A NEW EFFICIENT METHOD OF CONVERGENCE CALCULATION FOR ADAPTIVE FILTERS USING THE SIGN ALGORITHM WITH DIGITAL DATA INPUTS

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ABSTRACT

This paper proposes a new method of theoretical calculation of the expected convergence process for adaptive filters using the Sign Algorithm with *digital data as the input reference* signal. In the analysis use is made of *Gaussian approximated conditional pdf of the error signal* to derive a set of difference equations. The results of the experiment show sufficient accuracy of the proposed method for practical use, while significantly reducing the computing time in comparison with the previous methods.

1. INTRODUCTION

This paper proposes a new method of theoretical calculation of the convergence process for adaptive filters using the Sign Algorithm with digital data as the input reference signal.

Adaptive filters are widely used in the communications field. Examples are acoustic echo canceller, noise canceller, data echo canceller, and decision feedback equalizer, to name a few.

Among the many control algorithms for the adaptive filters, LMS Algorithm seems most popular. However, Sign Algorithm, derived from the LMS algorithm, is also attractive for its simplicity in implementation and robustness against disturbances. Fig.1 shows a data echo canceller for use in a two-wire bidirectional transmission system, while Fig.2 depicts a decision feedback equalizer to eliminate the intersymbol interference due to transmission distortion. In either of these examples, the adaptive filter using the sign algorithm has *digital data as the input reference* signal.

Many literatures are found which analyze the convergence of adaptive filters. Most of them assume Gaussian input reference process and Gaussian additive noise [1]-[3]. However, these analyses do not apply, in theory, to the adaptive filters in the figures above, since the input reference is digital data with discrete values, such as Binary, AMI and 2B1Q code.

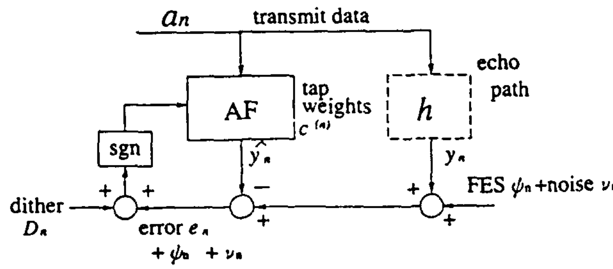

Fig.1 Data echo canceller.

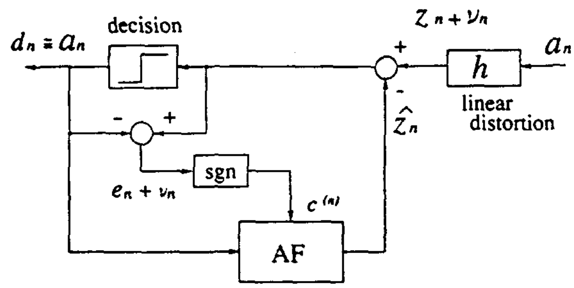

Fig.2 Decision feedback equalizer.

Claasen *et al.* analyze adaptive filters with general input reference process and non-Gaussian additive noise[4]. The analysis restricts the input reference to a *white* process, which results in a less accurate theoretical value of the filter convergence for a *colored* input reference process.

Reference [5] develops analysis of a data echo canceller with binary data as the input reference, based on jointly Gaussian distributed tap weights newly proposed. The theoretically calculated convergence is in an excellent agreement with the empirical one. However, the calculation requires averaging over all possible patterns of the input reference signal vector with the length equal to the number of the filter taps. Thus the amount of computation for this averaging, using the error function, becomes prohibitively large as the number of the taps increases.

Therefore, efficient, namely less time-consuming, method of convergence calculation with sufficient accuracy is desired for adaptive filters using the sign algorithm, when the input reference is a digital data signal including a *colored* process.

Section 2 of the paper proposes a new method of convergence calculation using *conditional pdf* of the error signal plus digital data and Gaussian noise. In Section 3 experimental results are given for three examples where we compare simulation and theoretical calculation of the filter convergence. Section 4 concludes the paper.

2. ANALYSIS

The tap weight update equation for the Sign Algorithm is given by

$$c^{(n+1)} = c^{(n)} + \alpha_c sgn(e_n + \psi_n + \nu_n)\mathbf{a}^{(n)} \quad (1)$$

where $\mathbf{a}^{(n)} = [\mathrm{a}_n, \mathrm{a}_{n-1}, \ldots, \mathrm{a}_{n-N+1}]^T$ is stationary input reference signal vector, $c^{(n)}$ is tap weight vector, ψ_n is digital data signal (Far End Signal in a data echo canceller), ν_n is additive Gaussian noise, N is number of taps, and α_c is step size.

