ADAPTIVE DFE ALGORITHMS FOR IS-136 BASED TDMA CELLULAR PHONES

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ABSTRACT

Optimum detection in randomly time-varying channels requires an efficient adaptive receiver structure. The complexity of signal processing in the receiver is limited by the amount of processing power and power consumption of the receiver. Therefore, efficiency and convergence of adaptive matched filtering and equalization techniques are very important. We extend common results in tracking theory for system identification to the equalization case for systems with short impulse response. Unlike in system identification where a steady-state error energy is minimized, the optimization criterion here is the minimum of the BER. However, they are related by a monotone function and therefore minimizing the BER is equivalent to minimizing the steady-state-error energy. Optimum parameters for LMS as well as RLS algorithms are derived and simulation results indicate that under the conditions defined in the TDMA standards and small delay spread, the performance of the two methods is comparable.

1. INTRODUCTION

Burst transmission of data through frequency and time selective fading channels requires fast and efficient adaptive signal processing techniques. The receiver should be able to detect the data with relatively low signal to noise ratio and in the presence of Rayleigh distributed multiplicative disturbances and fast variation of the channel impulse response due to the Doppler effect. Although it is known how a communication system with additive noise performs for an ideal equalizer, the tracking effects of the equalizer itself with a randomly time-varying channel have not been investigated. Usually, slow fading channels are assumed so that the effect of the equalizer is much smaller than the BER caused from the Rayleigh channel. The IS-136 standards however, require the 3% BER also for Doppler speeds up to 100Km/h. In this case the tracking noise of the equalizer becomes much larger than the error caused by the Rayleigh channel. Hence, any analysis of the performance of such a communication system must take the equalizer tracking noise into account. Note that tracking analyzes for LMS and RLS algorithms in the context of system identification can be found in literature (see[2,3]). In this paper we show

1. How to extend the tracking theory from [2,3] to the equalization case.

2. How to compute the steady-state-error energy and

3. minimize this energy with respect to the free parameters.

4. How to map the steady-state-energy to the BER.

2. THE DFE REFERENCE MODEL

In order to treat the equalization problem like an identification problem, we assume that there exists an optimal equalizer (model reference) with the structure depicted in Figure 1. The reference model consists of a linear filter that performs an equalization of the channel. Since a general channel can be perfectly equalized only by a filter of (double) infinite length, a finite (and general small) filter order can only achieve a rough equalization. The nonlinear decision device (slicer) following the equalizer guarantees that the outcome of the reference model equals the transmitted signal $s(k - D)$, where we allow for a delay of D samples. The difference of the reference model output $s(k - D)$ and the estimate $\hat{z}(k)$ leads to the error $e(k)$ that is used for updating the estimates $\hat{w}_k$. A final decision device delivers estimates $\hat{s}(k - D)$ of the transmitted sequence. In the following the delay D will be dropped for convenience. The outcome of the linear part is denoted $z(k) = u_k w_k + s_{k-1} a_k$ where the sampled receiver values have been combined in a row vector u_k and the filter taps in a column vector w_k for the feedforward and a_k for the feedback part. In a com-
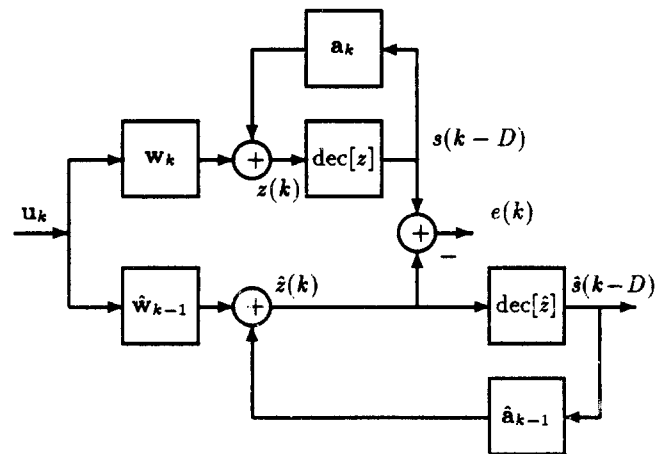

Figure 1: *Model reference structure for DFE equalizer.*

panion paper [4] we gave six assumptions on input statistics and channel characteristics in order to pursue the analysis.

