ANALYSIS OF A DELAYLESS SUBBAND ADAPTIVE FILTER

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ABSTRACT

In this paper, an analysis of a delayless subband adaptive digital filter (ADF) structure is presented. In this structure adaptive weights in each subband are computed by the LMS algorithm and then transformed into those in fullband by the Hadamard transform. The conventional subband ADF has transmission delay and aliasing effects associated with the filter bank. However in this manner such defects are avoided while retaining the computational and convergence speed advantages of subband decomposition. In addition the overall transfer function of the novel type of subband ADF is strictly equivalent to that of the Wiener filter for the fullband ADF. Also, a characteristic equation is derived to discuss stability of the adaptation algorithm. Some numerical results show good performance of this scheme.

1. INTRODUCTION

Recently, there have been many works concerning subband adaptive digital filters (ADF) [1][2]. Due to the improvement of the condition number of the covariance matrices of the input signals, the subband ADF with LMS adaptive algorithm gives better convergence property over the fullband ADF [2]. However, it is shown in [1][2] that the overall transfer function of the conventional subband ADF is that of the optimal Wiener filter for the fullband ADF multiplied by the filter bank delay term. This is a serious defect especially when the delay is large for good lowpass and highpass filters.

A delayless subband ADF has been proposed by Morgan and Thi [3]. They have presented some *qualitative* analysis for both the open and closed loop schemes. Their idea is that the tap coefficients in subbands are transformed into an equivalent set of fullband ones using the frequency sampling method, that is, DFT and inverse DFT. Transmission delay is avoided since the signal for cancelling the desired signal is computed by the fullband filter. With the closed loop scheme the fullband Wiener solution is a stationary point of the LMS type algorithm for adapting the tap coefficients in subbands. However, for maximally decimated filter bank, since the corresponding transformation matrix is singular its range space may not contain the above solution. If not, it is not so easy to calculate stationary points analytically.

In this paper we analyze the closed loop scheme in Figure 1 more *quantitatively* and propose a new adaptation algorithm. First we describe the delayless subband ADF scheme proposed in [3] and propose a modified transformation based on the frequency sampling method to avoid the above defect. In the case of 2-band, this coincides with the original method in [3] for even taps. Using the approach in [4] it is found that a stationary point in fullband is not still changed and the overall transfer function of the delayless subband ADF is strictly equal to that of the fullband Wiener filter. Hence aliasing effects characteristic of the conventional subband ADF are eliminated independently of the frequency response of nonideal analysis filters. It is an additional advantage that computational complexity for the proposed transformation is less than for the frequency sampling method. Also, we briefly describe stability of the adaptation algorithm by deriving a characteristic equation. For general 2^p-band case, repeating the transformation in the 2-band case recursively, we note that subband taps are transformed into fullband ones by the Hadamard transform. Some simulation results show that the proposed method gives better performance than the original one.

2. 2-BAND DELAYLESS SUBBAND ADF

We consider the 2-band delayless subband ADF with the closed loop structure in Figure 1, where $H_i(z)$ and $G_i(z)$ $(i = 0, 1)$ are analysis and subband adaptive filters, respectively, and $x(n)$ and $d(n)$ are stationary input and desired signals with zero mean, respectively. For simplicity each signal is assumed real. $x(n)$ and the error signal $e(n)$ are decomposed into sets of subband signals using lowpass and highpass filters, and in each subband the signals are decimated by a factor 2. Here we define the subband weight and input vectors of dimension $N/2$ as

$$\boldsymbol{g}_i(n) = [g_i(0)\ g_i(1)\ \dots\ g_i(\frac{N}{2}-1)]^T|_{at\ sub-time\ n}, \quad (1)$$

$$\boldsymbol{x}_i(n) = [x_i(n)\ x_i(n-1)\ \dots\ x_i(n-\frac{N}{2}+1)]^T, \quad (2)$$

respectively, where N is the number of the fullband taps. Then the vector $\boldsymbol{g}_i(n)$ is updated using the LMS

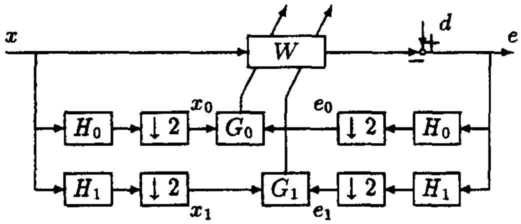

Figure 1. 2-band delayless subband ADF

type algorithm

$$g_i(n+1) = g_i(n) + \mu_i x_i(n) e_i(n) \quad (3)$$

where μ_i ($0 < \mu_i \ll 1$) is the adaptive step size.

