# ADAPTIVE SUB-CHANNEL EQUALIZATION IN MULTICARRIER TRANSMISSION

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### ABSTRACT

In multicarrier data transmission using filter banks, adaptive equalizers can be introduced in the receiver in every sub-channel, to achieve high bit rates. Following conventional data transmission techniques, two approaches can be envisaged, namely the double sampling equalizer (DSE) and the critical sampling equalizer (CSE). Both schemes are discussed and assessed in the present paper, in the multicarrier context. Estimations are given for the lengths of the equalizers as a function of the channel distortion and of the roll-off factor of the prototype filter in the receiver filter bank. Simulation results associated with two channel models are given to support the theoretical analysis.

## 1. INTRODUCTION

Filter banks can improve the spectral efficiency of multicarrier transmission schemes, since they provide a good level of separation between the sub-bands. In fact, the prototype filter, on which the uniform filter bank is based, can be designed in such a way that a given subchannel overlaps with its two neighbouring channels only.

The multicarrier approach is intended to be able to withstand severe channel distortions and it is the justification for the cost related to its complexity [1]. In such conditions, equalization is needed in the sub-channels and adaptive techniques are employed if the transmission channel is time varying.

Adaptive sub-channel equalization for that specific purpose has been reported in several papers [2-4]. A three branch equalizer per sub-channel is involved,. One branch being used to equalize the sub-channel under consideration itself and the two remaining branches having the objective of cancelling the interference from the two neighbouring subchannels.

The purpose of the present paper is to introduce a single branch equalizer to perform the same task. Since the sampling rate in the sub-channel has to be at least twice the symbol rate in the sub-channel, the device is called a double sampling equalizer (DSE). The paper is organized as follows. The double sampling equalizer is analysed in the next section and the specificities with respect to the conventional Fractionally spaced equalizer (FSE) for data communication are underlined [5]. Then, a comparison, in terms of computational complexity and performance objectives, with the 3-banch critical sampling equalizer is provided in section III. Finally, simulation results are given in section IV and discussed in the last section.

## 2. THE DOUBLE SAMPLING EQUALIZER (DSE)

According to the multicarrier concept, the transmission channel bandwidth is divided into $N$ equal sub-bands. The uniform filter bank which performs that task is derived from a prototype real low-pass filter H(z), which satisfies the classical Nyquist criterion for data transmission. Thus, independent data sequences can be transmitted in the subchannels.

The transmission model for subchannel $i$ is given in Fig.1. The filtering function is assumed to be equally split between the emitter and the receiver. In order to maximize the total bit rate, the symbol rate in every subchannel is $1/N$, and, accordingly, the cut-off frequency of the prototype real filter is $1/2N$. Due to the filter transition band $\Delta f$, the neighbouring subchannels $i$-$1$ and $i+1$ overlap with subchannel $i$. The transmission channel is modelled by the function $C(z)$, which has to be compensated by its inverse in the receiver. In fact, it is the objective if the equalizer perform that compensation.

The complex input data in each subchannel can be represented by their Z-transform $D_i(Z^N)$ and the signal $X_i(z)$ at the output of the receiver filter bank, before equalization, can be expressed as a sum of three terms:

$$X_i(z) = \left(D_i(z^N)H(zW^i) + D_{i+1}(z^N)H^{1/2}(zW^i)H^{1/2}(zW^{i+1}) \right. \quad (1)$$
$$\left. + D_{i-1}(z^N)H^{1/2}(zW^i)H^{1/2}(zW^{i-1})\right)C(z)$$

where $W = \exp\left(-j2\pi/N\right)$.

The overlapping of the subchannels is illustrated by the three branches in the emitter section in Fig;1 , and, therefore, it is straightforward to devise an equalizer with the same configuration of three branches in the receiver section.

The overlapping of the spectra in the signal $X_i(z)$ is shown in Fig. 2. Clearly, the frequency band occupied by $X_i(f)$ is $1/N+\Delta f$ and the transition band $\Delta f$ of the prototype filter $H(z)$ is assumed to be smaller than $1/N$. In those conditions, aliasing is avoided if the complex signal is sampled at the rate $2/N$.

In the absence of channel distortion, interference between neighbouring subchannels is avoided by interleaving the symbols in the "in-phase" subchannel and in the "quadrature" subchannel. It is the so called O-QAM technique, in which the impulse responses of neighbouring channels *i-1* and *i+1* are exactly zero at the sampling time of subchannel *i* [2,3].