These assumptions will be listed briefly without further explanation. The reader is referred to [4].

1. **Input Statistics**
   The white symbol sequence $s(k)$ is linearly filtered by the channel C and thus the received sequence $u(k) = C[s(k)] + v(k)$ is expected to be correlated. We assume that the received sequence has nearly Gaussian statistics. We also assume the additive noise $v(k)$ to be a Gaussian random process.

2. **Approximation**
   The output error is given by

   $$e(k) \approx e_a(k) + g[z(k)]. \qquad (1)$$

   where the *a-priori* error

   $$e_a(k) \triangleq z(k) - \hat{z}(k) = \mathbf{u}_k[\mathbf{w}_{k-1} - \hat{\mathbf{w}}_{k-1}] = \mathbf{u}_k\tilde{\mathbf{w}}_{k-1}, \qquad (2)$$

   and the error function describing the equalizer imperfection is defined as

   $$g[z(k)] \triangleq \text{dec}[z(k)] - z(k). \qquad (3)$$

3. **Equalizer Imperfection**
   When (1) is compared to a linear system identification scheme, the term $g[z]$ plays the role of additive noise. Note that $g[z]$ can also be expressed as

   $$g[z] = (1 - WC - A)[s(k)] - W[v(k)], \qquad (4)$$

   where W is an operator that describes the feedforward part of the equalizer coefficients. The first term in (4) describes the equalization effect. If the equalization is perfect, it is close to zero. This is rarely the case, however, and we therefore have to deal with equalizer imperfection.

4. **Channel Model**
   An approximate model for each path of the channel is an AR process with autocorrelation function

   $$R_n(\tau) = J_0(2\pi f \tau) \qquad (5)$$

   In order to simplify matters we assume a channel model with a simple one order process describing its dynamics and combine the driving terms into one new white noise term.

   $$\mathbf{c}_k = f\,\mathbf{c}_{k-1} + \sqrt{1 - f^2}\,\mathbf{q}_k \qquad (6)$$
   $$u(k) = \mathbf{s}_k \mathbf{c}_k + v(k) \qquad (7)$$

   where $\mathbf{q}_k$ is a white complex-valued Gaussian vector random process of unit variance, i.e., $E[\|\mathbf{q}_k\|_2^2] = 1$ for a one path model or twice that amount for a two path model. The row vector $\mathbf{s}_k$ consists of the transmitted symbol sequence $s(k)$.

5. **Reference Model Fluctuations** We assume that the dynamic of the reference model equalizer behaves like the channel, i.e., it follows the same model (6):

   $$\mathbf{w}_k = f\,\mathbf{w}_{k-1} + \sqrt{1 - f^2}\,\mathbf{q}_k. \qquad (8)$$

   This is true in particular when only one coefficient f is used with values close to one (as it is the case here). The variance of the fluctuations becomes

   $$\Delta_w = E[\|\mathbf{w}_k - \mathbf{w}_{k-1}\|^2] = 2(1 - f)E[\|\mathbf{q}_k\|^2]. \qquad (9)$$

6. **Correlations**
   In order to continue we assume that all processes $u(k), g[z(k)]$, and $\mathbf{q}_k$ are mutually independent. Note that if the equalizer imperfection is caused mainly by the linear part $(1 - WC - A)$, it will be correlated with the received sequence $u(k)$, and this assumption does not hold. We thus consider two cases: one for which the linear part is small but uncorrelated to the signal, the other where it is part of the signal (see (30) ahead). We will later give the results in form of bounds between these two cases.