On the other hand, the equivalence between the subband filters $G_i(z)$ and the fullband one $W(z)$ is given by

$$W(z) = \begin{cases} G_0(z^2) & \text{(lowpass band)} \\ G_1(z^2) & \text{(highpass band)} \end{cases} \quad (4)$$

in the frequency domain [2]. Hence, using the frequency sampling method we obtain the transformation of the subband taps into the fullband one $w(k)$ ($k = 0, 1, \ldots, N-1$) as

$$w(k) = \frac{1}{N} \sum_{m=0}^{N-1} W(e^{j\frac{2\pi}{N}m}) e^{j\frac{2\pi}{N}km} \quad (5)$$

$$= \begin{cases} \frac{1}{2}\{g_0(\frac{k}{2}) + g_1(\frac{k}{2})\} & (k: \text{even}) \\ \frac{1}{N} \sum_{l=0}^{\frac{N}{2}-1} \sum_{m=-\frac{N}{4}+1}^{\frac{N}{4}-1} \{g_0(l) - g_1(l)\} e^{-j\frac{2\pi}{N}(2l-k)m} . & (k: \text{odd}) \end{cases} \quad (6)$$

It can be easily shown that the fullband Wiener solution satisfies $E[x_i(n) e_i(n)] = 0$ ($i = 0, 1$), which makes the adaptation algorithm stationary. However we found that since the corresponding transformation matrix in (6) is singular, this solution may not belong to its range space. If not, the tap coefficients never converge to the solution and then the error signal is not minimum in the least mean square sense. To avoid this defect we propose a nonsingular transformation

$$w(k) = \frac{1}{2}\{g_0([\frac{k}{2}]) + (-1)^k g_1([\frac{k}{2}])\} \quad (7)$$

where $[x]$ denotes the largest integer not exceeding x. This is based on the above transformation and coincides with (6) for even k. Then a stationary point in fullband of the adaptation algorithm is the same as the Wiener solution. In addition, computational complexity is significantly reduced to a total of $N \log 2$ multiplications per subband input sample compared to a total of about $2N \log N$ for the original case.

Next, we investigate the overall transfer function of the delayless subband ADF in stationary state. In the frequency domain, from (7) the fullband filter is related to the subband one as

$$W(z) = \frac{1}{2}[(1 + z^{-1}) G_0(z^2) + (1 - z^{-1}) G_1(z^2)]. \quad (8)$$

This is consistent with (4) only for $\omega = 0, \pi$. On the other hand, $e(n)$ is stationary since $x(n)$ and $d(n)$ are stationary. From the facts about stationary processes in [4], the cross spectral density between $e_i(n)$ and $x_i(n)$ $S_{e_i x_i}(z)$ is expressed as

$$S_{e_i x_i}(z) = \frac{1}{2}[|H_i(z^{\frac{1}{2}})|^2 S_{ex}(z^{\frac{1}{2}}) + |H_i(-z^{\frac{1}{2}})|^2 S_{ex}(-z^{\frac{1}{2}})] \quad (9)$$

where $S_{ex}(z)$ is the cross spectral density between $e(n)$ and $x(n)$ given by

$$S_{ex}(z) = S_{dx}(z) - W(z) S_{xx}(z). \quad (10)$$

Also, $S_{xx}(z)$ and $S_{dx}(z)$ are the spectral density of $x(n)$ and the cross spectral density between $d(n)$ and $x(n)$, respectively. Substituting (10) into (9) and solving the linear equations $S_{e_i x_i}(z) = 0$ for $i = 0, 1$, which is equivalent to $E[x_i(n-k) e_i(n)]$ for all k, we obtain the optimal subband filters

$$G_0(z^2) = \frac{z}{2}[(1+z^{-1}) \frac{S_{dx}(z)}{S_{xx}(z)} - (1-z^{-1}) \frac{S_{dx}(-z)}{S_{xx}(-z)}], \quad (11)$$

$$G_1(z^2) = \frac{z}{2}[(1+z^{-1}) \frac{S_{dx}(-z)}{S_{xx}(-z)} - (1-z^{-1}) \frac{S_{dx}(z)}{S_{xx}(z)}]. \quad (12)$$

