VOLTERRA SERIES BASED MODELING AND COMPENSATION OF NONLINEARTIES IN HIGH POWER AMPLIFIERS

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ABSTRACT

Demands for higher data rates coupled with increased competition for the available RF bandwidth demand communications systems with greater bandwidth efficiency. Bandwidth efficient modulation techniques, such as QAM, require highly linear amplifier performance to achieve acceptable bit error rates. The nonlinear distortion which results when an amplifier is operated near saturation may preclude their use. One attractive option is to predistort the signal by placing a nonlinear filter in the signal path which compensates for the distortion introduced by the amplifier. In this paper, we present a predistortion technique which uses an inverse amplifier model based on a Volterra series approach. The input-output data from the amplifier is used to develop the parameters for a discrete Volterra series. The RLS adaptive filter technique is utilized to provide periodic updates to the inverse filter which allows tracking of amplifier variations. Results are presented for the case of 64 QAM modulation using a TWT amplifier operated at saturation.

1. INTRODUCTION

This paper introduces an adaptive predistortion technique which is based on Volterra series modeling of the RF amplifier to compensate for nonlinearties in High Power Amplifiers (HPA). The technique allows bandwidth efficient modulation schemes to be used with nonlinear HPAs with small degradation in system BER performance relative to a purely linear transmitter system. In addition, the technique adapts to changes in the nonlinearity of the HPA which could be caused by changes to the input power, carrier frequency, aging, temperature, or component replacement.

Several other techniques have been applied to this problem, the most obvious being to use a linear class A HPA or to operate a nonlinear HPA sufficiently far from saturation to obtain near linear performance. These techniques tend to require larger, heavier, more expensive, and less efficient HPAs than necessary. Other predistortion techniques include analog feedback [1], analog feed-forward [2], and digital table-lookup mapping [3]. The analog techniques have been limited by narrow operating bandwidths, extreme sensitivity to HPA variations, instabilities, and in some techniques the requirement for additional RF amplifiers. The digital mapping techniques have been limited by the massive amount of storage required for a sufficiently accurate mapping to be stored. In addition digital techniques based on neural networks [4] have developed.

In this paper, we present a predistortion technique which uses an inverse amplifier model based on a Volterra series analysis. The input-output data from the amplifier is used to develop the parameters for a discrete Volterra series. The RLS adaptive filter technique is utilized to provide periodic updates to the inverse filter which allows tracking of amplifier variations. Section 2 describes the signal and amplifier models. In Section 3, the application of the Volterra series to the amplifier modeling problem is presented. Section 4 develops an adaptive technique to allow the algorithm to track changes in HPA response. Section 5 describes the results of simulations using a 64 QAM modulation scheme, and Section 6 presents conclusions.

2. SIGNAL AND AMPLIFIER MODEL

A digital communication RF signal can be expressed as

$$x(t) = r(t)\cos[\omega_0 t + \Psi(t)] \tag{1}$$

where ω_0 is the carrier frequency, $r(t)$ is the modulated envelope, and $\Psi(t)$ is the modulated phase. In this paper, we will consider a commonly used model for the nonlinear distortion created by an RF amplifier. This model has been shown to be quite accurate for many types of amplifiers [5]. Using this model, the output of the HPA is described by

$$z'(t) = A[r(t)]\cos\{\omega_0 t + \Psi(t) + \Phi[(r(t))]\} \tag{2}$$

where $A(r)$ describes the AM-AM distortion, and $\Phi(r)$ describes the AM-PM distortion created by the amplifier. This model assumes that the amplifier is memoryless and that its response can be completely described by these two functions. For a TWT amplifier in particular, the two distortion functions can be modeled as [5]

$$\begin{aligned} A(r) &= \frac{\alpha_a r}{1+\beta_a r^2} \\ \Phi(r) &= \frac{\alpha_\Phi r^2}{1+\beta_\Phi r^2} \end{aligned} \tag{3}$$

In this paper, we will use a normalized amplifier (i.e. saturation point is 0 dBm) with coefficients: $\alpha_a = 1.96$, $\beta_a = 0.99$, $\alpha_\Phi = 2.53$, $\beta_\Phi = 2.82$. Figure 1 shows the distortion of a 64-QAM symbol constellation upon application of a typical TWT amplifier .

