

A Hybrid LMS-LMF Scheme for Echo Cancellation

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Abstract

The coefficients of an echo canceller with a near-end section and a far-end section are usually updated with the same updating scheme, such as the LMS algorithm. In this paper we propose a novel scheme for echo cancellation that is based on the minimization of two different cost functions, i.e., one for the near-end section and a different one for the far-end section. Two approaches are addressed and only one of them lead to a substantial improvement in performance over the LMS algorithm when it is applied to both sections of the echo canceller. The proposed scheme is also shown to be robust to noise variations, which is not the case for the LMS algorithm.

1. Introduction

The two most widely used algorithms for adaptive filters are the least mean-squares (LMS) [1] and the recursive least-squares (RLS) [2] algorithms. These are used to estimate the echo path in an echo canceller structure [3], and out of the two, the LMS algorithm is the most widely used. The LMS algorithm consists of minimizing the square of the error. In [4], the least mean fourth (LMF) algorithm was suggested where it arose as a special case of the more general family of steepest descent algorithms [2] with $2k$ error norms, k being a positive integer. However, no application of a combined LMS-LMF algorithm to echo cancellation has been reported before.

As it is known that in non-Gaussian environments the LMS algorithm is non-optimal. However, the LMF algorithm enjoys better convergence than the LMS algorithm in the non-Gaussian environments. As a result, an adaptive algorithm behaving robustly in Gaussian and non-Gaussian environments is required.

In this work, a new algorithm applied to long echo cancellers with two sections, the near-end (NE) and the far-end (FE) sections, separated by a bulk to considerably reduce the number of coefficients of the filter

is proposed. The proposed algorithm consists of applying the LMS algorithm in the NE section of the echo canceller and the LMF algorithm in its FE section, and it will be henceforth called the least mean square-fourth (LMSF) algorithm.

It is shown in this work that the LMSF algorithm leads to a lower minimum mean square error, hence results in less misadjustment, a faster convergence compared to the one obtained by the LMS algorithm when applied to both sections of the echo canceller [5]-[6], and more importantly behaving robustly in Gaussian and non-Gaussian environments.

Similar approach [7] is also used in the design of decision feedback equalizers (DFE). Substantial improvement in performance is obtained through the use of this method.

After presenting, in Section 2, the proposed cost functions, Sections 3 and 4 deal with the respective algorithms resulting from these cost functions. Performance evaluation of the resulting algorithms is given in Section 5.

2. Development of the minimization criteria

In this work the proposed performance criteria, are defined, respectively, for the NE section and the FE section as:

$$J_N(n) = E[e^2(n)], \quad (1)$$

$$J_F(n) = E[e^4(n)], \quad (2)$$

the error is $e(n) = d(n) + w(n) - y(n)$, where $d(n)$ is the desired value, $y(n)$ is the output of the adaptive system, and $w(n)$ is the additive noise, Fig. 1 depicts this clearly.

A similar approach is reported in [8], with a modification made to the FE section where a mixed-controlled LMS-LMF algorithm is applied.

Both of the above cost functions, Equations (1) and (2), are convex functions, as a result of these

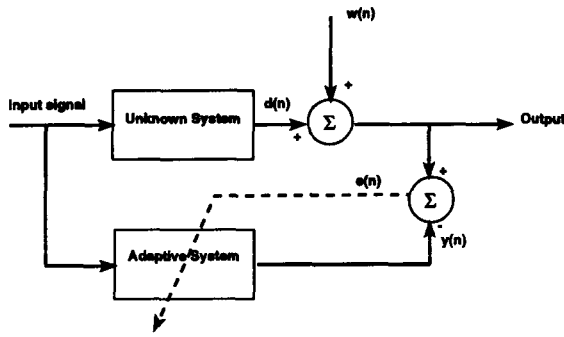


Figure 1: Block diagram of adaptive system identification.

minimization criteria a global minimum will be obtained. An adaptive process, such as the steepest descent method [2], can be used to seek this point.

Two approaches are considered for the analysis of the proposed cost functions. One would be updating both sections of the echo canceller with the same step size μ and hence the algorithm will be called LMSF Type I, and the other would be using two different step sizes for each section of the canceller μ_1 and μ_2 , respectively, and therefore called LMSF Type II.

Based on this motivation, the LMSF algorithm for recursively adjusting the tap coefficients of the NE canceller, $C_N(n)$, and those of the FE canceller, $C_F(n)$, is derived.