Substituting (11) and (12) into (8) again, we finally get $W(z) = S_{dx}(z)/S_{xx}(z)$. This is just the Wiener filter for the fullband ADF. Hence, aliasing effects are eliminated independently of the frequency response of nonideal analysis filters and the proposed delayless subband ADF is equivalent to the fullband ADF without transmission delay which is a disadvantage of the conventional subband ADF.

3. STABILITY OF ADAPTATION ALGORITHM

Here we examine the convergence condition of the LMS algorithm (3). Defining the *learning* (at time n) and *optimal* weight vectors of dimension N as $w([n/2])$ (the argument $n/2$ is due to decimation) and w_{opt}, respectively, we have the expression of the error signal as

$$e(n) = [w_{opt} - w([\frac{n}{2}])]^T x(n) + v(n) \quad (13)$$

where $x(n)$ is the input vector of dimension N expressed as

$$x(n) = [x(n)\ x(n-1)\ \ldots\ x(n-N+1)]^T \quad (14)$$

and the additive noise $v(n)$ is uncorrelated with $x(n)$. The subband signals are given by $x_i(n) = \sum_m h_i(m) x(2n-m)$ and a similar expression for $e_i(n)$.

Assuming that $\tilde{\boldsymbol{y}}$ and $\bar{\boldsymbol{y}}$ denote the even and odd tap vectors of $\boldsymbol{y}$, respectively, we reexpress (3) as

$$\begin{aligned} g_i(n+1) =& g_i(n) + \mu \sum_{lm} h_i(l) h_i(m) \tilde{x}(2n-l) \\ & \cdot [\boldsymbol{w}_{opt} - \boldsymbol{w}(n - [\frac{m}{2}])]^T \boldsymbol{x}(2n-m) \\ & + \mu \sum_{lm} h_i(l) h_i(m) \tilde{x}(2n-l) v(2n-m) \end{aligned} \quad (15)$$

where $\mu := \mu_0 = \mu_1$. Using (7), we obtain

$$\tilde{\boldsymbol{w}}(n) = \frac{1}{2}[g_0(n) + g_1(n)], \quad (16)$$

$$\bar{\boldsymbol{w}}(n) = \frac{1}{2}[g_0(n) - g_1(n)]. \quad (17)$$

Defining the weight-error vector as $\boldsymbol{\epsilon}(n) = \boldsymbol{w}(n) - \boldsymbol{w}_{opt}$ and using (15), (16), and (17), we have

$$\begin{aligned} \tilde{\epsilon}(n+1) =& \tilde{\epsilon}(n) - \mu \sum_{lm} a_0(l,m) \tilde{\boldsymbol{x}}(2n-l) \\ & \cdot \boldsymbol{x}^T(2n-m) \boldsymbol{\epsilon}(n - [\frac{m}{2}]) \\ & + \mu \sum_{lm} a_0(l,m) \tilde{\boldsymbol{x}}(2n-l) v(2n-m) \end{aligned} \quad (18)$$

where $a_i(l,m) = 1/2\{h_0(l)h_0(m) + (-1)^i h_1(l)h_1(m)\}$ $(i = 0,1)$. Assuming that the input is statistically independent of the weight-error, it follows that [5]

$$\begin{aligned} E[\tilde{\epsilon}(n+1)] =& E[\tilde{\epsilon}(n)] - \mu \sum_{lm} a_0(l,m) \\ & \cdot \{\tilde{\boldsymbol{R}}_{lm} E[\tilde{\epsilon}(n - [\frac{m}{2}])] + \bar{\boldsymbol{R}}_{lm} E[\bar{\epsilon}(n - [\frac{m}{2}])]\} \end{aligned} \quad (19)$$