The large QAM constellations which provide good bandwidth efficiency are most affected by amplifier nonlinearities. The nonlinear amplifier causes: a) clustering phenomena from ISI, b) warping of the symbol constellation, and c) spectral spreading, which can result in an unusable system performance.

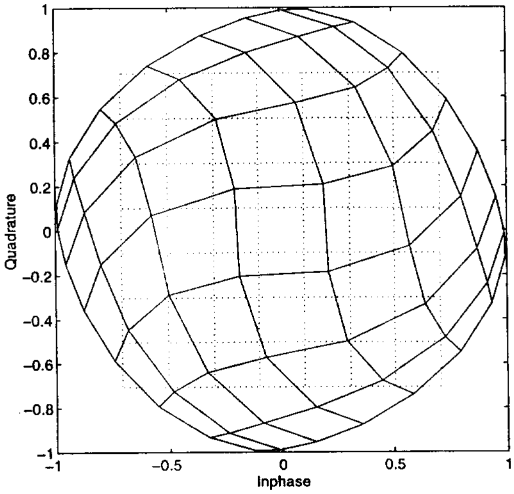

Figure 1 Effects of Nonlinear Amplification

3. VOLTERRA SERIES

The Volterra series is a generalization of the Taylor series and has been used frequently for the modeling of nonlinear functions [6]. The basic form of the series is:

$$y(t) = \sum_{n=0}^{\infty} H_n[x(t)] \tag{4}$$

where

$$H_n[x(t)] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \ldots, \tau_n) x(t-\tau_1) x(t-\tau_2) \ldots x(t-\tau_n) d\tau_1 d\tau_2 \ldots d\tau_n \tag{5}$$

The series may also be expressed in discrete form as

$$y(n) = \sum_{n=0}^{\infty} H_n[x(n)] \tag{6}$$

where

$$H_n[x(n)] = \sum_{i_1=-\infty}^{\infty} \cdots \sum_{i_n=-\infty}^{\infty} h^{(n)}_{i_1,i_2,\ldots,i_n} x(n-i_1) \ldots x(n-i_n) \tag{7}$$

In practical applications, the Volterra series must be simplified to avoid the summations over an infinite number of terms. A sufficiently accurate model can be obtained for many applications by using a finite number of terms and less than infinite memory. The resulting model may be classified by the number of terms included in the series (P), which characterizes the order of the nonlinear model, and by the amount of memory used when calculating each term of the series. In the case of modeling an HPA, the Volterra series may be solved by placing the equations into matrix form

$$\mathbf{y} = \mathbf{X}\mathbf{h} \tag{8}$$

where $\mathbf{y}$ consists of the HPA output data, $\mathbf{h}$ contains the coefficients used to construct the Volterra kernel , and $\mathbf{X}$ is a data matrix constructed from the input to the HPA. The optimal coefficients in the least mean square sense may then be found by

$$\mathbf{h} = \mathbf{X}^{+}\mathbf{y} \tag{9}$$

where $\mathbf{X}^{+}$ is the pseudo-inverse of the data matrix $\mathbf{X}$.

For the Volterra series based predistorter, a set of coefficients which describe the inverse of the HPA is desired. In this case the matrix version of the Volterra equations becomes

$$\mathbf{x} = \mathbf{Y}\mathbf{g} \tag{10}$$

where $\mathbf{x}$ is the amplifier input, $\mathbf{g}$ is the coefficients of the Volterra inverse HPA model, and $\mathbf{Y}$ is a data matrix constructed from the amplifier output. The optimal set of coefficients, $\mathbf{g}$, may then be found by

$$\mathbf{g} = \mathbf{Y}^{+}\mathbf{x} \tag{11}$$

Here, we refer to the technique of finding the parameters in this fashion as the fixed data method.