Now, if the channel distortion $C(f)$ is introduced, it becomes necessary to include an equalizer in the link. The corresponding block diagram for the receiver is shown in Fig. 3. In each subchannel, the equalization operation is performed on the frequency band of the signal at the output of the analysis filter bank. In order to minimise the computation rate, it is recommended to use the smallest possible sampling frequency and $2/N$ is adequate to prevent aliasing.

The decimation by $N/2$ operates a frequency shift and the signal shown in Fig.2 is moved to the frequency band $\left[-\left(\frac{1}{2N}+\frac{\Delta f}{2}\right)\left(\frac{1}{2N}+\frac{\Delta f}{2}\right)\right]$. Therefore, the transfer function of the equalizer needed, $E_i(Z)$, can be obtained as the product of the shifted inverse channel transfer function $C^{-1}(zW^{-i})$ by the transfer function $H_d(Z)$ of a decimation low-pass real FIR filter with maximal transition band $\left[\left(\frac{1}{2N}+\frac{\Delta f}{2}\right)\frac{2}{N}-\left(\frac{1}{2N}+\frac{\Delta f}{2}\right)\right]$:

$$E_i(z)=C^{-1}(zW^{-i})H_d(z) \qquad (2)$$

This decimation filter prevents aliasing to occur when the equalizer $E_i(Z)$ is undersampled by the factor $N/2$. The maximal transition bandwidth of the filter $H_d(Z)$ is $\Delta f_d = 1/N - \Delta f$ and its length $N_d$ is given by the classical estimation for FIR filters [6]

$$N_d=\frac{2}{3}\log_{10}\left(\frac{1}{10\delta^2}\right)\frac{1}{1/N-\Delta f} \qquad (3)$$

The scalar $\delta$ is the filter in-band and out-of-band ripple, which is related to the equalization accuracy. The length $N_e$ of the undersampled equalizer $E_i(Z^{N/2})$ in subchannel $i$ can be expressed as

$$N_e=(N_{c1}+N_d-1)\frac{2}{N} \qquad (4)$$

where $N_{c1}$ is the length of the ideal equalizer $C^{-1}(Z)$.

Estimation (4) is valid for all the subchannels, however, the coefficients are different, because of the frequency shift concerning $C^{-1}(Z)$. From an adaptive filtering point of view, it is interesting to notice that, in the input signal to the equalizer, $D_i(Z^N)$ drives the adaptation, while the other components appear as disturbing signals and they contribute to increase the residual output error after convergence, as will be shown in the simulations.

As concerns the implementation aspects, the double sampling is compatible with the interleaving of the symbols in neighbouring channels. In fact, the analysis filter bank in the receiver must be operated at twice the symbol rate and it is an important constraint of that kind of approach.

Referring to the classical fractionally spaced equalizer in data communication [5], at first glance, the same advantages can be expected for the DSE, in terms of adaptation speed and accuracy, robustness to timing offsets, ease of implementation. However, the situation is specific here, due to the overlapping of the subchannels which modifies the conditions of the adaptation process and the presence of the analysis filter bank in the receiver.

## 3. THE CRITICAL SAMPLING EQUALIZER (CSE)

If the signal $X_i(Z)$ in the receiver is sampled at the symbol rate $1/N$, aliasing occurs and the two interferring signals must be cancelled. The corresponding block diagram is shown in Fig.4. Two additional branches are included, which are sometimes called cross-terms [4]. In order to find their length, the interference impulse response must be determined. Assuming that the prototype filter in the bank has a transition band close to a raised cosine, the interference filter $G_i(f)$ takes the form of the following cosine function:

$$|G_i(f)|=\frac{1}{2}\cos\left(\pi\frac{f-\left(\frac{2i+1}{2N}\right)}{\Delta f}\right) \text{ for } \left|f-\frac{(2i+1)}{2N}\right|\le\frac{\Delta f}{2}$$

$$|G_i(f)|=0 \quad \text{for } \left|f-\frac{(2i+1)}{2N}\right|>\frac{\Delta f}{2} \qquad (5)$$

The envelop of the corresponding impulse response is

$$g(t)=\frac{\Delta f}{2\pi}\frac{\cos(\pi\Delta f t)}{1/4-\Delta f^2t^2} \qquad (6)$$

and the length of the interference impulse response can be estimated by $2/\Delta f$ which leads to the following number of coefficients in the cross-terms:

$$N_i=\frac{2}{\Delta f}\cdot N \qquad (7)$$

As concerns the middle section, its frequency response must include the aliasing effect, which covers the frequency band $\left[\frac{1}{2N}-\frac{\Delta f}{2},\frac{1}{2N}\right]$. Using a similar approach as above, the length of the corresponding impulse response can be estimated by $2\cdot 2/\Delta f$. Now, including the compensation of the channel distortion, the number of coefficients $N_{co}$ of the middle section is expressed by

$$N_{eo} = \left[ N_{c1} + \frac{4}{\Delta f} \right] \cdot \frac{1}{N} \quad (8)$$

The computational complexities of the DSE and CSE can be compared using estimation (4) and (8). A key observation is that the prototype filter transition band $\Delta f$ has opposite impacts in (3) and (8). For the DSE it must be kept small, while a large value is preferable for the CSE and the filter bank as well.