3. TRACKING THEORY
3.1. Tracking of the LMS Algorithm
We first write the LMS update equation

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mu(k)\mathbf{u}_k^* e(k) \qquad (10)$$

in terms of $\tilde{\mathbf{w}}_k = \mathbf{w}_k - \hat{\mathbf{w}}_k$ as

$$\tilde{\mathbf{w}}_k = \Delta\mathbf{w}_k + \tilde{\mathbf{w}}_{k-1} - \mu(k)\mathbf{u}_k^* e(k), \qquad (11)$$

with $\Delta\mathbf{w}_k = \mathbf{w}_k - \mathbf{w}_{k-1}$. As long as no error occurs we simply have $e(k) = \mathbf{u}_k\tilde{\mathbf{w}}_{k-1} + g[z(k)]$. We replace the step-size $\mu(k)$ by the projection step-size $\alpha\bar{\mu}(k)$ with

$$\bar{\mu}(k) = 1/\|\mathbf{u}_k\|_2^2$$

and calculate the quadratic l_2- norm of the parameter error vector

$$
\begin{aligned}
\|\tilde{\mathbf{w}}_k\|_2^2 &= \|\Delta\mathbf{w}_k\|_2^2 + \|\tilde{\mathbf{w}}_{k-1}\|_2^2 + \alpha^2\bar{\mu}(k)|e(k)|^2 \\
&+ 2\,\text{Re}\{\tilde{\mathbf{w}}_{k-1}^*\Delta\mathbf{w}_k - \alpha e^*(k)\bar{\mu}(k)\mathbf{u}_k\Delta\mathbf{w}_k \\
&\quad -\alpha e^*(k)\bar{\mu}(k)\mathbf{u}_k\tilde{\mathbf{w}}_{k-1}\}.
\end{aligned}
\qquad (12)
$$

We take the expectation of both sides and use the following terms:

$$E[\bar{\mu}(k)|\mathbf{u}_k\tilde{\mathbf{w}}_{k-1}|^2] = \gamma\|\tilde{\mathbf{w}}_{k-1}\|_2^2 \qquad (13)$$
$$E[\bar{\mu}(k)e^*(k)\mathbf{u}_k\Delta\mathbf{w}_k^*] = \gamma E[\tilde{\mathbf{w}}_{k-1}\Delta\mathbf{w}_k^*] \qquad (14)$$
$$E[\text{Re}\{\tilde{\mathbf{w}}_{k-1}\Delta\mathbf{w}_k\}] = \frac{-(1-f)^2}{1-f(1-\alpha\gamma)}E[\|\mathbf{q}_k\|^2],$$

with γ between zero and one depending on the correlation of the received sequence. The derivation for the last line is somewhat lengthy and therefore not given here.

With these assumptions, (12) now reads

$$
\begin{aligned}
E[\|\tilde{\mathbf{w}}_k\|_2^2] &= \Delta_w + (2\alpha\gamma)E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2] \\
&+ \alpha^2\left[\gamma E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2] + E[\bar{\mu}(k)]\sigma_g^2\right] \\
&- 2\frac{(1-\alpha\gamma)(1-f)^2}{1-f(1-\alpha\gamma)}E[\|\mathbf{q}_k\|^2] \\
&= \Delta_w + E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2][1 + \gamma\alpha(\alpha-2)] \\
&- 2\frac{(1-\alpha\gamma)(1-f)^2}{1-f(1-\alpha\gamma)}E[\|\mathbf{q}_k\|^2] \\
&+ \alpha^2\sigma_g^2 E[\bar{\mu}(k)].
\end{aligned}
\qquad (15)
$$

In steady-state, $E[\|\tilde{\mathbf{w}}_k\|_2^2] = E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2]$ and (15) becomes

$$E[\|\tilde{\mathbf{w}}_k\|_2^2] = \frac{\Delta_w + \alpha^2\sigma_g^2 E[\bar{\mu}(k)] - 2\frac{(1-\alpha\gamma)(1-f)^2}{1-f(1-\alpha\gamma)}E[\|\mathbf{q}_k\|^2]}{\gamma\alpha(2-\alpha)}. \qquad (16)$$

Substituting for Δ_w from (9), we finally get

$$E[\|\tilde{w}_k\|_2^2] = \frac{\Delta_w \frac{\alpha\gamma}{1-f(1-\alpha\gamma)} + \alpha^2\sigma_g^2 E[\bar{\mu}(k)]}{\gamma\alpha(2-\alpha)} . \quad (17)$$