The computational cost of determining the Volterra coefficients may be reduced by limiting the size of the data matrix. This can be accomplished by using smaller nonlinearity order (fewer terms in the series) and by using less memory. In addition, the bandpass nature of the system under consideration results in all of the even terms of the Volterra series being identically zero. These terms may therefore be ignored, and the size of the data matrix considerably reduced.

4. ADAPTIVE TECHNIQUES

The characteristics of an HPA are expected to vary due to various factors including: aging of components, temperature, changes to the input power, carrier frequency and component replacement. To design a predistorter that can track such changes we now consider adaptive techniques based upon the Recursive Least Squares algorithm [7]. The method attempts to minimize the error between the estimated signal output and the actual signal output. For the Volterra series predistorter, the error can be represented as

$$e(n) = x(n) - \hat{x}(n) = x(n) - \mathbf{g}^{H}(n)\mathbf{y}(n)$$

After calculating the error, the Kalman gain vector is determined by

$$K(n) = \frac{P(n-1)\mathbf{y}^{*}(n)}{\lambda + \mathbf{y}^{H}(n)P(n-1)\mathbf{y}^{*}(n)}$$

Then the inverse correlation matrix is updated

$$P(n) = \frac{1}{\lambda}[P(n-1) - K(n)\mathbf{y}^{H}(n)P(n-1)]$$

and lastly the Volterra series coefficients for the amplifier inverse model are updated by

$$\mathbf{g}(n) = \mathbf{g}(n-1) + K(n)e(n)$$

5. SIMULATION RESULTS

The algorithms developed in this paper were tested using computer simulations. The response of the HPA was simulated by using the model develop by Saleh [5]. The results are presented in two subsections; the first describing the fixed data Volterra series technique developed in Section 3, and the second describing the adaptive techniques developed in Section 4.

5.1. Fixed Data Predistorter

The computational complexity of these techniques varies strongly with the size of the matrices required to be manipulated, so we first present the relationship between performance and the order of nonlinearity and amount of memory utilized. Table 1 illustrates how the number of parameters required to be determined varies with the number of memory delays for nonlinearity orders of $P = 5$, and $P = 7$. The number of terms required increases rapidly with larger amounts of memory in the model.

System order	Delays	# of terms
5	1	18
5	2	73
5	3	224
5	4	565
7	1	38
7	2	223
7	3	924

The dependence of performance upon memory is presented in Figure 2 for a constant nonlinearity order of $P = 7$. The MSE represents the difference between the original 64 QAM symbol constellation and the result of passing the signal through the predistorter and amplifier in series. The performance is relatively independent of memory size for this application. This is not surprising after considering that the underlying transmitter model consists of a memoryless nonlinearity.

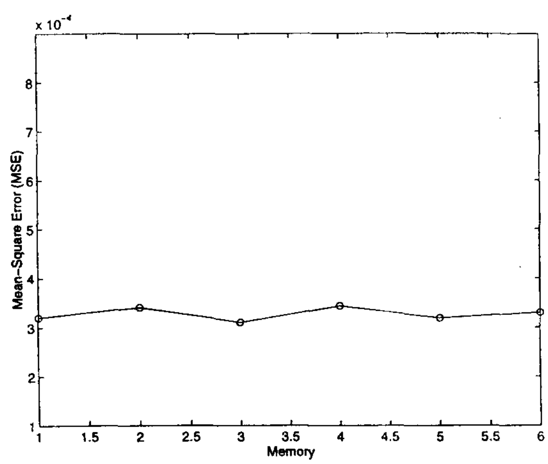

Figure 2 System Performance versus Memory Size

Next we examine the relationship between system performance and the order of nonlinearity utilized in the model. Figure 3 presents results for a constant memory $Q = 2$. The performance of the technique improves with increasing nonlinearity order until reaching a minimum when $P \cong 7$. These results indicate that a desirable operating point for this amplifier lies at $P = 7$, and $Q = 1$. These values will be used throughout the remainder of the simulation results.