## 4. Simulation Results

In order to validate the above results by simulation, the transition band of the prototype filter is chosen as $\Delta f = \frac{1}{2N}$, which corresponds to a roll-off factor $\alpha = N \cdot \Delta f = 0.5$. It looks as a convenient value for both DSE and CSE.

The number of filters in the bank is $N=8$ for DSE with 129 coefficients and $N=16$ for CSE with 63 coefficients. The test signals consist of 4-QAM complex data for the DSE and one-bit real data for the CSE. As concerns the channel, two models are considered, a minimum phase (MP) channel model

$$C_1(z) = \frac{1}{1 - 0.5z^{-1}} \quad (9)$$

and a linear phase channel model:

$$C_2(z) = 0.5 - z^{-1} + 0.5z^{-2} \quad (10)$$

to reflect multipath propagation.

The steady state Mean Square Error (MSE) at the output of the equalizer is shown in Fig. 5 for the DSE, as a function of the number of equalizer coefficients, for the two channel models and two configurations of input signals. In the one input" case, the signal is present in channel $i$ only and there is no residual interchannel interference, the subchannel equalization is tested separately.

On the contrary, when independent input signals are fed to channel $i$ and neighbouring channels $i+1$ and $i-1$, the residual interference adds up and it accounts for the difference between the continuous and dotted curves in Fig. 5.

It is worth pointing out that the performance obtained by the DSE with channel $C_1(z)$ is significantly better than with channel $C_2(z)$. In fact, the transversal equalizer attempts a minimum phase (MP) approximation of the inverse of $C_2(z)$, in the frequency band of subchannel $i$, which results in a long filter.

Fig.6 is devoted to the CSE. The results confirm the estimated length for the cross-terms, namely $2/\alpha = 4$. In the simulations, independent data are present in channel 2, 3 and 4, and the measurements are made in subchannel $i=3$. With channel $C_1(z)$, where a small amount of distortion is introduced in the subchannels, the performance is insensitive to the length of the middle section. On the contrary, with channel $C_2(z)$, it is essentially the middle section which controls the output MSE and $N_{eo}=9$ coefficients are needed to reach 70 dB, which is in good agreement with estimation (8).

The Least Mean square (LMS) algorithm is used in the adaptation procedure for the equalizers and measurements are taken once the system has reached the steady state.

## 5. Conclusion

Two approaches have been investigated for subchannel equalization in a multicarrier system and estimations have been derived for the length of the double sampling equalizer with a single branch and the critical sampling equalizer with 3 branches.

Clearly, both schemes can meet the performance objectives. More work is certainly needed to reach a detailed and fair comparison of the two approaches, however, the above results give interesting preliminary indications.

With a roll-off $\alpha = 0.5$ for the prototype filter of the analysis-synthesis filter banks, the number of equalizer coefficients is almost the same in both cases for similar output steady state mean square error levels. Considering that the computation rate in the receiver filter bank must be doubled for the DSE, the CSE has a clear advantage in arithmetic complexity.

Now, if different values of the roll-off factor are considered, it must be pointed out that the length of the DSE increases with $\alpha$, while it is the opposite for the CSE and the filter banks.

To pursue and complete the comparison, other respects should be considered and more distortions should be included. In particular, the sensitivity to timing offset in the receiver must be investigated.

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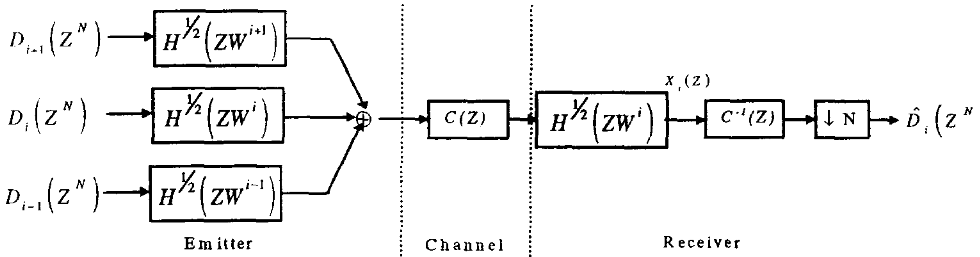