For the update error $e(k)$ we use the relation

$$E[|e(k)|^2] = \gamma M \sigma_u^2 E[\|\tilde{w}_{k-1}\|_2^2] + \sigma_g^2 ,$$

and obtain for $\sigma_u^2 = 1$

$$\lim_{k\to\infty} E[|e(k)|^2] = \frac{\gamma M \Delta_w}{(1-f(1-\alpha\gamma))(2-\alpha)} \quad (18)$$
$$+ \frac{2-\alpha(1-ME[\bar{\mu}(k)])}{2-\alpha}\sigma_g^2 .$$

If we compare this result with the one from [1] we note that in the first term $1/\alpha$ is replaced by $1/[1-f+f\alpha\gamma]$. In fact, as $\alpha \to 0$, the steady-state error energy will not grow beyond all limits but will remain bounded by the equalizer fluctuations Δ_w. Thus, (18) is a more accurate description.

Let us consider the flat Rayleigh fading channel for which the receiver sequence $u(k)$ can be assumed to be a white random process with $\gamma = 1/M$. If we further recall that the compound noise $\sigma_g^2 = E[|W[v(k)] + e_e(k)|^2] = E[|w_k(0)|^2]\sigma_v^2 + \sigma_e^2$, and that $\Delta_w = 2(1-f)E[|w_k(0)|^2]$, we find the final expression for flat Rayleigh fading

$$\lim_{k\to\infty} E[|e(k)|^2] = \frac{\Delta_w}{(1-f+\frac{\alpha f}{M})(2-\alpha)} \quad (19)$$
$$+ \frac{2-\alpha(1-ME[\bar{\mu}])}{2-\alpha}\left[\sigma_v^2 E[|w_k(0)|^2] + \sigma_e^2\right] .$$

Thus, it is possible to compute the optimal step-size for minimal steady-state error. We obtain a quadratic equation in the step-size α

$$\alpha^2 + 2\alpha A + B = 0 , \quad (20)$$

with the terms

$$A = \left[\frac{M(1-f)}{f}\right](1+C)$$

$$B = \left[\frac{M(1-f)}{f}\right](A-2C)$$

$$C = \frac{|w_k(0)|^2}{ME[\bar{\mu}(k)][|w_k(0)|^2\sigma_v^2 + \sigma_e^2]} .$$

The optimal solution is then given by

$$\alpha_{opt} = -A + \sqrt{A^2 - B} . \quad (21)$$

3.2. Tracking of the RLS Algorithm

The same method that has been applied for the LMS algorithm can also be applied for the RLS algorithm. The following terms can be computed beforehand:

$$E[P_k^{-1}] = \lambda E[P_{k-1}^{-1}] + R_{uu} = \frac{R_{uu}}{1-\lambda} , \quad (22)$$

$$E[\bar{\mu}(k)] = E\left[\frac{1}{u_k P_k u_k^*}\right] = \frac{1}{M}\frac{1}{1-\lambda} , \quad (23)$$

where for both we assumed steady-state, i.e, the initial terms $P_{-1} = \delta I$ has deceased. The RLS update equation reads in terms of the parameter error

$$\tilde{w}_k = \tilde{w}_{k-1} + \Delta w_k - P_k u_k^* e(k) . \quad (24)$$

Different to the LMS algorithm we now consider $E[\tilde{w}_k^* P_k^{-1} \tilde{w}_k]$ and following the same procedure we obtain

$$\tilde{w}_k^* P_k^{-1} \tilde{w}_k = \tilde{w}_{k-1}^* P_k^{-1} \tilde{w}_{k-1} + \Delta w_k^* P_k^{-1} \Delta w_k$$
$$+ (\bar{\mu}(k))^{-1}|e(k)|^2 + 2\text{Re}\{\tilde{w}_{k-1}^* P_k^{-1} \Delta w_k\}$$
$$- 2\text{Re}\{e^*(k)\bar{\mu}(k)u_k(\Delta w_k + \tilde{w}_{k-1})\} . \quad (25)$$

