SIGNAL RESTORATION BY STATISTICAL SOFT MORPHOLOGY

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ABSTRACT

A new set of non linear signal and image processing operators is presented. Their definition is based on the introduction of the statistical properties of Bayesian reconstruction in soft morphological operators.

Statistical soft operators represent a trade-off between the noise cleaning properties of statistical morphology and the shape preservation properties of soft morphology. The main characteristic of these operators is the individualization of two parts within each structuring element (SE) according to soft morphology (i.e. "hard" and "soft" SEs), and to define on this basis a probabilistic estimation model which is a generalization of the Statistical Morphology model.

Results are presented to show that the statistical soft morphological operators can be considered robust to structured noise, i.e. noise showing both statistical (e.g. additive Gaussian noise) and morphological (e.g. noise with a particular shape) structure.

1 INTRODUCTION

The definition of statistical soft morphological operators is based on soft morphological operators [1] and statistical morphological operators [2]. Soft morphology, like mathematical morphology (MM), considers the signal that has to be processed as a set, whereas the statistical morphology is based on a functional approach.

A relationship between these operators has been derived for binary signals [3]. This paper aims at providing a more general formulation, that is valid for multilevel signals.

Soft morphology is an extension of mathematical morphology; the main differences of soft morphology with respect to MM are:

- the definition of a composed structuring set which individuates two structuring elements (SE): the hard SE $A \subset B$ and the soft one $B \setminus A$,; besides, it is defined by a r parameter that is related to the rank of an order filter;

- the elementary operations, which are in the soft morphological case, a sort of rank order filters; e.g. the output of soft dilation at m-th input is defined as the r-th largest value in the set

$$\{r \Diamond f(a): a \in A_m\} \cup \{f(b): b \in (B \setminus A)_m\} \qquad (1)$$

where $r \Diamond f(a)$ denotes the repetition operation of $f(a)$ and A_m is the structuring element A shifted at m-th position, whereas in mathematical dilation we consider the maximum value in the set

$$\{f(b): b \in (B)_m\}$$

The result of this formulation is that a greater weight is associated with the input values that fall inside A with respect to those which fall inside B\A; this property allows one to obtain less sensitive operators to small shape variations of the object that has to be processed.

Statistical morphology (SM) is also an extension of the mathematical morphology. Like MM, it considers a structuring element B as an observation window on the signal, but the output is defined according to a function-based approach that is based on a probabilistic signal model [2]. This method uses the Bayesian approach in order to define a probabilistic generalisation of mathematical morphological max-min operators. A parameter, β, is introduced to give different weights to each element from the input set. The behaviour of statistical operators changes by varying the β parameter; for example, for statistical dilation, if $\beta \to 0$ all the input elements have the same weight, so a linear filter is obtained. When $\beta \to \infty$ statistical erosion and dilation become mathematical erosion and dilation. Using an intermediate β value, it is possible to obtain operators determining a good smoothing level, that is necessary for noise elimination, without degrading high variations (such as the image edges).

Statistical soft morphology (SSM), which combines the two above-mentioned techniques, is here defined. Statistical soft morphological operators show the property of being able to clean a signal that is corrupted by a structured noise, which presents both statistical and morphological properties. An example of this kind of noise is the speckle noise [6], which is a kind of moltiplicative correlated noise affecting, for example, Synthetic Radar Aperture (SAR) images.

2 ELEMENTARY STATISTICAL SOFT OPERATORS

The basic morphological operations, *erosion* and *dilation,* consist in replacing the value of an input sample by the largest, or smallest, value in a neighbourhood set that is related to that sample. Similar operations are performed in soft morphology. In the case of SSM, as in SM [2], we suppose that input samples values are I_i and the neighbourhood of each pixel i is $N_i = N_{i,j}$ with the property that $N_{i,j} = 1$ if the j-th sample falls inside the SE B that is centred on the i-th sample.

To take into account the hard and soft part of the SE, the neighbourhood set is divided into two complementary sets N_i^h and N_i^s that are related to hard and soft SE, with the following properties:

$$N_i = N_i^h \cup N_i^s \quad , \quad N_i^h \cap N_i^s = \varnothing$$

$$N_{i,j}^h = \begin{cases} 1 & \text{if } j \in N_i^h \\ 0 & \text{otherwise} \end{cases} \qquad N_{i,j}^s = \begin{cases} 1 & \text{if } j \in N_i^s \\ 0 & \text{otherwise} \end{cases}$$

Let us consider the following events:

$$j \in N_i^h = \{\text{the } j\text{-th sample falls inside hard SE}\}$$

$$j \in N_i^s = \{\text{the } j\text{-th sample falls inside soft SE}\}$$

Owing to the neighbourhood set characteristic, it is possible to state that the $j \in N_i^h$ and $j \in N_i^s$ events are mutually exclusive, so the probability that a sample falls inside the SE B is given by:

$$P(j \in N_i) = P(j \in N_i^h) + P(j \in N_i^s) \qquad (2)$$

To obtain the winner-take-all (*statistical soft dilation*) for the i-th sample, with analogy to SM, we introduce binary decision units $V_{i,j}$ such that $V_{i,j}$=1 if the j-th sample is selected as winner, otherwise $V_{i,j}$=0.