*Fig.1 Transmission model for sub-channel i.*

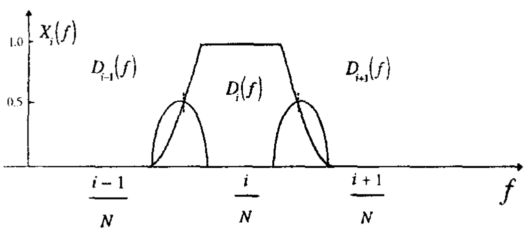


*Fig.2 Spectrum of the i-th output of the analysis filter bank.*

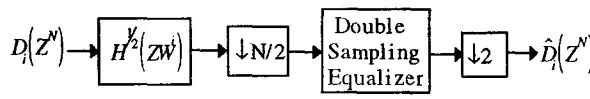


*Fig.3 Receiver with double sampling Equalizer.*

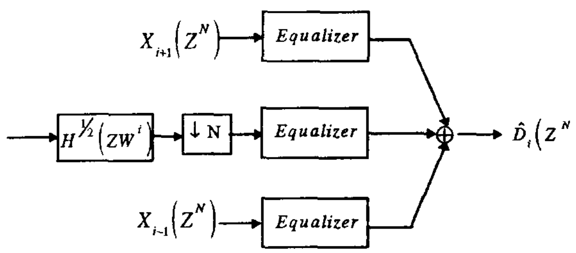


*Fig.4 Critical sampling equalizer for sub-channel i.*

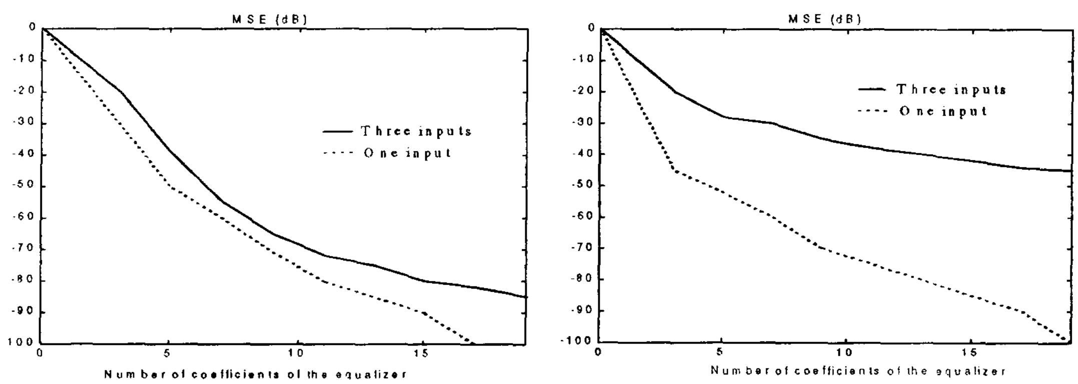


*Fig. 5 DSE: output MSE versus number of coefficients for channel model 1 and 2 with $\alpha = 0.5$*

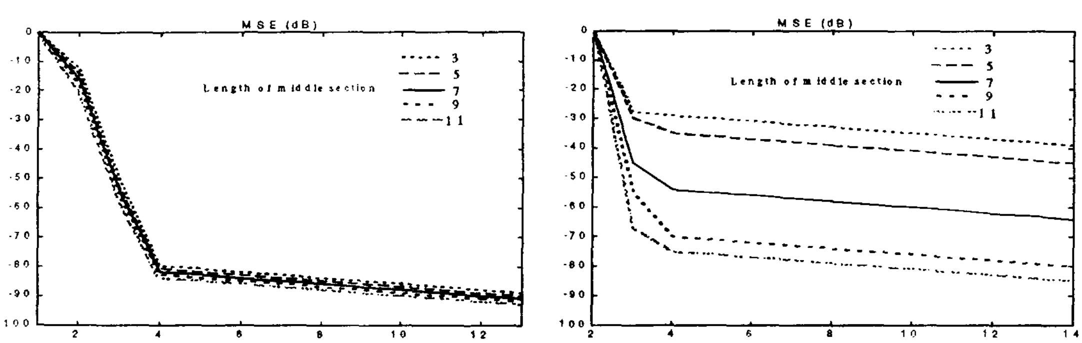


*Fig. 6 CSE: output MSE versus number of coefficients in the cross-terms for channel model 1 and 2 with $\alpha = 0.5$*