where $\tilde{\boldsymbol{R}}_{lm} = E[\tilde{\boldsymbol{x}}(2n-l)\tilde{\boldsymbol{x}}^T(2n-m)]$ and $\bar{\boldsymbol{R}}_{lm} = E[\tilde{\boldsymbol{x}}(2n-l)\bar{\boldsymbol{x}}^T(2n-m)]$. A similar equation is derived for $E[\bar{\epsilon}(n)]$. Combining these updating equations, finally a characteristic equation is given by

$$\left| (z-1)\boldsymbol{I} + \mu \sum_{lm} \left[\begin{array}{c|c} a_0(l,m)\tilde{\boldsymbol{R}}_{lm} & a_0(l,m)\bar{\boldsymbol{R}}_{lm} \\ \hline a_1(l,m)\bar{\boldsymbol{R}}_{lm} & a_1(l,m)\tilde{\boldsymbol{R}}_{lm} \end{array} \right] z^{-[\frac{m}{2}]} \right| = 0. \quad (20)$$

If all the roots of (20) are inside the unit circle, the tap coefficients converge. The stability of the adaptation algorithm depends on the frequency response of analysis filters and the autocorrelation of the input signal. However the order of the equation (20) is so large that it is not so easy to find the stabilizing parameter μ analytically.

4. MULTI-BAND DELAYLESS SUBBAND ADF

We consider a 4-band delayless subband ADF with tree structure. Here we define the subband taps in the i-th band as $g_i(k)$ $(i = 0,1,2,3;\ k = 0,1,\ldots,N/4-1)$. Reducing this scheme to the 2-band one, as in the Section 2 the subband taps are transformed as

$$w_i(k) = \frac{1}{2}\{g_{2i}([\frac{k}{2}]) + (-1)^k g_{2i+1}([\frac{k}{2}])\} \quad (21)$$

for $i = 0,1$, $k = 0,1,\ldots,N/2-1$ where $w_0(k)$ and $w_1(k)$ are the taps in the lowpass and highpass bands, respectively. Next, we transform $w_i(k)$ into the fullband taps $w(k)$ using

$$w(k) = \frac{1}{2}\{w_0([\frac{k}{2}]) + (-1)^k w_1([\frac{k}{2}])\} \quad (22)$$

for $k = 0,1,\ldots,N-1$. From (21) and (22), the transformation of the subband taps into the fullband ones is expressed as

$$\begin{bmatrix} w(4k) \\ w(4k+1) \\ w(4k+2) \\ w(4k+3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} g_0(k) \\ g_2(k) \\ g_1(k) \\ g_3(k) \end{bmatrix} \quad (23)$$

for $k = 0,1,\ldots,N/4-1$. We note that the matrix at the right-hand side of (23) is the 4×4 Hadamard matrix and the order of band in the subband tap vector is changed appropriately.

In an M-band case where $M = 2^p$ $(p = 1,2,\ldots)$, repeating the above operation recursively, the subband taps are transformed into the fullband one as

$$\boldsymbol{w}_f(k) = \frac{1}{M}\boldsymbol{H}(p)\boldsymbol{w}_s(k) \quad (24)$$

for $k = 0,1,\ldots,N/M-1$ where

$$\boldsymbol{w}_f(k) = [w(Mk)\ w(Mk+1)\ \ldots\ w(Mk+M-1)]^T, \quad (25)$$

$$\begin{aligned} \boldsymbol{w}_s(k) =& [\ g_0(k)\ g_{M/2}(k)\ g_{M/4}(k)\ g_{3M/4}(k)\ g_{M/8}(k) \\ & g_{5M/8}(k)\ g_{3M/8}(k)\ g_{7M/8}(k)\ \ldots\ g_{M-2}(k)\ g_1(k) \\ & g_{M/2+1}(k)\ g_{M/4+1}(k)\ g_{3M/4+1}(k)\ \ldots\ g_{M-1}(k)\ ]^T, \end{aligned} \quad (26)$$

$$\boldsymbol{H}(p+1) = \left[\begin{array}{c|c} \boldsymbol{H}(p) & \boldsymbol{H}(p) \\ \hline \boldsymbol{H}(p) & -\boldsymbol{H}(p) \end{array} \right], \quad \boldsymbol{H}(1) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (27)$$

Changing the order of band in the subband tap vector appropriately, we can transform the tap coefficients in subbands into those in fullband using the Hadamard transform. The transformation requires $M \log M$ multiplications for each block and a total of $N \log M$ per subband input sample. Since $M \leq N$ generally, computational complexity in the proposed method is less than in the frequency sampling method.