Defining "tap error" vector $\theta^{(n)} = h - c^{(n)} = [\theta_0^{(n)}, \theta_1^{(n)}, \cdots, \theta_{N-1}^{(n)}]^T$ (h : unknown system response to be estimated), we find update equation for the tap errors

$$\theta^{(n+1)} = \theta^{(n)} - \alpha_c sgn(e_n + \psi_n + \nu_n)\mathbf{a}^{(n)} \quad (2)$$

and the error signal

$$e_n = \mathbf{a}^{(n)T}\theta^{(n)}. \quad (3)$$

Here a_n and ψ_n are digital data processes such as Binary, AMI and 2B1Q.

From eqs. (2) and (3) the difference equations for the mean and the covariance of $\theta^{(n)}$ can be calculated as

$$m^{(n+1)} = m^{(n)} - \alpha_c p^{(n)} \quad (4)$$

$$\mathbf{R}^{(n+1)} = \mathbf{R}^{(n)} - \alpha_c(\mathbf{U}^{(n)} + \mathbf{U}^{(n)T}) + \alpha_c^2(\mathbf{Ra} - p^{(n)}p^{(n)T}) \quad (5)$$

where

$$\mathbf{Ra} = E\left[\mathbf{a}^{(n)}\mathbf{a}^{(n)T}\right]$$

: covariance matrix of the input reference $\mathbf{a}^{(n)}$

$$m^{(n)} = E\left[\theta^{(n)}\right],\ \mathbf{R}^{(n)} = E\left[\Delta^{(n)}\Delta^{(n)T}\right],\ \Delta^{(n)} = \theta^{(n)} - m^{(n)},$$

$$p^{(n)} = E\left[sgn(e_n + \psi_n + \nu_n)\mathbf{a}^{(n)}\right] = \left[p_0^{(n)}, p_1^{(n)}, \cdots, p_{N-1}^{(n)}\right]^T \quad (6)$$

and

$$\mathbf{U}^{(n)} = E\left[sgn(e_n + \psi_n + \nu_n)\mathbf{a}^{(n)}\Delta^{(n)T}\right] = \left[U_{k\kappa}^{(n)}\right] \quad (7)$$

Now let $z = e_n + \psi_n + \nu_n$ and consider a random variable $z|\mathrm{a}_{n-k}$ given the values of ψ_n and $\theta^{(n)}$. Then

$$p_k^{(n)} = E_{\mathrm{a}_{n-k},\psi_n}\left[E_\theta\left[E_{z|\mathrm{a}_{n-k}}\left[sgn(z)\,\middle|\,\psi_n, \theta^{(n)}\right]\right]\mathrm{a}_{n-k}\right] \quad (8)$$

and

$$U_{k\kappa}^{(n)} = E_{\mathrm{a}_{n-k},\psi_n}\left[E_\theta\left[E_{z|\mathrm{a}_{n-k}}\left[sgn(z)\,\middle|\,\psi_n, \theta^{(n)}\right]\Delta_\kappa^{(n)}\right]\mathrm{a}_{n-k}\right] \quad (9)$$

(E_x [.] denotes expectation with respect to x.)

In the former analyses, $e_n + \nu_n$ was assumed to be Gaussian distributed [3]. Though a_{n-k} is a random variable taking on discrete values, it is reasonable to assume taht $z|\mathrm{a}_{n-k}$ (z given a_{n-k}) is Gaussian.

Assuming the conditional *pdf* $p(z|\mathrm{a}_{n-k})$ to be *Gaussian distribution*, one can calculate eqs. (8) and (9) to obtain

$$p_k^{(n)} \cong 2E_{\mathrm{a}_{n-k},\psi_n}\left[\mathrm{a}_{n-k}erf\left(\mu_k^{(n)}/\sigma_k^{(n)}\right)\right] \quad (10)$$

and

$$U_{k\kappa}^{(n)} \cong \left(2/\sigma_k^{(n)}\right) \times E_{\mathrm{a}_{n-k},\psi_n}\left[\mathrm{a}_{n-k}\left(\mu_{ak\kappa}^{(n)}\mathrm{a}_{n-k} + \mu_{\psi k\kappa}^{(n)}\psi_n\right)p_N\left(\mu_k^{(n)}/\sigma_k^{(n)}\right)\right] \quad (11)$$

where

$$\mu_k^{(n)} = \left(r_{ak}^T m^{(n)}/\sigma_a^2\right)\mathrm{a}_{n-k} + \psi_n$$

$$\left(\sigma_k^{(n)}\right)^2 = trace\left(\mathbf{Ra}_{(k)}\mathbf{K}^{(n)}\right) + \sigma_\nu^2$$

$$= \varepsilon^{(n)} - r_{ak}^T\mathbf{K}^{(n)}r_{ak}/\sigma_a^2 + \sigma_\nu^2,$$