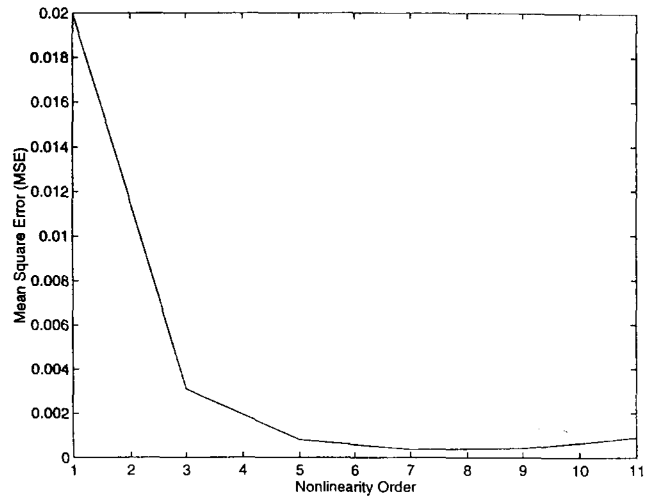

Figure 3 System Performance versus Nonlinearity Order

Figure 4 displays the results of passing the original 64 QAM symbol constellation through the predistorter and Figure 5 presents the symbol constellation after passing through both the predistorter (7th order) and the HPA. The predistorter corrects for most of the distortion caused by the amplifier and yields a $MSE = 2.4 \times 10^{-4}$. Figure 6 shows the corrected symbol constellation when an adaptive predistorter is used.

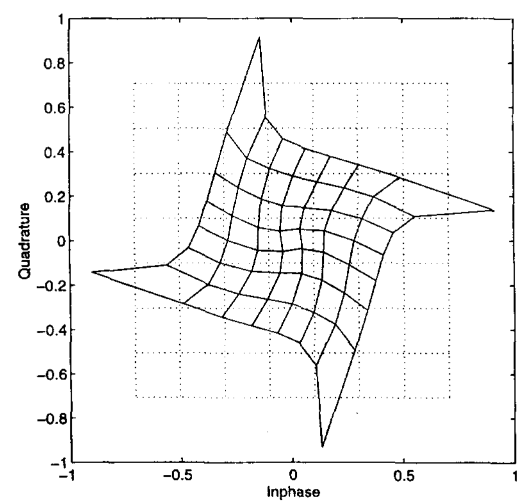

Figure 4 Predistorted Signal Constellation; 7th Order Fixed Data Predistorter.

6. COMMUNICATIONS SYSTEM SIMULATION

The fixed data and adaptive predistorters were included in the baseband section of a 64-QAM communications system as table look up functions. The probability of symbol error, P_s, versus the singal-to-noise ratio (SNR) is plotted for both types of predistorters. Figure 7 compares the performance of the QAM-64 system (with predistorter) for various orders of nonlinearity againt the ideal (no nonlinearities present) QAM-64 system (the solid line with asterisks). The various cases compared are 5th and 7th order fixed data predistorters, a 7th order adaptive predistorter, and a 7th order fixed predistorter with limited Volterra coefficients. In obtaining each plot in Figure 7, 100,000 symbol points are transmitted through the system. The 7th order fixed predistorters (both full and limited coefficient case) provided the best performance; there is very little performance difference between the full and the limited coefficient solutions. Both 7th order predistorters required 2 dB more power than ideal 64-QAM system to achieve a P_s of 10^{-5}. The adaptive predistorter required 2.5 dB more than the fixed predistorter of the same order, but 2.5 dB less power than the 5th order fixed case.

