-2\pi k-\pi s}{N})} \\ &\quad \cdot \frac{sin(\omega_2 - 2\pi l - \pi t)}{sin\frac{1}{2}(\frac{\omega_2-2\pi l-\pi t}{N})}| \end{aligned} \quad (17)$$

where $\omega'_{1k} = \frac{\omega_1 - 2\pi k}{N}$, $\omega'_{2l} = \frac{\omega_2-2\pi l}{N}$, and we assume that $X(\omega'_{1k},\omega'_{2l}) = 1$ for simplicity.

For example, when $N = 2$, $s = 1$ and $t = 0$, Eq.(17) yields

$$\begin{aligned}|X^f_{10}(\omega_1,\omega_2)| &= |\frac{1}{4}\sum_{k=0}^{1}\sum_{l=0}^{1}\frac{sin(\omega_1 - 2\pi k - \pi)}{sin\frac{1}{2}(\frac{\omega_1-2\pi k-\pi}{2})} \\ &\quad \cdot \frac{sin(\omega_2 - 2\pi l)}{sin\frac{1}{2}(\frac{\omega_2-2\pi l}{2})}| \end{aligned} \quad (18)$$

We can find an alias free point at the frequency $\omega_1 = \pi$,$\omega_2 = 0$. By evaluating Eq.(17) for $s, t = 0, 1, 2, 3$, Table 1 is obtained.

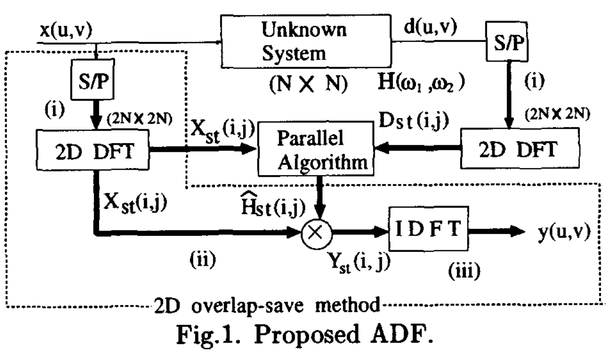

Fig.1. Proposed ADF.

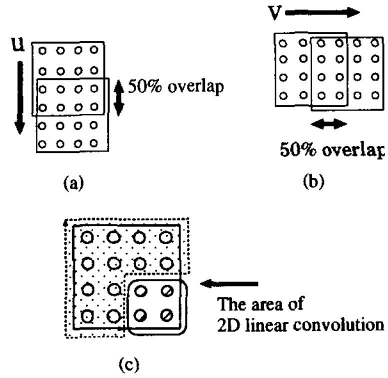

Fig.2. 2-D overlap-save method.

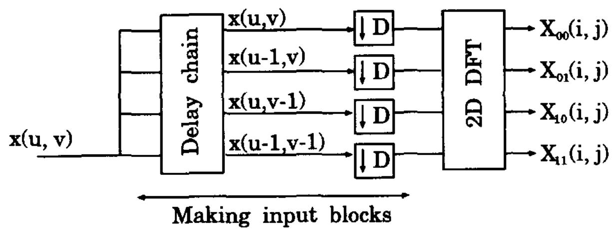

Fig.3. 2-D DFT of the image.

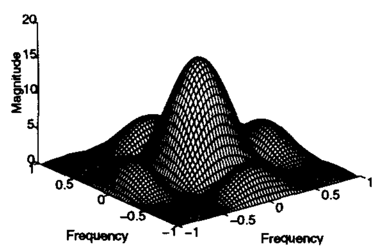

Fig.4. Frequency response of the 2-D DFT-FSF.

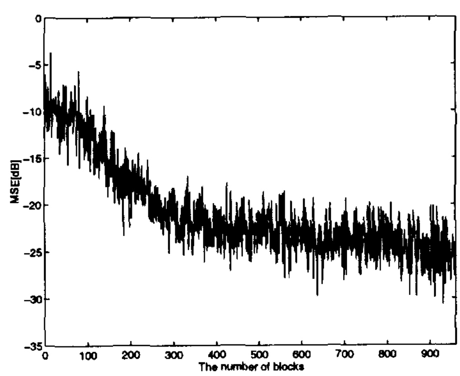

Fig.5. Mean squared error curve.