The cost function that has to be minimized is similar to the statistical dilation; taking into account the two different disjoint SE, it is possible to write the cost function in the following way:

$$E_w[\{V_{i,k}\}] = E_w[\{V_{i,k}\}, k \in N_i^h] + E_w[\{V_{i,k}\}, k \in N_i^s] = = -\sum_j V_{i,j} N_{i,j}^h I_j - \sum_j V_{i,j} N_{i,j}^s I_j \qquad (3)$$

where $\{V_{i,j}\} = (V_{i,1}, \dots, V_{i,n})$, and n is the number of samples that as to be processed.

We have to minimize the cost function with the constraint that $\sum_j V_{i,j}$=1, because only a winner is allowed, so we obtain $V_{i,w} = 1$ when $I_w \geq I_j$ for $j \neq w$ and $V_{i,j} = 0$ otherwise.

Analogously to SM, we have to define a statistical distribution for $\{V_{i,j}\}$; owing to the property (2), it is possible to write:

$$P_w[\{V_{i,k}\}] = P_w[\{V_{i,k}\}, k \in N_i^h] + P_w[\{V_{i,k}\}, k \in N_i^s] \qquad (4)$$

For the definition of the joint probabilities $P_w[\{V_{i,k}\}, k \in N_i^h]$ and $P_w[\{V_{i,k}\}, k \in N_i^s]$, according to Soft Morphology we have to take into account the greater weight associated to the hard SE, that is due to the repetition of the falling elements inside it. First, we consider that the elements in hard SE have the same weight as the elements in soft SE (this means that $r = 1$); with analogy to SM, it is possible to write:

$$P_w[\{V_{i,k}\}, N_i^h] = \frac{1}{Z} \cdot \left\{\exp\left[-\beta_1 E_w\left[\{V_{i,j}\}, j \in N_i^h\right]\right]\right\} \qquad (5)$$

If we consider that the falling elements in hard SE are repeated by r times, it is possible to associate with them a r times higher probability. This consideration allows one to write the following probabilities:

$$P_w[\{V_{i,k}\}, N_i^h] = \frac{r}{Z} \cdot \left\{\exp\left[-\beta_1 E_w\left[\{V_{i,j}\}, j \in N_i^h\right]\right]\right\} \qquad (6)$$

$$P_w[\{V_{i,k}\}, N_i^v] = \frac{1}{Z} \cdot \left\{\exp\left[-\beta_2 E_w\left[\{V_{i,j}\}, j \in N_i^v\right]\right]\right\} \qquad (7)$$

Z is a normalization constant and β_1 and β_2 are positive parameters representing the inverse of the temperature in the hard and soft SE respectively. Following the method used by Yuille *et al.*[2], it is possible to obtain the partition function at location i:

$$Z = \sum_j \left[r \cdot \exp(\beta_1 N_{i,j}^h I_j) + \exp(\beta_2 N_{i,j}^s I_j) \right] \qquad (8)$$

Now we can define the output of the dilation operator at the i-th sample in the following way:

$$O_w(\beta_1, \beta_2, r) = \sum_k N_{i,k} I_k P_w\left[V_{i,k} = 1, V_{i,j} = 0 \ \forall \, j \neq k\right] \qquad (9)$$

From the property that is shown by eq. (2), it follows that it is possible to express the output of the statistical soft dilation at location i by means of two terms that are related to the hard and soft SE:

$$O_w(\beta_1, \beta_2, r) = \sum_k \left\{ N_{i,k}^h I_k P_w\left[V_{i,k} = 1, V_{i,j} = 0 \ \forall \, j \neq k, \ k \in N_i^h\right]\right\} + + \sum_k \left\{ N_{i,k}^s I_k P_w\left[V_{i,k} = 1, V_{i,j} = 0 \ \forall \, j \neq k, \ k \in N_i^s\right]\right\} \qquad (10)$$

From eqs. (6), (7) and (10) we obtain:

$$O_w(\beta_1, \beta_2, r) = = \sum_k \frac{r \cdot N_{i,k}^h I_k \exp(\beta_1 N_{i,k}^h I_k) + N_{i,k}^s I_k \exp(\beta_2 N_{i,k}^s I_k)}{\sum_j [r \cdot N_{i,j}^h I_j \exp(\beta_1 N_{i,j}^h I_j) + N_{i,j}^s I_j \exp(\beta_2 N_{i,j}^s I_j)]} \qquad (11)$$