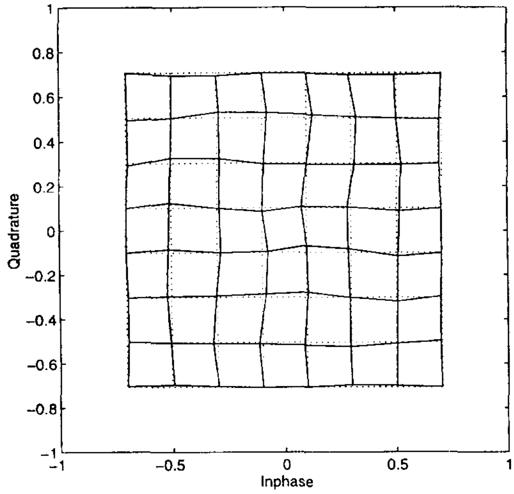

Figure 5 Corrected Signal Constellation; 7th Order Fixed Data Predistorter.

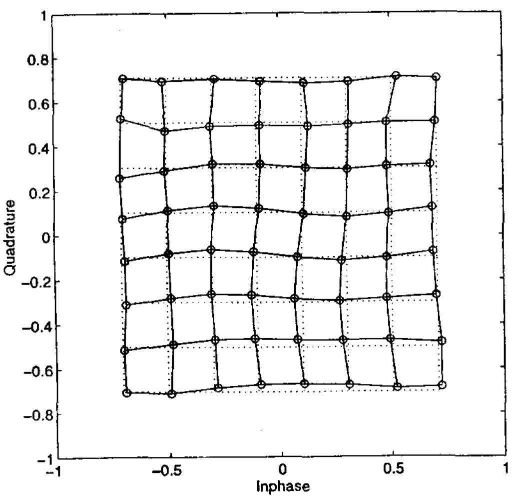

Figure 6 Corrected Signal Constellation; 7th Order Data Adaptive Predistorter.

7. CONCLUSIONS

In this paper we have examined the use of the discrete Volterra series to model the inverse of an HPA. This model formed the basis of a nonlinear filter used to predistort the baseband signal in order to linearize the transmitter system. The results above show that the discrete Volterra series approach can generate a satisfactory model for the inverse of a TWT HPA response. Furthermore, significant linearization of the transmitter system is achieved when this nonlinear filter is inserted in the signal path. This allows large, bandwidth efficient modulation schemes to be successfully employed despite the nonlinear distortion which results from an HPA operated near saturation. Use of these techniques have significant potential to allow greatly increased data rates utilizing existing equipment and bandwidth allowances. In addition, the adaptive techniques developed from the RLS algorithm allow this technique to track amplifier performance through changing conditions.

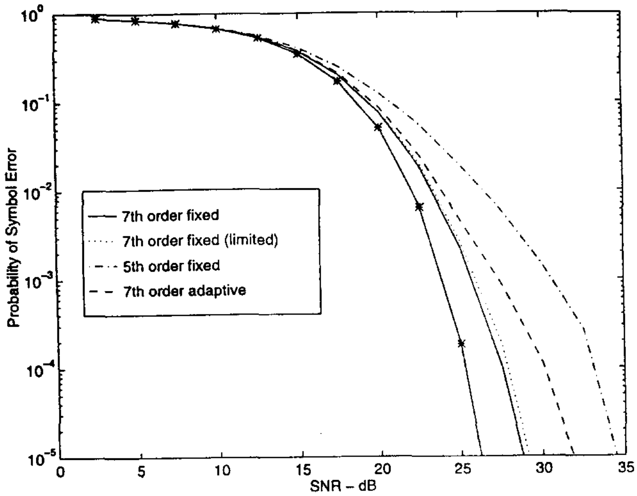

Figure 7 Probability of Symbol Error Versus Signal-to-Noise Ratio (SNR).

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