$$\mu_{ak\kappa}^{(n)} = r_{ak}^T r_\kappa^{(n)}/\sigma_a^2 + \left(r_{ak}^T m^{(n)}/\sigma_a^2\right)\mu_{\psi k\kappa}^{(n)},$$

$$\mu_{\psi k\kappa}^{(n)} = -m^{(n)T}\mathbf{Ra}_{(k)}r_\kappa^{(n)}/\left(\sigma_k^{(n)}\right)^2,$$

$$\mathbf{Ra}_{(k)} = \mathbf{Ra} - r_{ak}r_{ak}^T/\sigma_a^2,$$

$$\mathbf{K}^{(n)} = E\left[\theta^{(n)}\theta^{(n)T}\right] = m^{(n)}m^{(n)T} + \mathbf{R}^{(n)}$$

: second – order moment of $\theta^{(n)}$,

$\varepsilon^{(n)} = trace\left(\mathbf{Ra}\,\mathbf{K}^{(n)}\right)$: *Mean Squared Error* (MSE),

r_{ak}^T : the k - th row vector of the matrix $\mathbf{Ra}$,

$r_\kappa^{(n)}$: the κ - th column vector of the matrix $\mathbf{R}^{(n)}$,

$\sigma_a^2 = E\left[a_n^2\right]$: variance (power) of the input reference a_n,

$\sigma_\nu^2 = E\left[\nu_n^2\right]$: variance (power) of the additive Gaussian noise ν_n,

$erf(x) = \int_0^x p_N(t)dt$

and

$$p_N(x) = exp\left(-x^2/2\right)/\sqrt{2\pi}.$$

The mean and the covariance of the tap errors $\theta^{(n)}$ can now be repetitively calculated using the difference equations (4) and (5) with (10) and (11). In calculating eqs. (10) and (11), only averaging over the possible patterns of a_{n-k} and ψ_n is required, a considerable

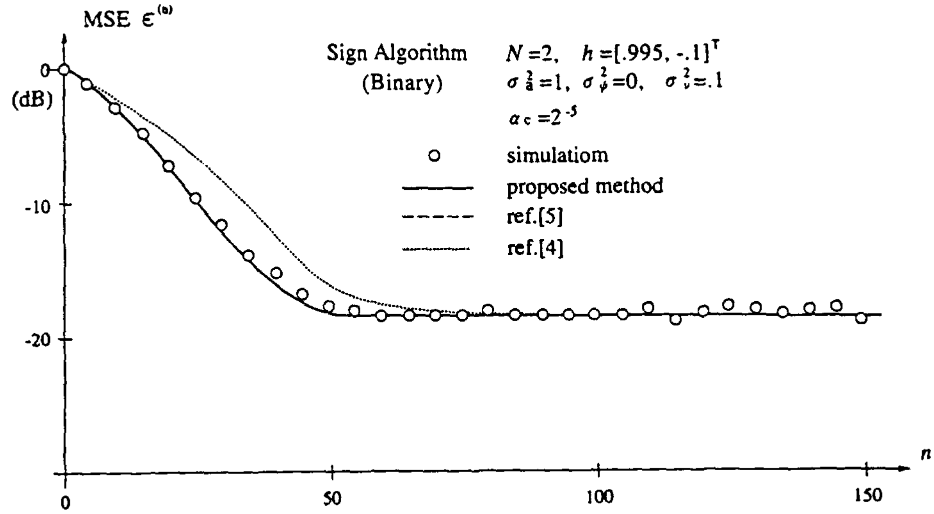

Fig.3 Adaptive filter convergence (Example #1).