From eq. (11), we notice that it is possible to make the problem easier by reducing the number of parameters; in fact, if we consider the parameter β_1^* that is defined as

$$\beta_1^* = \beta_1 + \ln(r) \qquad (12)$$

the output of the statistical soft dilation at location i can be defined depending on two parameter only, i.e. β_1^* and β_2:

$$O_w(\beta_1^*,\beta_2)=\sum_k \frac{\exp(\beta_1^* N_{i,k}^h I_k)+N_{i,k}^s I_k \exp(\beta_2 N_{i,k}^s I_k)}{\sum_j [\exp(\beta_1^* N_{i,j}^h I_j)+N_{i,j}^s I_j \exp(\beta_2 N_{i,j}^s I_j)]} \quad (13)$$

With similar computations, it is possible to obtain the output of the *statistical soft erosion* at location i:

$$O_l(\beta_1^*,\beta_2)=\sum_k \frac{\exp(-\beta_1^* N_{i,k}^h I_k)+N_{i,k}^s I_k \exp(-\beta_2 N_{i,k}^s I_k)}{\sum_j [\exp(-\beta_1^* N_{i,j}^h I_j)+N_{i,j}^s I_j \exp(-\beta_2 N_{i,j}^s I_j)]} \quad (14)$$

Combining the basic operation it is possible to obtain different statistical soft operators; for example it is possible to define the *statistical soft opening (closing)* by performing a statistical soft erosion (dilation) that is followed by a soft statistical dilation (erosion) in cascade.

3 EXPERIMENTAL RESULTS

In this section experimental results showing the capability of the presented operators are shown.

Experiments have been performed on a signal which represents a row of the *Lena* image (Fig.1). The signal was corrupted by different type of noise:

a. i.i.d. Gaussian noise G(0,400);
b. 20% corrupting impulsive noise;
c. structured noise; This type of noise was derived from noise G(0,400) filtered by a mathematical opening; for this filtering, two sizes of SE B were chosen: card(B)=3 and card(B)=5 (Fig.2).

The following operations were performed to evaluate and compare some filters:

a. smoothing operations through M filtering window such that card(M)=3 (*smooth 1*) and card(M)=5 (*smooth 2*);
b. median filtering through M filtering window such that card(M)=3 (*median 1*) and card(M)=5 (*median 2*);
c. statistical soft openings with $\beta_1^*=1.69$, $\beta_2=0.1$ and structuring set such that card(B)=5 and card(A)=3 (*stat. soft opening 1*) or card(B)=7 and card(A)=5 (*stat. soft opening 2*)

The evaluations have been done with SNR (*Signal to noise Ratio*) value, MSE (Mean Square Error) value and with the *shape1_error* of order γ by the masking element W that is defined by *Kuosmanen et al.* [5] in the following way:

$$sh1\_err=\frac{1}{N}\sum_i \sum_{k\in W} |X(i)-X(k)-(Y(i)-Y(k))|^\gamma$$

where N is the number of samples that have to be processed, X represents the reference signal and Y the signal that has to be evaluated. We have considered card(W)=3 or card(W)=5, and γ =2. Shape preservation is better when this parameter is smaller. The SNR and MSE values show the noise level in the output signal, whereas the *shape1_error* shows the shape difference between the uncorrupted signal and the processed signal.

Experimental results show that smoothing filter provide good noise cleaning when the signal is corrupted by Gaussian noise, whereas median filters provide good results for signal that are corrupted by impulsive noise, as expected.

These filters fail when the signal is corrupted by structured noise. Tables 1 and 2 show that the best results are obtained by applying to the corrupted signals a statistical soft opening with a structuring set such that the hard SE that contains completely the structuring mask that was used to correlate the noise (i.e. $A \subseteq M$).

Further experiments were performed on a row of a synthetic SAR image that was corrupted by a simulated 2-look speckle noise and the related results to noise cleaning effect are shown in table 3. The speckle noise is a moltiplicative noise; due to this characteristic, it is not easy to predict the expected local shape preservation properties of the various filters, and, consequently to evaluate them accordingly. Therefore, we chose to evaluate only noise cleaning properties by means of a global measure such as SRR. In this sense, from table 3 it is possible to notice that statistical soft operators provide better results then other filters.