We are now faced with one mayor difference compared to the LMS algorithm. Because of the inner matrix P_k^{-1} in the middle of the expressions, the time variations effect the terms in a different manner. In other words we cannot expect to compute $E[\tilde{w}_{k-1}^* P_k^{-1} \Delta w_k]$ as easily as before. If we, however, focus on the flat Rayleigh fading case, the acf matrix $R_{uu} = I$ and the expressions become very similar. We only have to translate the forgetting factor λ into an equivalent step-size α by the formula

$$\alpha = (1-\lambda)M \quad (26)$$

and we obtain

$$\lim_{k\to\infty} E[|e(k)|^2] = \frac{\Delta_w}{(1-f+\alpha\frac{f}{M})(2-\alpha)} + \frac{2}{2-\alpha}\sigma_g^2 . \quad (27)$$

The compound noise $\sigma_g^2 = \|W\|^2\sigma_v^2 + \|1-WC-A\|^2$ or for a flat Rayleigh channel even simpler, $\sigma_g^2 = E[\sigma_v^2|c_k(0)|^2/[\sigma_v^2 + |c_k(0)|^2]^2] + E[\sigma_v^4/[\sigma_v^2 + \|c_k(0)\|^2]^2]$.

4. COMPUTING THE BER

Although the steady-state-error energy is a good measure for the tracking performance of an equalizer, one is more interested in the final BER for such systems. We therefore need to relate the steady-state error energy to the BER. To this end, we first note that the outcome of the linear equalizer filter $\hat{z}(k)$ is an estimate of $s(k)$, i.e.,

$$\hat{z}(k) = s(k) - e(k) . \quad (28)$$

In the steady-state situation the error term $e(k)$ plays the role of additive noise. Using (1) and (28) we have

$$\hat{s}(k) = \text{dec}[\hat{z}(k)] = \text{dec}[s(k) - e(k)] \quad (29)$$
$$\approx \text{dec}[s(k) - g[z(k)] - e_a(k)] . \quad (30)$$

For a Rayleigh fading channel and additive Gaussian noise the BER for the differentially encoded case is given by [5,6]

$$\text{BER} = \frac{1}{2+\text{SNR}} . \quad (31)$$

For the LMS algorithm the SNR is given by

$$\text{SNR}_{LMS}^{-1} = \frac{2(1-f)}{(1-f+\frac{\alpha f}{M})(2-\alpha)} - \frac{\sigma_v^2}{E[|w_k(0)|^2]} \quad (32)$$
$$+ \frac{2-\alpha(1-ME[\bar{\mu}])}{2-\alpha}\left[\sigma_v^2 + \frac{\sigma_e^2}{E[|w_k(0)|^2]}\right] .$$

The optimal step-size for minimum BER can be obtained by differentiating (31) with respect to α. The result for RLS is similar and can be obtained by substituting $2 - \alpha(1 - ME[\bar{\mu}])$ by 2 and $\lambda = 1 - \alpha/M$ in the above expression.

5. SIMULATION RESULTS

We are now able to determine optimal step-sizes for minimizing the steady-sate error. Figure 2 depicts the MSE $E[|e(k)|^2]$ as a function of the step-size α for flat fading Rayleigh channel and speeds of 8,100 and 237Km/h, when using a T-spaced transversal equalizer of order $M = 3$. The optimal step-sizes for an SNR=19.2dB and these speeds are $\alpha_{opt} = 0.02, 0.21$ and 0.42, respectively. Also the MSE is relatively flat around the optimal value so that hitting precisely the optimal value is not a very critical issue. Com-
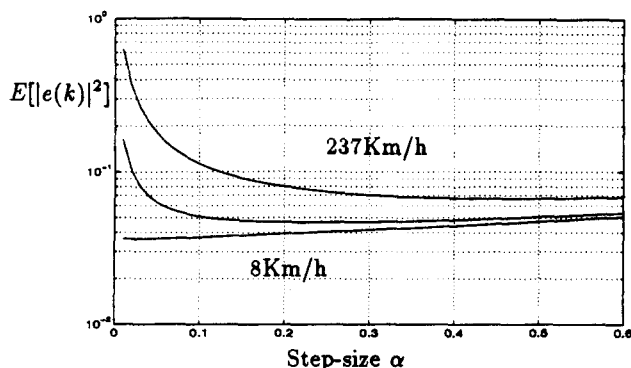
Figure 2: *Optimal step-size for DFE-LMS algorithm.*

parisons of these theoretical values with simulation results and optimal step-sizes for minimizing the BER will be discussed in a section further ahead together with the DFE-RLS algorithm.