5. SIMULATION RESULTS

We consider system identification problems and assume that the unknown system to be identified is a 15th order FIR filter and is corrupted by a white noise with SNR = 40 dB. The analysis filter bank is the 2-band CQF bank

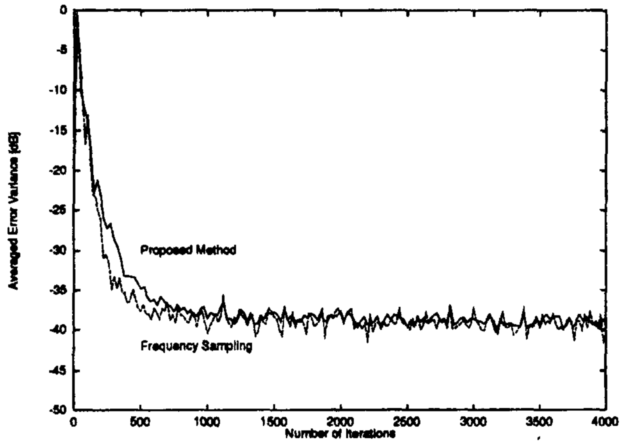

Figure 2. Learning curve of error signal (I)

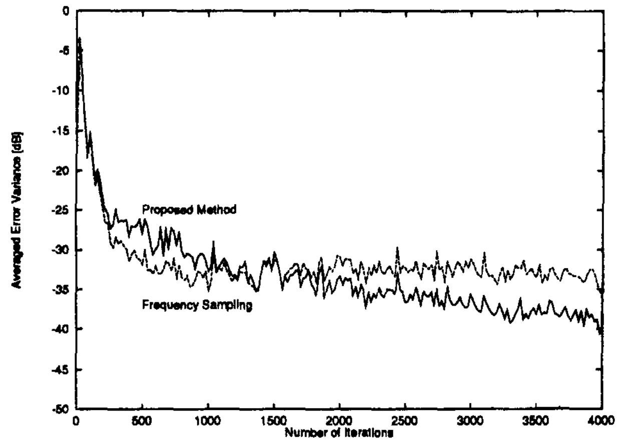

Figure 3. Learning curve of error signal (II)

Table 1. Averaged error variance after convergence for each unknown system (dB)

	proposed	original	conventional
I	−39.0	−39.0	−37.7
II	−38.8	−33.4	−33.6

with 16 taps in [6] and the input signal is generated by AR(1) with coefficient 0.9.

Figure 2 shows the learning curves of the error signal in the proposed method (by solid line) and the frequency sampling method (by dashed line). From this figure and the upper row of Table 1, which shows the averaged error variances after convergence in the conventional subband filtering method as well as in the above two methods, both of the delayless methods have better performance. Moreover in these methods the unknown system is almost completely identified and then there are few aliasing effects. However, the curve in the proposed method converges somewhat slower since in the frequency domain (8) represents the true fullband filter less equivalently than (4).

For another unknown system, the results are shown in Figure 3 and the lower row of Table 1. Consequently the proposed method has the best performance. However the frequency sampling method shows nearly equal performance to the conventional one. The difference arises since the fullband Wiener solution does not belong to the range space of the transformation matrix corresponding to (6). In the proposed method the unknown system is almost completely identified in this case as well. In addition, it is straightforward to calculate the theoretical values of the tap coefficients in subbands, and these values are approximately equal to the experimental ones, which are averaged after convergence.

6. CONCLUSION

A delayless type of subband ADF has been analyzed *quantitatively* and a nonsingular transformation of adaptive tap coefficients has been proposed. This scheme avoids the defects of the conventional type, aliasing effects as well as transmission delay. The frequency response of nonideal analysis filters has an effect only on convergence property. Moreover simulation results show advantages of this scheme. Whatever the desired signal, it can be completely cancelled, while in the frequency sampling method it is possible only if the fullband Wiener solution belongs to the range space of the corresponding transformation. Now we need to examine the convergence condition of the adaptation algorithm more, which is briefly stated in Section 3.

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