3. The LMSF Type I algorithm

For the LMSF Type I algorithm, then, the updating scheme is:

$$C_N(n+1) = C_N(n) + \mu e(n)X_N(n), \quad (3)$$

$$C_F(n+1) = C_F(n) + 2\mu e^3(n)X_F(n), \quad (4)$$

where $X_N(n)$ and $X_F(n)$ are the input signals in the NE and the FE sections of the canceller, respectively. Sufficient condition for convergence of the algorithm is therefore given by:

$$0 < \mu < \frac{2}{\lambda_{max}(\mathbf{A})}, \quad (5)$$

where $\lambda_{max}(\mathbf{A})$ is the largest eigenvalue of the following matrix:

$$\mathbf{A} = \begin{bmatrix} \mathbf{R}_N & \mathbf{0} \\ \mathbf{0} & 6E[w^2(n)]\mathbf{R}_F \end{bmatrix}, \quad (6)$$

where \mathbf{R}_N and \mathbf{R}_F are, respectively, the autocorrelation matrices of the NE and the FE sections of the echo canceller, and $E[w^2(n)]$ is the measurement noise power.

4. The LMSF Type II algorithm

For the LMSF Type II algorithm, the updating scheme is expressed in the following form

$$C_N(n+1) = C_N(n) + \mu_1 e(n)X_N(n), \quad (7)$$

$$C_F(n+1) = C_F(n) + 2\mu_2 e^3(n)X_F(n). \quad (8)$$

and therefore sufficient conditions for convergence of the algorithm are [9]-[10]:

$$0 < \mu_1 < \frac{2}{N_1 \sigma_x^2}, \quad (9)$$

and

$$0 < \mu_2 < \frac{2}{6N_2 \sigma_x^2 E[w^2(n)]}, \quad (10)$$

where N_1 and N_2 are the lengths of the NE the FE cancellers, respectively, and σ_x^2 is the signal power.

The results in (5), (9), and (10) are obtained when the analysis for the convergence in the mean of the LMSF algorithm is used. Table 1 summarizes main parameters for this algorithm [9]. The first μ_{max} and the second one correspond, respectively, to the convergence in the mean and the mean-square of the LMSF Type II algorithm, and μ_{opt} is the optimal value of the step size. The i th time constant in this table is denoted by τ_i .

5. Simulation results

The performance of the LMSF algorithm (Type I & II) is compared with three other algorithms. These are: the LMS algorithm that is based on the minimization of the MSE, i.e., $J_1(n) = E[e^2(n)]$, the LMF algorithm that is based on the minimization of the mean fourth error (MFE), i.e., $J_2(n) = E[e^4(n)]$, and the third algorithm is based on the minimization of two functions, a MFE for the NE section of the canceller, i.e., $J_{N3}(n) = E[e^4(n)]$, and a MSE for its FE section, i.e., $J_{F3}(n) = E[e^2(n)]$. The latter algorithm will be called the least mean fourth-square (LMFS) algorithm.

The input signal is binary ($x_i = \pm 1$), the additive noise is a Gaussian white noise with a variance of -30 dB with respect to unity, and the channel considered has one zero at the origin and one pole at 0.9. Finally, the step sizes for the above stated algorithms are chosen so that their respective convergence is the fastest.

The convergence performance of all five algorithms is illustrated in Fig. 2. The LMSF Type II algorithm clearly outperforms the other four algorithms and provides less weight noise than the LMS algorithm. Also, as it is observed from Fig. 2 that the worst algorithm happens to be the LMSF Type I. This suggests that

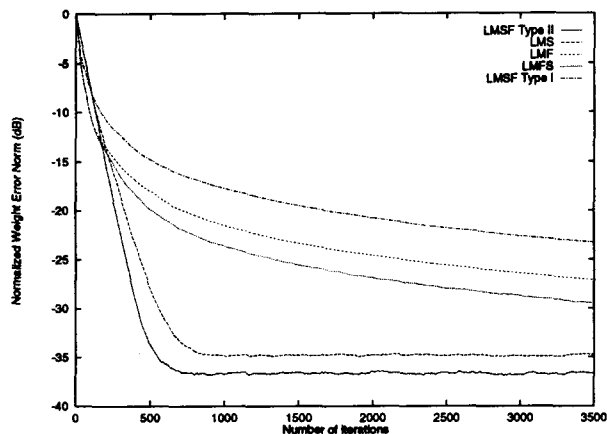


Figure 2: Learning curves for the five algorithms.

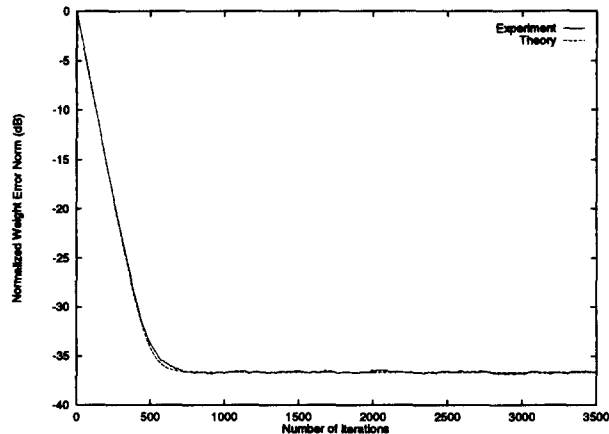


Figure 3: Theoretical and experimental learning curves for the LMSF Type II algorithm.

for the LMSF algorithm to perform well separate step sizes should be used.