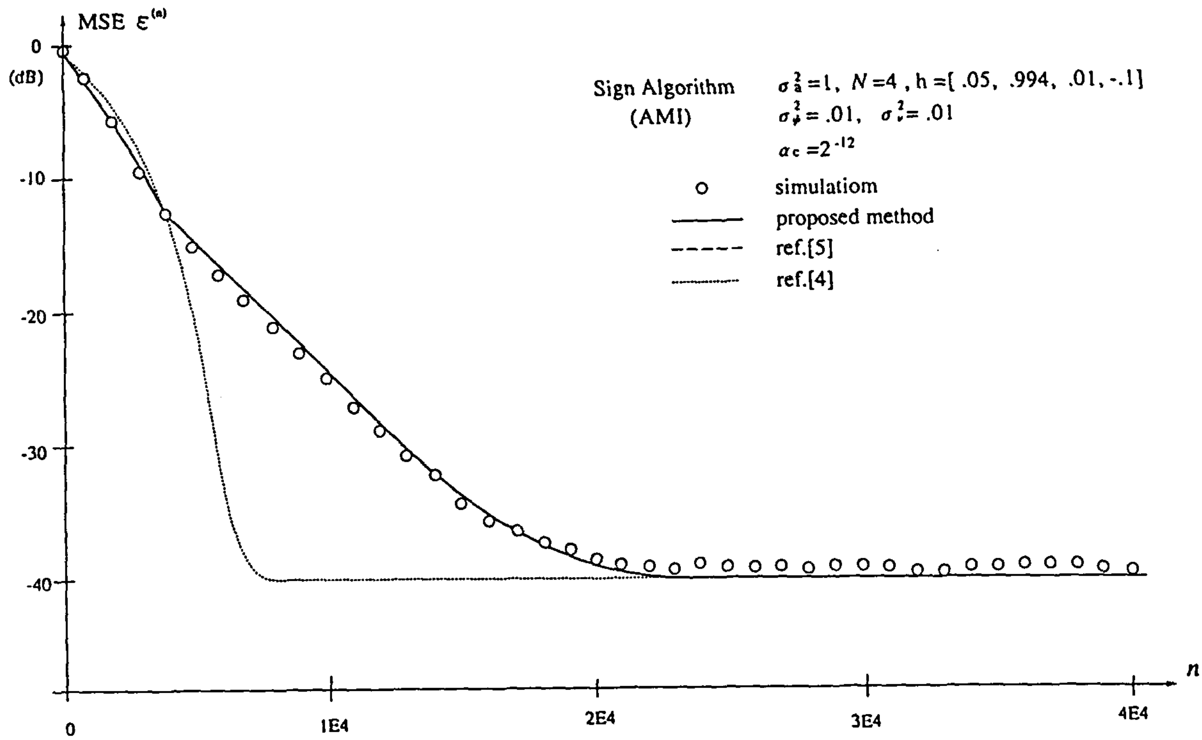

Fig.4 Adaptive filter convergence (Example #2).

reduction in the computing time. For example, with 2B1Q code, the proposed method requires $4 \times 4 \times N$ computations of the function *erf*( ) (dominant in computing time), while the method in [5] requires $4^N \times 4$. Even for $N=4$, the reduction (ratio) is as much as *16*.

3. EXPERIMENT

Three examples of adaptive filters are prepared with different types of digital data for a_n and ψ_n. Simulations and theoretical calculations of the convergence process are performed and compared.

Example #1 : input reference a_n is Binary and white process
$N=2$, with additive Gaussian noise ν_n

Example #2 : input reference a_n is AMI code
$N=4$, with AMI FES ψ_n
and additive Gaussian noise ν_n

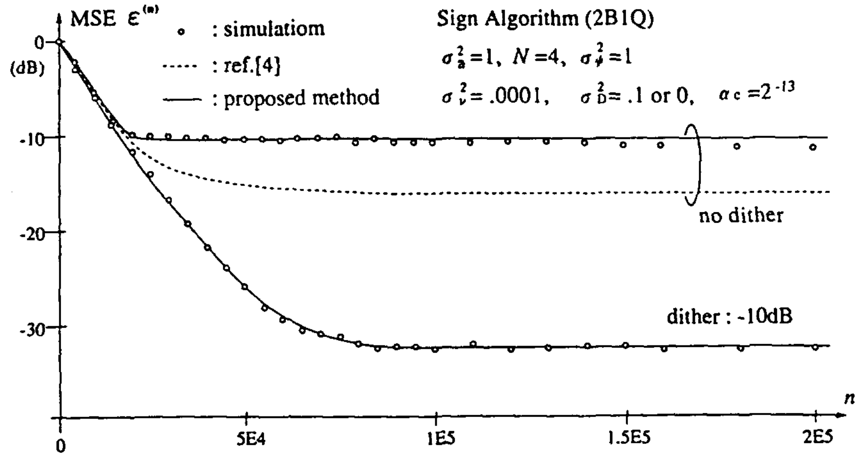

Fig.5 Adaptive filter convergence (Example #3).

Example #3 : input reference a_n is 2B1Q code
N=4, with 2B1Q FES ψ_n and additive
Gaussian noise ν_n.
with and without Gaussian "Dither" signal D_n

Figs 3 throngh 5 show the results of the experiment for Examples #1 through #3, respectively. Also plotted in the figures are the results of the convergence calculations using the methods given in [4] and [5]. The proposed method exhibits sufficient accuracy with significant improvement in efficiency as described in Section 2.

4. CONCLUSION

A new efficient method for theoretically calculating the expected convergence of adaptive filters using the Sign Algorithm with *digital data inputs* is proposed The previous methods have been less accurate or required a large amount of computation. The proposed method uses a *Gaussian approximated conditional pdf of the error signal* to derive difference equations for repetitive calculations of the mean and the covariance of the "tap errors", which further estimate the convergence of the mean squared error (MSE).

The results of the experiment show sufficient accuracy of the proposed method for practical use, while reducing the computing time by a scale of magnitude.

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