Similar to the optimal step-sizes we are also able to determine optimal forgetting factors λ for the RLS algorithm. Figure 3 depicts the MSE as a function of the forgetting factor λ for Rayleigh fading channel with small delay spread and speeds of 8, 100 and 237Km/h. The optimal forgetting factors for an SNR =19.2dB and these speeds are $\lambda_{opt} = 0.99, 0.95$ and 0.9, respectively. Also the MSE is relatively flat around the optimal value so that hitting precisely the optimal value is not a very critical issue.
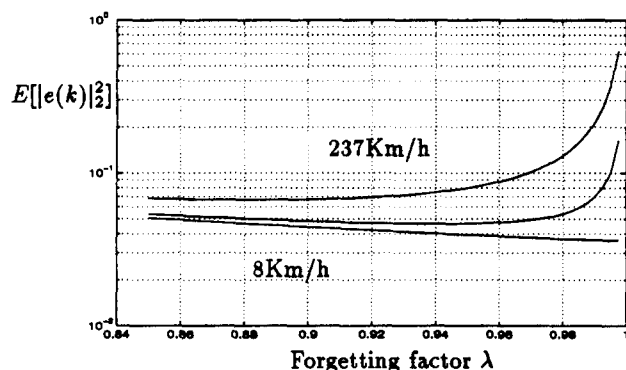
Figure 3: *Optimal forgetting factor for DFE-RLS algorithm.*

Figure 4 compares simulation and theoretical result over a wide range of SNR when a DFE equalizer structure is deployed. The theoretical results can readily be extended to this equalizer structure. The experiment includes impulse shaping by a square-root raised cosine filter. We compared a pure LMS, a pure RLS solution (upper and lower continuous lines) and a hybrid LMS/RLS solution (dashed line). As

the figure shows their difference is very small and very well described by our theoretical bound (upper dotted line). The value denoted by 'x' in the figure defines the required BER according to the standards. The algorithm will be used in a hand-phone that is currently under development.
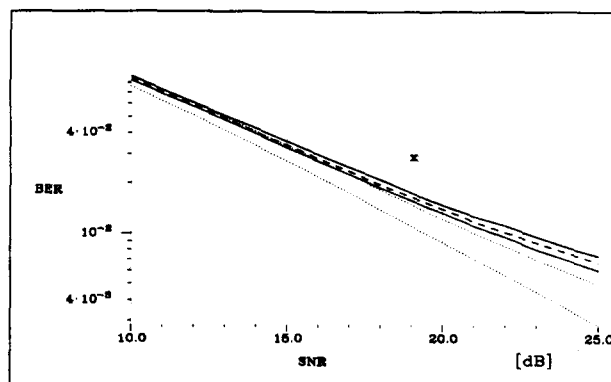
Figure 4: *BER for flat fading channel at speed=100Km/h. DFE-RLS and DFE-LMS: continuous lines. DFE-RLS/LMS: dashed line, optimal equalizer: lower dotted line, theoretical equalizer bound: upper dotted line.*

6. CONCLUSIONS

Our theoretical investigations as well as our simulations indicate that the LMS and the RLS algorithms exhibit similar performance in their tracking behavior when used under IS-136 conditions. Both meet the 3% BER bound as required by standard for Doppler speeds up to 100Km/h and small delay spread. Hence the LMS algorithm with much lower complexity can be applied for low delay spread environment. Finally, in order to meet higher Doppler effects such as that of PCS bands some modification of the adaptive schemes are required and will be presented in a future report.

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