The close agreement between theory and experiment for the convergence rate of the LMSF Type II algorithm is depicted in Fig. 3.

The LMSF Type II algorithm converges faster than the LMS algorithm in two respects: when the convergence rates are the fastest for each algorithm, and when both algorithms converge to the same steady state value. The latter one is depicted in Fig. 4.

The effect of the noise variance on the convergence behavior of the LMSF Type II algorithm has also been studied [9]. It is found that, as predicted by theory, as long as the noise variance is close to or much greater than the actual value the convergence of the algorithm to the optimum solution is always guaranteed.

Since the LMSF algorithm is a hybrid algorithm, its performance will be affected by the noise distribution as it is the case for the LMF algorithm [4], this depicted in Fig. 5. As it is expected from theory, the uniform distribution will give a lower misadjustment error than its Gaussian counterpart. Also, as can be noticed from this figure that in both noise distributions the LMSF Type II outperforms the LMS algorithm (even with Gaussian noise), hence the robustness of the LMSF Type II.

Finally, the LMSF Type II algorithm requires only three more multiplications per update than the LMS algorithm.

6. Conclusion

In conclusion, a new adaptive scheme for echo cancellation has been introduced. It is shown that minimizing both the mean square criterion and the mean fourth over the NE and the FE sections of the echo canceller with different step sizes, respectively, led to superior performance over the commonly used mean

square error alone, and to other proposed algorithms as well. Moreover, the algorithm is shown to be robust to Gaussian and non-Gaussian environments.

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Table 1: Main parameters of the LMSF Type II algorithm.

| | NE Section | FE Section |
|---------------|--|--|
| Cost function | $J_N(n) = E[e^2(n)]$ | $J_F(n) = E[e^4(n)]$ |
| μ_{max} | $\frac{2}{\lambda_{N_{max}}(\mathbf{R}_N)}$ | $\frac{2}{6E[w^2(n)]\lambda_{F_{max}}(\mathbf{R}_F)}$ |
| μ_{max} | $\frac{2}{N_1\sigma_z^2}$ | $\frac{E[w^2(n)]}{5N_2\sigma_z^2E[w^4(n)]}$ |
| μ_{opt} | $\frac{1}{N_1\sigma_z^2}$ | $\frac{E[w^2(n)]}{10N_2\sigma_z^2E[w^4(n)]}$ |
| τ_i | $\frac{1}{\mu_1\lambda_{N_i}(\mathbf{R}_N)}$ | $\frac{1}{6\mu_2E[w^2(n)]\lambda_{F_i}(\mathbf{R}_F)}$ |

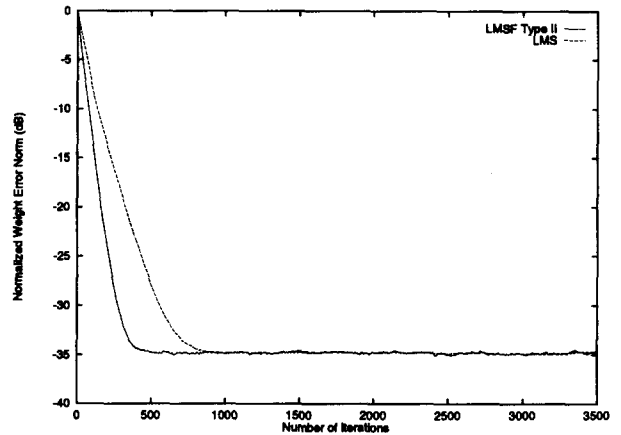


Figure 4: Learning curves for the LMSF Type II and the LMS algorithms with the same steady state value.

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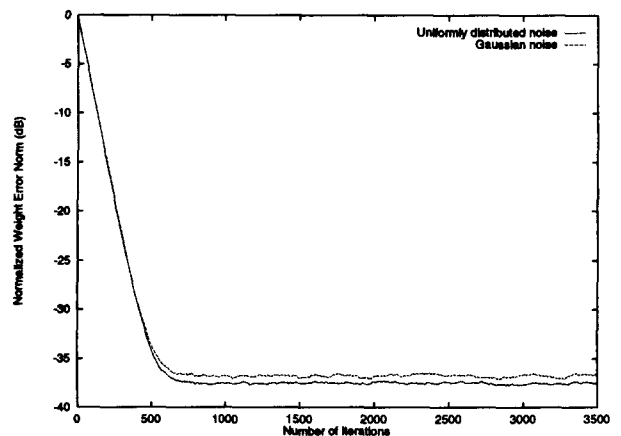


Figure 5: Effect of noise distribution on the convergence behavior of the LMSF Type II algorithm.