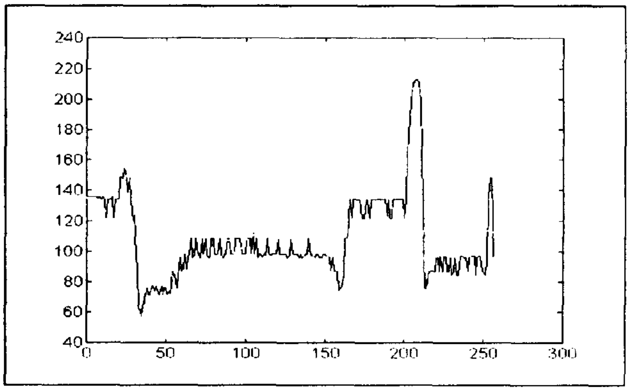

Fig.1 *Original signal (256 samples)*

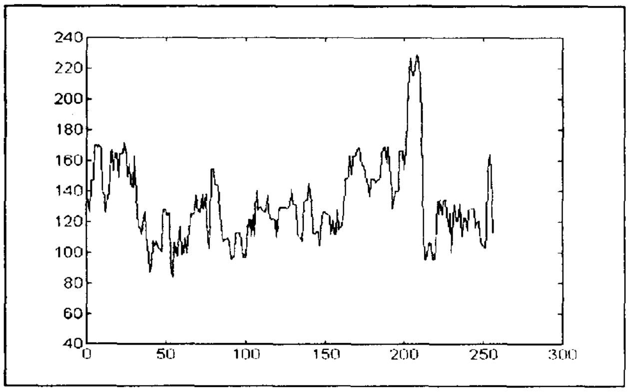

Fig.2 *Signal corrupted by an additive structured noise (i.i.d. Gaussian noise G(0,400) that is correlated by means of a mathematical opening with a structuring element B such that card(B)=5).*

4 CONCLUSIONS

Experimental results allow us to conclude that statistical soft morphological operators are able to clean the structured noise corrupting the signal preserving both the signal value (good SNR or MSE are obtained) and the signal shape (good *Shapel_error*). Results from signal that is corrupted by a speckle noise suggest that a possible application of the method could consist in the SAR image restoration.

From the performed experiments we have noticed that very different results are obtained if the parameter values are changed, so it would be possible to obtain good results even applying statistical soft operators on a signal that is corrupted by Gaussian or impulsive noise.

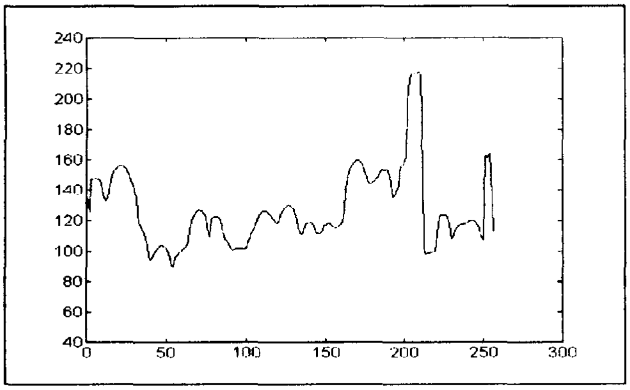

Fig.3 *Signal resulting from applying a statistical soft opening* ($\beta_1^* = 1.69$, $\beta_2 = 0.1$, *structuring set such that card(B)=7, card(A)=5) to the signal of fig.2.*

	SNR [dB]	MSE	*shl_err W=3*	*shl_err W=5*
Noisy signal	13.66	537.25	3380	9325
Stat. soft opening 1	16.37	288.23	3728	8398
Stat. soft opening 2	**16.59**	**273.79**	**2994**	**5338**
Smooth 1	14.12	484.21	3475	8921
Smooth 2	14.34	459.72	3345	7873
Median 1	13.72	530.03	3272	9286
Median 2	13.75	527.18	3000	9011

Table 1 *Results obtained from a signal corrupted by structured noise with a M correlating mask such that card(M)=3. Bold characters represent the best results.*

	SNR [dB]	MSE	*shl_err W=3*	*shl_err W=5*
Noisy signal	12.24	745.08	4003	9325
Stat soft opening 1	13.23	593.18	3578	8398
Stat. soft opening 2	**13.75**	**526.28**	**3160**	**5338**
Smooth 1	12.32	732.19	3507	8921
Smooth 2	12.34	728.95	3240	7873
Median 1	12.19	754.87	3971	9286
Median 2	12.28	738.57	3732	9011

Table 2 *Results obtained from a signal that is corrupted by structured noise with a M correlating mask such that card(M)=5. Bold characters represent the best results.*

	SNR [dB]	MSE
Noisy signal	8.23	1842.5
Stat soft opening 1	12.94	730.87
Stat. soft opening 2	**13.76**	**605.70**
Smooth 1	10.41	1310
Smooth 2	10.86	1181.8
Median 1	10.17	1382.5
Median 2	10.69	1227.6

Table 3 *Results obtained from a signal that is corrupted by 2-look speckle noise. Bold characters represent the best results.*

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